

Gamut Mapping using Color-Categorical Weighting Method

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Abstract

Categorical color mapping using color-categorical weighting method is proposed for gamut mapping. This method determines the most optimal mapping point in a destination space by shifting LCH (Luminance, Chroma, Hue) of a source color with color-categorical weighting. The proposed method is compared with GCUSP and clipping method on gamut mapping from a CRT monitor to an inkjet printer for evaluating its performance. Categorical color mapping is ranked first with statistically significant difference on Z-score derived from comparison experiments.

Categorical Color Matching

A matching technique has been utilized to develop various knowledge and techniques in the field of color science and engineering. A color reproduction system has been also designed for colorimetric or appearance matching between an original and a reproduction. On the other hand, gamut mapping does not have a certain matching criterion due to difference in device gamut between a source device and a destination device. From the viewpoint of matching-basis, gamut mapping is a critical issue in comparison with other techniques.

Categorical color mapping has been proposed to design a universal gamut mapping.[1] It keeps relative color-categorical relationship between an original and a reproduction so that the following objectives are realized: 1) color name matching as keeping color-categorical property, 2) preserving relative relationship between the points being inside a given color-categorical cluster. This new matching criterion is named 'categorical color matching'. Middle point mapping with categorical normalized distance realizes categorical color matching as follows:[2]

$$\mathbf{D_t} = \mathbf{V}\mathbf{D_s} \qquad (1)$$

where $\mathbf{D}_s$ is Mahalanobis' distance vector of a source color, $\mathbf{V}$ is Categorical color matching operator consists of scaling matrix and weighting vector, $\mathbf{D}_t$ is Mahalanobis' distance vector of the most optimal mapping point in a destination space based on categorical color matching. A average vector and a covariance matrix for calculating Mahalanobis' distance are derived from categorical color naming experiment.[1,2]

Categorical Color Mapping using Color-Categorical Weighting Method

This paper proposes a new type of categorical color mapping, which is named 'color-categorical weighting method'. It consists of two mapping operations, which are pre-mapping and main mapping as illustrated in Fig.1. The pre-mapping works so that a source device gamut is pushed into a destination device gamut. The main mapping relocates pre-mapped point to the most optimal mapping point by color-categorical control.

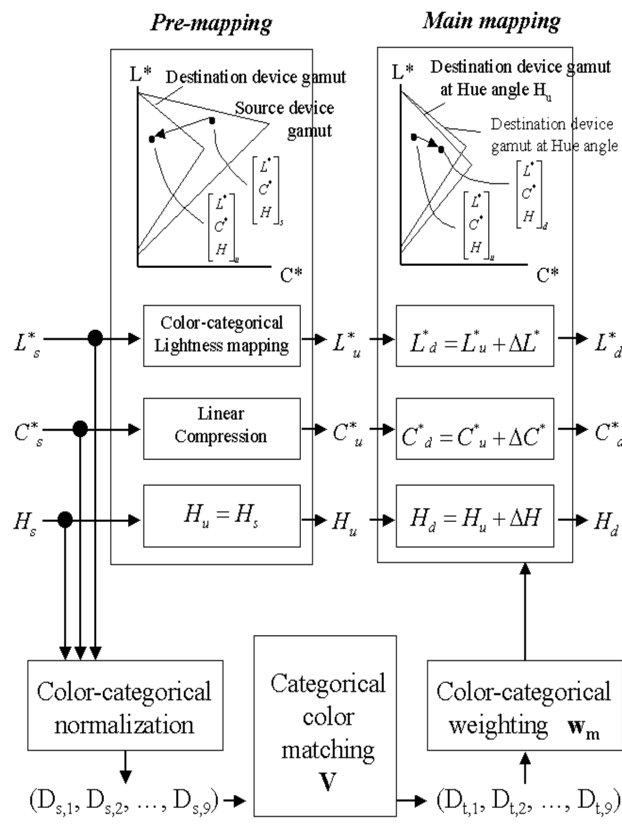

Figure 1. Structure of color-categorical weighting method.

