Optimization of Camera Spectral Sensitivities

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Abstract

In the evaluation and optimal design of digital camera spectral sensitivity (*SS*), it is useful to have a metric of goodness that agrees closely with the perceived color accuracy of human visual system. In this paper, Vora et al's μ-factor based on the orthonormal color matching function space and a universal measure of goodness (*UMG*) based on CIE $L^*a^*b^*$ space with noise consideration were introduced to evaluate and optimize the camera spectral sensitivities. General physical constraints on spectral sensitivities were discussed in the paper and three feasible approaches to the optimal design of camera spectral sensitivities were proposed, compared and verified with simulated experiments.

Introduction

Recent years applications of digital cameras and high-resolution color scanners are widely spreading in the home and office environments. The principle of digital camera is usually a CCD or CMOS sensor array with a set of filters before it to mimic human visual system. Human visual responses to color stimuli have been determined by psychophysical experiments and are officially recommended as color matching functions by CIE, which characterizes spectral distributions of object colors by tristimulus values since the human eye has three types of cones with different spectral sensitivities. Most imaging systems are therefore set up with three channels and the device sensitivities are initially designed to approximate human visual system.

There are two folders in optimizing spectral sensitivities. One is the measure; the other is how to implement the optimization in real world. Theoretically, the spectral sensitivities for color imaging devices should satisfy the Luther condition, that is, the spectral sensitivities need be a nonsingular transformation of color matching functions. In practice, it is not always possible to make filters that satisfy the Luther condition due to the physical limitations of fabricating process. Furthermore, recording noise will degrade the color accuracy even when spectral sensitivities fulfill the Luther condition. A measure of goodness or quality factor for evaluating and designing spectral sensitivities of color imaging devices under real condition is therefore desirable.

Vora and Trussell introduced μ-factor, which describe the difference between the orthonormal subspaces of color matching functions and the spectral sensitivity space. This measure can be related to a mean-squared error metric in CIEXYZ space. Recently, Wolski et al[7] proposed the use of local linearization of CIELAB space to reduce the computational complexity with preserving the desirable property of perceptual uniformity. Sharma and Trussell[6] presented a new figure of merit for color scanners, which is also based on an error metric in linearized CIELAB space but incorporates a model for measurement noise. This measure has high degree of perceptual relevance and also accounts for noise performance of different filters.

The higher the quality factor for the imaging device, the more accurate color reproduction is expected. One approach to improve the color is to use an increased number of color channels. As the number of color channels is increased, additional information about the object color is obtained, but cost and fabrication difficulty is likely to increase. Four-channel system is suggested to be a good tradeoff.[10] This paper demonstrates a method to compute the optimal transmittance of a fourth filter by maximizing the total quality factor of the system.

In this paper, the introduction of quality factors including μ-factor and UMG is followed by discussion of the physical constraints for practical spectral sensitivities. Finally, we present several feasible approaches to the optimal design of camera spectral sensitivities.

In our paper, all spectral distributions are sampled at 10nm intervals from 400nm to 700nm and represented as N-element column vectors (N=31).

μ-Factor of A Set of Spectral Sensitivities

Let *M* denote the matrix of *K* scanning spectral sensitivities (including detector sensitivity and the transmittance of filter and the camera optical path), $M = [m_1, m_2, \cdots, m_K]$. Let $A = [x, y, z]$ denote the human visual space (color matching functions) to be approximated. When multiple illuminants are involved, we may define *M* and *A* as:

$$\begin{aligned} M &\Rightarrow \begin{bmatrix} L_1 M & L_2 M & \dots & L_n M \end{bmatrix} \\ A &\Rightarrow \begin{bmatrix} L_1 A & L_2 A & \dots & L_n A \end{bmatrix} \end{aligned} \quad (1)$$

where $L_1, L_2, ..., L_n$ are the diagonal matrices of the power spectrum of the illuminants involved.