Pre-mapping

The pre-mapping compresses chroma of a source color $[L^*, C^*, H]_s$ linearly and lightness of the source color by categorical lightness mapping as shown in Equation 2

$$\begin{bmatrix} L^* \\ C^* \\ H \end{bmatrix}_u = \begin{bmatrix} (1-C_r)L^*_{u,center} + C_r L^*_{u,surface} \\ C_r C^*_{d,\max} \\ H_s \end{bmatrix}, \quad C_r = \frac{C^*_s}{C^*_{s,\max}} \qquad (2)$$

where $L^*_{u,center}$ is compressed lightness on a lightness axis, $L^*_{u,surface}$ is compressed lightness on a gamut surface. C_r works for a linear interpolation between the lightness axis and the gamut surface along a chroma direction.

L*u,center is defined according to color-categorical distributions of achromatic components. Fig.2 shows boundaries between a CRT monitor (as a source device) and an inkjet print (as a destination device). The point A is the boundary between Black and Gray. The point B is the boundary between Gray and White. Additionally, the point C is a device black point and the point D is a device white point. A nonlinear regression technique derived the following equation to optimize these boundary relationships including the point C and D with correlation coefficient 1.0:

$$L^*_{u,center} = -2.259 L^{*\,3}_s + 3.643 L^{*\,2}_s - 0.577 L^*_s + 0.193 \qquad (3)$$

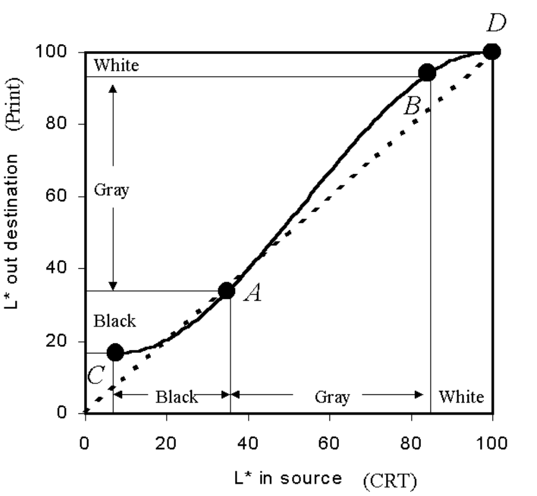

Figure 2. Cubic equation for categorical lightness mapping on a lightness axis given as Eq.3.

L*surface is defined according to color-categorical distributions on a gamut surface. Cusps of a source device are mapped onto a gamut surface of a destination device in relation to the color-categorical distribution. Figure 3 shows color-categorical distribution on a gamut surface between H = 80 and H = 160: (a) CRT monitor, (b) print. Along a lightness direction, Yellow (open diamond) of the CRT monitor is distributed between 65 and 95. Yellow of the print is distributed between 80 and 95. Therefore, if lightness of the CRT monitor is mapped to the print with the same value, Yellow below L* = 80 is changed to Brown (cross) or Orange (close circle) on the print. Based on Fig.3, necessity for lightness control associated with color-categorical distribution can be understood. Figure 4 indicates one more reason for the necessity. One observer picked up the closest color patch to primary or secondary colors of a CRT monitor from a lot of samples. The samples located on a printer gamut surface. In Fig.4, the point G indicates the closest color to the green primary of a CRT monitor for the observer. The green secondary of a printer is closer to the green primary of CRT monitor than the point G in terms of hue angle. On the other hand, the green secondary of a printer is further to the Green primary of CRT monitor than the point G in terms of lightness. It is reasonable for understanding this results that the observer prioritized to minimize difference in lightness. This understanding coincides with Katoh's experiment.[3] On the other hand, the point Y denotes the closest color to the yellow secondary of a CRT monitor for the observer. The point Y is almost same as the yellow primary of print. The observer determined the closest color patches for all the primary and secondary colors (i.e. R, G, B, C, M, Y). Based on these results, the following trend was recognized: the wider color category is distributed in lightness direction, the further the closest color patch to the primary / secondary colors of CRT monitor is from a cusp line of a printer.

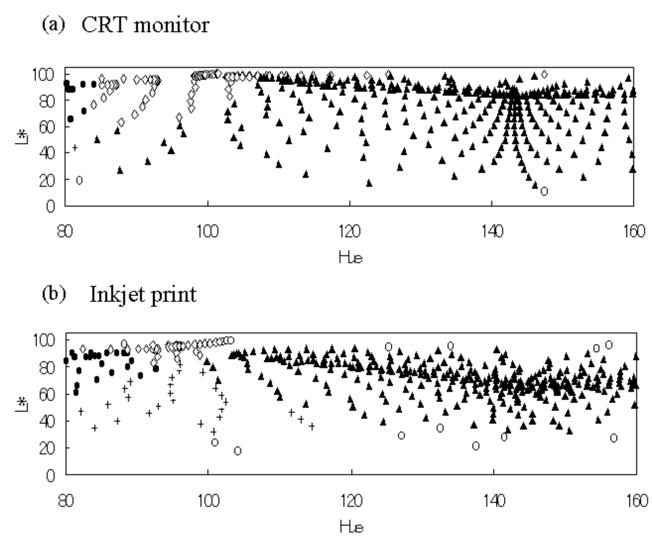

Figure 3. Color-categorical distribution on gamut surface: Green (solid triangle), Yellow (open diamond), Orange (closed circle), Brown (cross), Achromatic (open circle).

Based on the above ideas in relation to Fig.3 and Fig.4, cusp lightness of a source device $L^*_{s,cusp}$ is mapped onto a gamut surface of a destination device by the following equation:

$$L^*_{s,cusp,mapped} = L^*_{d,cusp} + te \qquad (4)$$
$$e = L^*_{s,cusp} - L^*_{d,cusp} \geq 0$$

where $L^*_{d,cusp}$ is cusp lightness at hue angle H_s in a destination space. t is controlled color-categorically by the following equation:

$$t = p_{s,a}\frac{H_{a+1} - H_s}{H_{a+1} - H_a} + p_{s,a+1}\frac{H_s - H_a}{H_{a+1} - H_a}, \quad (a = 1,2,...,6) \qquad (5)$$
$$p_{s,b} = \frac{s'_{LL,b}}{s'_{LL,\max}}, \quad (b = 1,2,...,7, \quad p_{s,7} = p_{s,1})$$
$$s'_{LL,b} = \frac{s'_{LL,b}}{\min(s'_{LL,c})} - 1$$
$$s'_{LL,\max} = \max\left[\frac{s_{LL,b}}{\min(s_{LL,c})} - 1\right], \quad (c = 1,2,...,6)$$

where a, b, c is the number to denote a primary / secondary color, $s_{LL,b}$ is variance of lightness of primary or secondary b in a destination space, H_a is the nearest hue angle that is lower than H_s, H_{a+1} is the nearest hue angle that is higher than H_s.

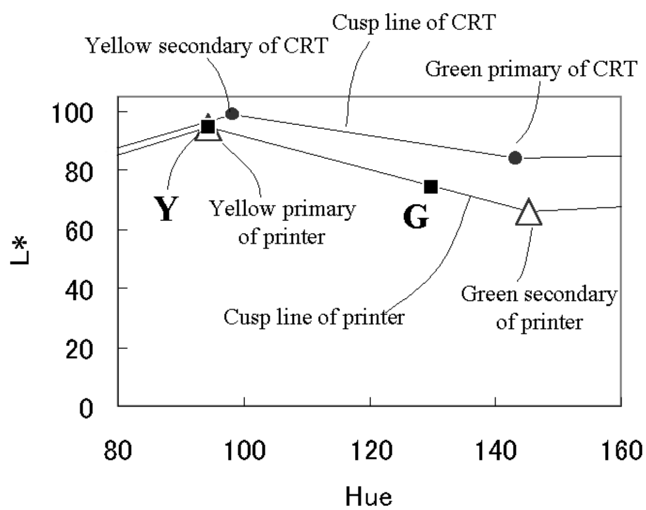

Figure 4. The closest color patch to the Green primary of CRT monitor and the Yellow secondary of CRT monitor for one observer.