An orthonormal basis for *A* is defined by $U = [u_1, u_2, \ldots, u_\alpha]$. The number of orthonormal vectors, α, is the rank of A. Similarly, an orthonormal basis for M is defined by $O = [o_1, o_2, \cdots, o_\beta]$. Also notice that β is the rank of M. The orthonormal basis U and O need not represent realizable spectral sensitivities. One can prove that $M(M^T M)^{-1} M^T = OO^T$ and $A(A^T A)^{-1} A^T = UU^T$.[8]

The purpose is to approximate A by a linear combination of M, that is, to minimize the merit function

$$\Delta = \|A - MQ\|_F^2,$$

where Q is the coefficient matrix to be optimized. This is a least-squares problem.

$$Q = (M^T M)^{-1} M^T A$$

is obtained with pseudo-inverse operation. And the residue $\Delta_{\min}$ is given by

$$\begin{aligned}\Delta_{\min} &= \|A - MQ\|_F^2 = \cdots \\ &= Trace\{A^T A\} - Trace\{A^T M (M^T M)^{-1} M^T A\}\end{aligned} \quad (2)$$

therefore,

$$\begin{aligned}\frac{\Delta}{\|A\|_F^2} &= 1 - \frac{Trace\{A^T M(M^T M)^{-1} M^T A\}}{Trace\{A^T A\}} \\ &= 1 - \mu_A(M)\end{aligned} \quad (3)$$

where

$$\mu_A(M) = \frac{Trace\{A^T M(M^T M)^{-1} M^T A\}}{Trace\{A^T A\}} \quad (4)$$

is the normalized measure of goodness for a set of spectral sensitivities M against color matching functions A. The higher $\mu_A(M)$, the closer M and A in some sense. Since quality factor $\mu_A(M)$ can be applied to any number of taking illuminants and viewing illuminants among $L_1, L_2, ..., L_n$, for convenience, we may name it as *M*-factor. When only single illuminant is present and the orthonormal subspace U of A is used, we have

$$\mu_U(M) = \frac{Trace\{O^T U U^T O\}}{\alpha} = \frac{\sum_{i=1}^{\beta} q(o_i)}{\alpha} \rightarrow \mu_U(O) \quad (5)$$

which is the original definition of μ-factor for a set of *SS* by Vora et al.[4]

Universal Measure of Goodness (*UMG*)

It is known that the color of an object is specified by its CIE *XYZ* tristimulus values:

$$t(r) = A^T L_v r = A_v^T r \quad (6)$$

where $t(r)$ is the 3×1 vector of CIE *XYZ* tristimulus values, A is the $N\times3$ matrix of CIE *XYZ* color matching functions, L_v is the $N\times N$ diagonal matrix with samples of the illuminant spectrum along the diagonal and $A_v = L_v A$.

The process of capturing object color with a *K*-channel camera can be expressed as:

$$t_c(r) = M^T L_c r + \varepsilon = G^T r + \varepsilon \quad (7)$$

where $t_c(r)$ is a $K\times1$ vector of camera measurements, M is the $N\times K$ matrix of camera total spectral sensitivity, L_c is the $N\times N$ diagonal matrix with samples of the taking illuminant spectrum along the diagonal, $G = L_c M$, and ε is the $K\times1$ measurement noise vector for the K channels.

The CIE XYZ tristimulus values may be estimated as linear transformations of the scanner measurements:

$$\hat{t}(r) = B t_c(r) \quad (8)$$

where the transformation B is determined by minimizing some type of color error, such as mean-squared color difference, maximal color difference etc.

For the case of single illuminant involved, the mean-squared color error may be defined as the merit function by:

$$\begin{aligned}\Delta(A_v, G, B) &= E\{\|F(t(r)) - F(\hat{t}(r))\|^2\} \\ &\approx E\{\|J_F(t)[t(r) - \hat{t}(r)]\|^2\}\end{aligned} \quad (9)$$

where F is the transformation from CIE *XYZ* to $L^*a^*b^*$, and $J_F(t)$ is the Jacobian matrix of the transformation F. By using this locally linear approximation and assuming the recording noise is zero mean and independent of the object spectra, one can derive

$$\Delta_{\min}(A_v, G, B) = \alpha(A_v) - \tau(A_v, G) \quad (10)$$

where $0 \le \tau(A_v, G) \le \alpha(A_v)$ and define the figure of merit for color imaging devices[6]

$$q_{FOM}(A_v, G) = \frac{\tau(A_v, G)}{\alpha(A_v)} \quad (11)$$

A simple correction can be made to the equation:

$$q = 1 - \sqrt{1 - \frac{\tau(A_v, G)}{\alpha(A_v)}} = 1 - \sqrt{1 - q_{FOM}} \quad (12)$$

so that the average color difference of an ensemble of spectra varies linearly versus quality factor as shown in Figure 1. A very linear relationship is expected when the camera *UMG* is rather high (greater than 0.8).