*L*u*,surface is given by categorical lightness mapping on gamut surface defined as follows:

$$L^*_{u,surface} = \begin{cases} L^*_{d,\min} + (L^*_{s,cusp,mapped} - L^*_{d,\min})\dfrac{L^*_s - L^*_{s,\min}}{L^*_{s,cusp} - L^*_{s,\min}} \\ \qquad when \quad L^*_s \leq L^*_{s,cusp,mapped} \\ L^*_{s,\max} - (L^*_{d,\max} - L^*_{s,cusp,mapped})\dfrac{L^*_{s,\max} - L^*_s}{L^*_{s,\max} - L^*_{s,cusp}} \\ \qquad when \quad L^*_s > L^*_{s,cusp,mapped} \end{cases} \qquad (6)$$

where $L^*_{s,min}$ is the minimum lightness of a source gamut, $L^*_{s,max}$ is the maximum lightness of the source gamut, $L^*_{d,min}$ is the minimum lightness of a destination gamut, $L^*_{d,max}$ is the maximum lightness of the destination gamut. Figure 5 shows cusp line mapped by Eq.4 from a source device (i.e. CRT monitor) to a destination device (i.e. print).

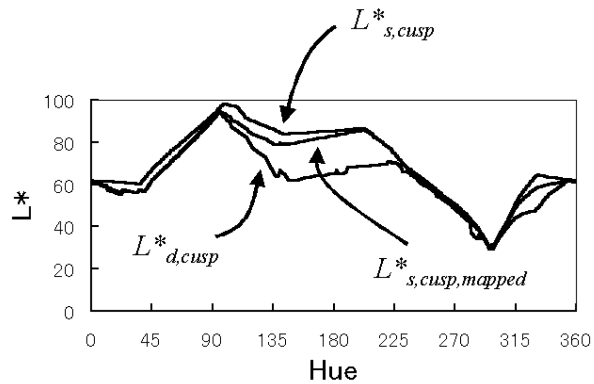

Figure 5. Categorical lightness mapping on a gamut surface.

Main Mapping

The main mapping relocates the pre-mapped vector [*L*, C*, H*]*u* into the most optimal mapping vector [L*, C*, H]d by shifting L, C, and H as follows:

$$\begin{bmatrix} L^* \\ C^* \\ H \end{bmatrix}_d = \begin{bmatrix} L^* \\ C^* \\ H \end{bmatrix}_u + \begin{bmatrix} \Delta L^* \\ \Delta C^* \\ \Delta H \end{bmatrix}$$
$$= \begin{bmatrix} L^* \\ C^* \\ H \end{bmatrix}_u + F\mathbf{M}\mathbf{W_m} + \mathbf{J}C_r(1-F)\mathbf{P}\mathbf{W_p} \qquad (7)$$

where **M** is Average difference matrix given in Appendix A, **P** is Gamut-surface constraint difference matrix given in Appendix B, $\mathbf{W_m}$ is weighting vector for **M**, $\mathbf{W_p}$ is weighting vector for **P**, F is MPM(Middle point mapping) operation factor given in Appendix C, and **J** is Chroma-fitting matrix given in Appendix D. **M** gives difference in *LCH* on average vectors and **P** gives difference in *LCH* on gamut-surface constraints. $\mathbf{W_m}$ and $\mathbf{W_p}$ balance powers of all the color categories based on a position of a source color. $\mathbf{W_m}$ is defined with color-categorical normalized distance of the most optimal mapping point from the viewpoint of categorical color matching as follows:

$$\mathbf{W_m} = \begin{bmatrix} w_{m,1} \\ w_{m,2} \\ \vdots \\ w_{m,n} \end{bmatrix}, \qquad w_{m,i} = \frac{\dfrac{1}{D_{t,i}}}{\sum_{j=1}^{n}\dfrac{1}{D_{t,j}}} \qquad (8)$$
$$if \quad D_{t,i} = 0 \quad then \quad w_{m,i} = 1, w_{m,k\neq i} = 0$$
$$if \quad D_{t,k\neq i} = 0 \quad then \quad w_{m,i} = 0, w_{m,k\neq i} = 1, w_{m,l\neq i\neq k} = 0$$

where $\mathbf{D}_{t,i}$ is derived from $\mathbf{D}_{s,i}$ by Categorical color matching operator $\mathbf{V}$ given by Eq.1. As well as $\mathbf{M}$, $\mathbf{P}$ has difference in gamut-surface constraints, which consists of device primary colors, device secondary colors, black, and white, between a source space and a destination space as shown in Appendix B. Then, as well as $\mathbf{W}_m$, $\mathbf{W}_p$ is defined to weight $\mathbf{P}$ according to a position of a source color as follows:

$$\mathbf{W}_p = \begin{bmatrix} w_{p,1} \\ w_{p,2} \\ \vdots \\ w_{p,q} \end{bmatrix}, \qquad w_{p,g} = \frac{\dfrac{1}{E_{s,p,g}}}{\displaystyle\sum_{j=1}^{q} \frac{1}{E_{s,p,j}}} \tag{9}$$

$$\text{if } E_{s,p,g} = 0 \text{ then } w_{p,g} = 1, w_{p,k \neq g} = 0$$

$$\text{if } E_{s,p,k \neq g} = 0 \text{ then } w_{p,g} = 0, w_{p,k \neq g} = 1, w_{m,l \neq g \neq k} = 0$$

where $E_{s,p,g}$ is Euclidian distance between a source color and gamut-surface constraint g. Gamut-surface constraint difference matrix $\mathbf{P}$ is in charge of description of gamut shape. As illustrated in Fig.6, average constraint mapping $\mathbf{MW}_m$ maps a source average to a destination average, if an original color coincides with the source average. Gamut-surface constraint mapping $\mathbf{PW}_p$ maps a source gamut-surface constraint to a destination gamut-surface constraint, if an original coincides with the source gamut-surface constraint. MPM operation factor F balances power of average constraint mapping $\mathbf{MW}_m$ and gamut-surface constraint mapping $\mathbf{PW}_p$ based on a position of a source color. C_r eliminates an effect of $\mathbf{PW}_p$ for keeping gray balance. Chroma-fitting matrix $\mathbf{J}$ controls chroma of a main mapped point. Since $\mathbf{PW}_p$ works globally in a color space with eight constraints, a local control on chroma is essential to follow $\mathbf{PW}_p$. $\mathbf{J}$ compresses chroma linearly in a destination space so that a main mapped point is replaced an inside of a destination gamut or just on its surface.

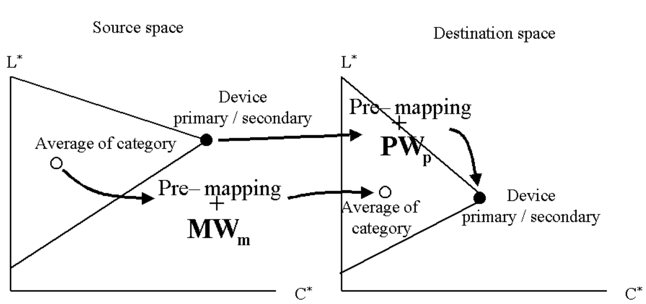

Figure 6. Average constraint mapping $\mathbf{MW}_m$ *and gamut-surface constraint mapping* $\mathbf{PW}_p$.