Since the taking (recording) and viewing illuminant may be different, we may define a quality factor for any taking-viewing illuminant pair. If the illuminant pair can be chosen from a set of illuminants $\{L_1, L_2, \cdots, L_n\}$, a quality factor matrix may be defined as:

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} & \cdots & q_{1n} \\ q_{21} & q_{22} & q_{23} & \cdots & \vdots \\ q_{31} & q_{32} & q_{33} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{n1} & q_{n2} & q_{n3} & \cdots & q_{nn} \end{bmatrix} \quad (13)$$

where q_{ij} is the quality factor with taking illuminant L_i and viewing illuminant L_j.

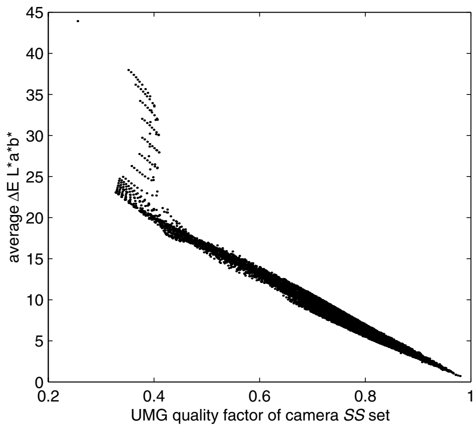


Figure 1. UMG versus CIE $L^*a^*b^*$ color difference

The so-called universal measure of goodness (*UMG*) for the illuminant set may be defined as the average of elements of one column, one row, diagonal, or all elements:

$$UMG = \frac{1}{n}\sum_{i=1}^{n} q_{ij} \tag{14a}$$

$$UMG = \frac{1}{n}\sum_{j=1}^{n} q_{ij} \tag{14b}$$

$$UMG = \frac{1}{n}\sum_{i=1}^{n} q_{ii} \tag{14c}$$

$$UMG = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij} \tag{14d}$$

depending on different types of problems involved. The value indicates the performance of sensitivities under single or multiple viewing and recording illuminants. Particularly, if the viewing illuminant and taking illuminant are the same as the CIE equi-energy illuminant and noise is omitted, the related *UMG* may be regarded as initial merit indicator for a given *SS* set.

Physical Constraints on Spectral Sensitivities

The optimal design of spectral sensitivities is usually to search a set of filters by maximizing some pre-defined measure of goodness while satisfying given physical constraints emerging from practical fabrication process. The measure of goodness such as the μ-factor defined by Vora et al[4] or the figure of merit by Sharma[6] or the aforementioned *UMG* can be used as criteria. The constraints usually imposed on camera spectral sensitivities are:

(1) Non-Negativity and Boundedness:

The transmittance of spectral sensitivity at each wavelength is non-negative; the transmittance at each wavelength cannot exceed one or some other constant. Different boundedness constraint may be exerted according to real world.

$$0 \le m_{(R,G,B)}(\lambda_i) \le 1 \tag{15}$$

(2) Smoothness:

The second derivative of the physical sensitivity can be used as a measure of curvature and therefore as a measure of smoothness of the sensitivity.

$$\left| m(\lambda_{i-1}) - 2m(\lambda_i) + m(\lambda_{i+1}) \right| \le \Delta_{\max} \tag{16}$$

(3) *Single Peak* (Optional, Preferred):

The transmittance of the filter has one global peak, and without multiple local peaks. There is more chance to fabricate a single-peaked filter than a multi-peaked filter.

Other constraints such as the range of the first derivative of the spectral sensitivities, symmetry etc. may also be included depending on specific problems.

Optimization of Spectral Sensitivities

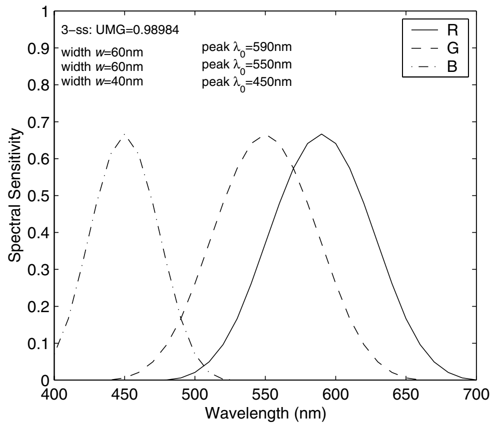
Since each spectral sensitivity has 31 variables (assume visible range defined on 400nm-700nm with an interval of 10nm), optimization problem with near 100 variables for a three-channel camera is very difficult to be implemented in reality. It still can be done theoretically, but it's hard to judge if the obtained optimum is really optimal since the optimization is much likely to be trapped in local valleys. Some simplification is necessary in practice.

1. An Optimal Subset of a Discrete Set of Filters

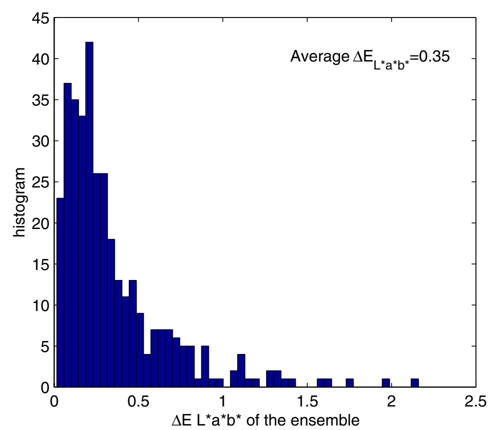
A simple formulation of the optimization problem is to determine the "best" set of K filters from a set of existing filters. Suppose the set S is the set of existing filters from which the best subset M0 of K filters is to be chosen. UMG may be maximized with respect to subsets of S, of size K, by exhaustive searching K filters at a time. If N is the size of set S, such a search will involve $C_N^K = N!/K!(N-K)!$ times of evaluations of the measure of goodness. For instance, Vora[11] selected optimal three-filter subset from the Kodak Wratten Filter Set, in light of μ-factor. The optimal set of the filters (23A, 48A and 52) has μ-factor of 0.912.

Another example is that we choose the best three hypothetical spectral sensitivities[12] from the complete combinations of cubic spline functions with varied peak wavelength and width. With exhaust searching, one can obtain the three hypothetical spectral sensitivities (*R*: peak (590nm), width *w*=60nm; *G*: peak (550nm), width *w*=60nm; and *B*: peak (450nm), width *w*=40nm), which contribute a *UMG* of 0.990. It is the highest among the discrete spectral sensitivity sets with cubic spline shape.

One can calculate the color difference of each sample in the Vrhel object color ensemble[9] between the measured reflectance spectra and the predicted one with this hypothetical optimal spectral sensitivity set. Under daylight D65, the overall average CIE $L^*a^*b^*$ color difference is 0.35. Based on criteria of either quality factor or average color difference, it's an optimal spectral sensitivity set.



(a)



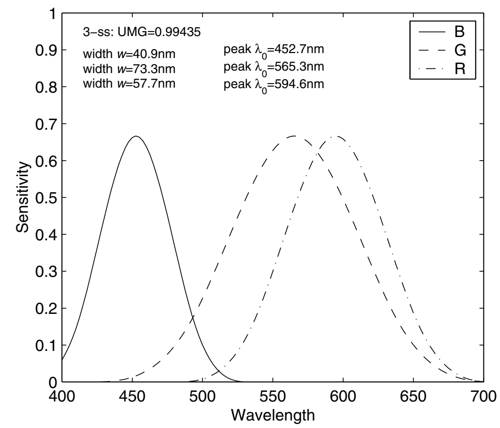
(b)

Figure 2. (a) An optimal set of spectral sensitivities by computing every possible combination; (b) Histogram of color difference of each sample of the ensemble

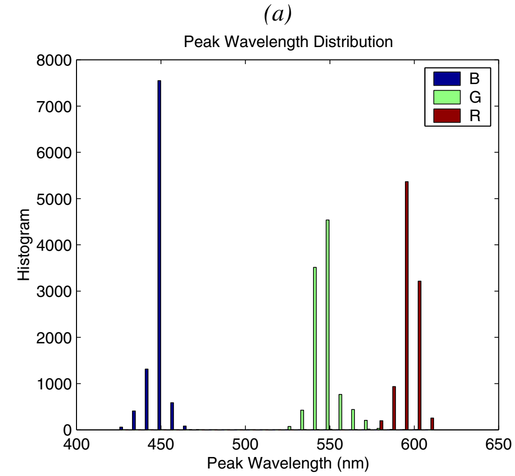
2. Parameterization of Filter Characteristics

One way of incorporating a manageable dynamic range and smoothness for filters is by modeling each filter in terms of smooth, nonnegative functions with a few parameters. As alternative way of modeling SS other than cubic spline functions, we can model the filters as single Gaussian or as the sum of two Gaussians. Other functions, such as sinusoidal functions can also be used. It is feasible that each filter has no more than 5 or 7 parameters (degree of freedom), resulting in tractable formulations of the optimization problem and in physically realizable filters. The functions were chosen for ease of implementation and efficiency of the optimization routine. For instance,