Experiment

Performance of color-categorical weighting method was evaluated by comparing with conventional gamut mapping techniques, which were GCUSP[4] and clipping method minimizing colorimetric difference with keeping hue angle, on gamut mapping in CIELAB space from a CRT monitor (5000K-white) to an inkjet print illuminated with 5000K fluorescent light.

The four kinds of color-categorical weighting method as listed in Table 1 were established to evaluate the following three viewpoints:

1) Categorical lightness mapping on lightness axis (Eq.2).
2) Average difference matrix M (Appendix A).
3) Gamut-surface constraint difference matrix P (Appendix B).

CCM#1 is the closest method to the basic algorithm of color-categorical weighting method among the four methods. The following linear scaling was applied to lightness mapping on a lightness axis for calculating $L^*_{u,center}$.

$$L^*_{u,center} = L^*_{d,\min} + (L^*_{d,\max} - L^*_{d,\min}) \frac{L^*_s - L^*_{s,\min}}{L^*_{s,\max} - L^*_{s,\min}} \tag{10}$$

Color matching experiment, which is applied in Fig.4, supplied the component $[L^*, C^*, H]_{p,d,i}^t$ of matrix $\mathbf{P}$. CCM#2 was designed by modifying CCM#1 based on two ideas. One of them was that colorimetric matching pairs were introduced to the component $[L^*, C^*, H]_{\cdot,d,i}^t$ of matrix M. The other was the component $[L^*, C^*, H]_{p,d,i}^t$ in matrix $\mathbf{P}$. To eliminate the color matching experiment applied in CCM#1, $H_{p,d,i}$ was supplied from $H_{s,i}$. $L^*_{p,d,i}$ was supplied from categorical lightness mapping on a gamut surface. Hue angle was constant in CCM#2. Among the four methods, this method is closest to the conventional gamut mapping methods from the viewpoint of 1) linear lightness mapping, 2) colorimetric matching on average constraints, and 3) hue constant. CCM#3 was established by modifying CCM#1 in matrix $\mathbf{M}$. Perceptual lightness matching pairs were applied to lightness component $L^*_{\cdot,d,i}$ in matrix $\mathbf{M}$. Chroma and hue were given as a colorimetric matching pair. CCM#4 applied categorical lightness mapping on a lightness axis in order to calculate $L^*_{u,center}$ in the pre-mapping.

Paired-comparison experiments were executed to derive a statistical measure, which indicates closeness to an original displayed on a CRT monitor, among the four kinds of categorical color mapping method and the conventional methods (GCUSP and the clipping method). Four kinds of image (i.e. 1. SCID Fruit basket, 2. SCID Musician, 3. Kodak Photo-CD Macaws, and 4. Mountain view) were applied for this experiment. The number of subjects was fifteen.

Result and Considerations

Z-score or V-score retrieved from paired-comparison experiments were shown in Fig.7. CCM#2 and CCM#3 were ranked first on average among the six kinds of gamut mapping method with a statistically significant difference as shown in Fig.7 (a). On the other hand, CCM#1 and

CCM#4 have no significant difference from the conventional techniques. Based on the above results, the average constraint mapping $\mathbf{MW}_m$ constructed by colorimetric matching pairs were important to gain color reproducibility. Furthermore, categorical lightness mapping on a lightness axis given by Eq.3 is not necessary for designing an optimal gamut mapping method. Since there is no significant difference between CCM#2 and CCM#3, brightness matching on lightness in matrix **M** is not mandatory. As mentioned before, color matching experiment is necessary for CCM#1, CCM#3, and CCM#4 to define the matrix **P**. Since CCM#2 realized the same performance to CCM#3, the color matching experiment can be omitted by applying categorical lightness mapping on a gamut surface to the matrix **P**.

Table 1. Specifications of the four kinds of categorical color mapping.