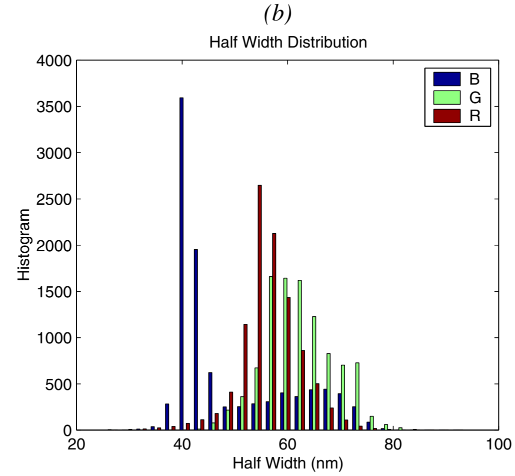
$$S_i(\lambda) = \exp\left(-\frac{(\lambda-\lambda_{i1})^2}{2\sigma_{i1}^2}\right) + \alpha_i \exp\left(-\frac{(\lambda-\lambda_{i2})^2}{2\sigma_{i2}^2}\right), \qquad (17)$$
$$i = R, G, B$$



(a)



(b)



(c)

Figure 3. (a) Optimization result of parametrization; (b) Histogram of optimal peak wavelength; (c) Histogram of optimal half width

The resulting “optimal” filters will be sums of Gaussians and, hence, easy to be fabricated.

Once again it is likely that the merit function has many local maximum, which implies that a particular solution is a function of the starting point and not necessary the really optimal set of filters. To minimize this effect, thousands of trials are attempted with different initial points used. The resulting sensitivity set is the one with maximal quality factor among those trials. Figure 3 is results of optimization of parameterization of filter characteristics. Each sensitivity has two parameters (peak position, half width), totally six for three sensitivities. It can be seen that the range of optimal peak position and width spreads widely but clusters on a few points, as shown in Figure 3(b, c).

3. Only One Channel Needs Optimization

There are cases when two or three spectral sensitivities are given and only the last is free to search for an optimal one. In this case, one has 31 variables totally (still too many), however the problem is much easier. The above two approaches may still be applied, but a direct optimization towards the 31 variables could be an interesting trial in practice, while a local optimal result is still likely to be obtained. In the following example, given three sensitivities with UMG of 0.827, a fourth channel is designed so that the total quality factor is improved. We assume the fourth SS satisfy the general three constraints with a smoothness tolerance of 0.025 (this number can be adjusted to make a tradeoff between smoothness and freedom). The unique peak position of the fourth SS slides from 400nm to 700nm by 10nm, totally 31 possible positions (since we don’t know which wavelength it should locate at). Final UMG of the set consisting of four SS can be as high as 0.959. The optimal peak position of the fourth spectral sensitivity locates at 610nm (Figure 4).

Discussions and Conclusions

It is hard to tell which kind of spectral sensitivity is easy to be fabricated in practice, the one with defined shape or the one with arbitrary shape but satisfaction of the three common constraints. Generally, optimization of the spectral sensitivity with predefined shape yields smooth curve and high quality factor but may omit those beyond this shape, which could be optimal set as well. Global optimization without shape limitation is likely trapped in local minima and produce intractable curve. Optimization though computing combination of discrete set is tedious, but produces feasible solution fastest from the available filters. As a next stage, we will incorporate more constraints from industrial viewpoints and carry out full trials by actually fabricating the optimal sensitivities.

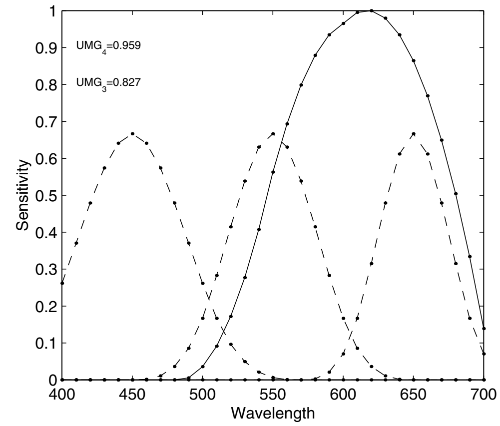


*Figure 4. Assume three sensitivities are given, for instance, cubic spline functions peak at 450*nm*, 550*nm *and 650*nm *with width parameter w of 60*nm*, 50*nm*, 40*nm*. A fourth sensitivity is optimized according to the physical constraints described above*

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Biography

Shuxue Quan received his B.S. and M.S. degree in Optical Engineering from Beijing Institute of Technology in 1994 and 1997. Since 1997 he has been a Ph.D. candidate in Imaging Science at Rochester Institute of Technology. His work primarily focuses on the design and optimization of spectral sensitivities for color imaging systems.

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