Method	*Lightness mapping for $L^*_{u,center}$*	*$[L^*, C^*, H]_{d,i}^t$ in matrix M*	*$[L^*, C^*, H]_{p,d,i}^t$ in matrix P*
CCM#1	Linear scaling	Color naming	Minimization of perceptual difference
CCM#2	Linear scaling	Colorimetric matching	L^*: categorical lightness mapping on gamut surface C^*: Maximum H: CRT primary / secondary
CCM#3	Linear scaling	L^*: Perceptual brightness matching C^*,H: colorimetric matching	Minimization of perceptual difference
CCM#4	Categorical lightness mapping on a lightness axis	Colorimetric matching	Minimization of perceptual difference

GCUSP and clipping method were inferior to CCM#2 and CCM#3 because of perceptual hue shift, which was recognized on blue sky in the image 'Mountain view' and dark bluish-gray in the image 'Musician' as hue change toward purple. CCM#2 is the closest method to the conventional gamut mapping techniques as mentioned before. The significant difference between CCM#2 and the conventional techniques means an ability of color-categorical weighting method for compensating the hue curvature in CIELAB space. It was ensured that color-categorical weighting method could linearize the hue curvature in CIELAB space.

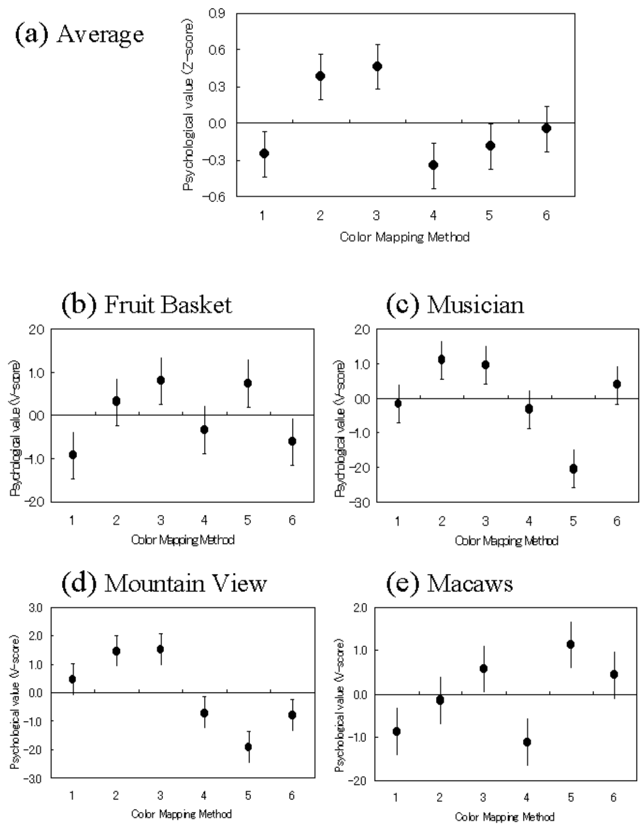

Figure 7. Results of paired-comparison experiment: Color mapping method 1 CCM#1, 2:CCM#2, 3:CCM#3, 4:CCM #4, 5:GCUSP, 6:Clipping.

Conclusions

Color-categorical weighting method was developed as a new type of categorical color mapping. Color reproducibility of the new categorical color mapping was evaluated on gamut mapping from a CRT monitor to a inkjet print by paired-comparison experiments. GCUSP and clipping method minimizing colorimetric difference with keeping hue angle were applied as conventional techniques. On averages among four kinds of test images, the categorical color mapping was ranked first with a significant difference from the conventional techniques. Colorimetric matching pairs were necessary for the matrix **M**. Categorical lightness mapping on a gamut surface is useful to reduce procedures for defining the matrix **P**.

Appendix

Appendix A: Definition of **M**

$$\mathbf{M}=\begin{bmatrix} L^*_{\mu,u,1}-L^*_{\mu,d,1} & L^*_{\mu,u,2}-L^*_{\mu,d,2} & \Lambda & L^*_{\mu,u,n}-L^*_{\mu,d,n} \\ C^*_{\mu,u,1}-C^*_{\mu,d,1} & C^*_{\mu,u,2}-C^*_{\mu,d,2} & \Lambda & C^*_{\mu,u,n}-C^*_{\mu,d,n} \\ H_{\mu,u,1}-H_{\mu,d,1} & H_{\mu,u,2}-H_{\mu,d,2} & \Lambda & H_{\mu,u,n}-H_{\mu,d,n} \end{bmatrix}$$

where $[L^*\ C^*\ H]_{\bullet,u,i}{}^{t}$ is a pre-mapped average vector of color name i, $[L^*\ C^*\ H]_{\bullet,d,i}{}^{t}$ is an average vector of color name i in a destination space.

Appendix B: Definition of **P**

$$\mathbf{P}=\begin{bmatrix} L^*_{p,u,1}-L^*_{p,d,1} & L^*_{p,u,2}-L^*_{p,d,2} & \Lambda & L^*_{p,u,q}-L^*_{p,d,q} \\ C^*_{p,u,1}-C^*_{p,d,1} & C^*_{p,u,2}-C^*_{p,d,2} & \Lambda & C^*_{p,u,q}-C^*_{p,d,q} \\ H_{p,u,1}-H_{p,d,1} & H_{p,u,2}-H_{p,d,2} & \Lambda & H_{p,u,q}-H_{p,d,q} \end{bmatrix}$$

where $[L^*\ C^*\ H]_{p,u,g}{}^{t}$ (g = 1,2,…,q) is a pre-mapped point g locating on a gamut surface, $[L^*\ C^*\ H]_{p,d,g}{}^{t}$ is a target point in a destination space.

Appendix C: Definition of F [2]

$$F=w_{f,1}f_1+w_{f,2}f_2+\ \Lambda\ +w_{f,n}f_n$$

$$f_i=\frac{\dfrac{1}{E_{s,f,i}}}{\displaystyle\sum_{j=1}^{q}\frac{1}{E_{s,p,j}}+\frac{1}{E_{s,f,i}}}$$

$$\text{if}\ \ E_{s,f,i}=0\ \ \text{then}\ \ f_i=1$$

$$\text{if}\ \ E_{s,p,t}=0\ \ \text{then}\ \ f_i=0$$

$$w_{f,i}=\frac{\dfrac{1}{D_{s,f,i}}}{\displaystyle\sum_{j=1}^{q}\frac{1}{D_{s,f,j}}}$$

$$\text{if}\ \ D_{s,f,i}=0$$

$$\text{then}\ \ w_{f,i}=1, w_{f,k\neq i}=0$$

$$\text{if}\ \ D_{s,f,k\neq i}=0$$

$$\text{then}\ \ w_{f,i}=0, w_{m,k\neq i}=1, w_{f,l\neq i\neq k}=0$$

where $E_{s,f,i}$ is Euclidian distance between an original and an average of category i, $E_{s,p,g}$ is Euclidian distance between an original and gamut-surface constraint g, $D_{s,f,i}$ is Mahalanobis' Mahalanobis' distance from an average of category i to an original color.

Appendix D: Definition of **J**

$$\mathbf{J}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & w_c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$w_c=\frac{C^*_{d,\max,target}-C^*_{c,s,\max,u}-F\mathbf{M_c}\mathbf{W_m}}{C_r(1-F)\mathbf{P_c}\mathbf{W_p}}$$

where $C^*_{d,max,target}$ is target chroma of a point on a destination gamut surface, $C^*_{c,s,max,u}$ is chroma of a point to which a given point on a source gamut surface is pre-mapped by Eq.2, $\mathbf{M_c}$ is row vector of matrix **M** corresponding to chroma component, $\mathbf{P_c}$ is row vector of matrix **P** corresponding to chroma component. Gamut-surface constraint mapping $\mathbf{PW_p}$ takes part in mapping a source gamut boundary to a destination gamut boundary globally by mapping gamut-surface constraints (i.e. device primary/secondary colors, white, and black). Chroma-fitting matrix **J** controls chroma so that a main mapped point locates inside of destination gamut or just its surface.

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