A Mathematical Model for Inkjet Printer Characterisation

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Abstract

An accurate, physically based mathematical model for characterisation of inkjet printers would allow the minimum set of observations to be used to derive well-conditioned workable printer characterisations with known error characteristics.

This paper seeks to present such a method of directly computing the characterisation of an inkjet printer from a limited set of colorimeter measurements. The method (called the **Double Neugebauer Model** by the author) employs a variation of the Neugebauer ink-mixing model and uses least squares to fit the model to the measured data. The method has been verified for a limited range of inkjet printers and shown to have an accuracy to the order of RMS $\Delta E = 1$ on those printers. The resulting characterisation can be inverted numerically and used to produce a printer characterisation profile when combined with gamut mapping techniques.

Introduction

Inkjet printer characterisation is presently done using a variety of techniques. Some of these techniques derive from theoretical considerations and others are heuristic.

Single Ink Models

Optical dot gain models employ the Yule-Neilsen equations to model optical dot gain as the dot size increases. This is not directly relevant to inkjet printers because inkjet printer dots are always the same size. Dot gain certainly affects the effective dot size but its effect is not straightforward because the dots overlap. Using the method outlined by Balasubramanian[2] did not produce a reasonable result for the Canon BJC 7000 or HyperPhoto inkjet printers when tested by the author.

I have seen suggestions that the colour of a single ink blend can be modelled with a polynomial, the coefficients of which are determined by least squares. This was tried and yielded inadequate results for various orders of polynomial. In particular, the function could not always be inverted due to a local minimum value occurring for certain inks (particularly black) close to the full ink colour.

Splines have also been used to model ink blends. The author expects that they can yield useful results. However, there is no physical basis for their use in printer characterisation.

Multi-Ink Models

The standard Neugebauer ink-mixing model has been used to model inkjet printers but it yields inadequate accuracy. It must be incorporated with a model for modelling a single ink blend. For inkjet printers, single ink blends are often modelled using direct interpolation from observed values when using this technique.

Variations on multi-variate polynomials[4] have also been employed but the results are not ideal.

Multi-variate B-Splines have also been used for modelling printer characterisations. Adequate results can be achieved this way[3] but again there is no physical basis for the procedure.

Heuristic methods can also be used. For example, starting from observations of ink blend samples and the Neugebauer ink-mixing model, it is possible to heuristically improve the model until it is of useful accuracy.

Equipment

The following equipment was used for the experiments described in this paper.

Minolta CR-321 Chroma Meter

This instrument claims an accuracy of $\Delta E = 1$ in its instruction booklet. This was not particularly verified but the experimental results are consistent. Other work done with this instrument in our office is also consistent with this accuracy claim.

Printers and Papers

The Canon BJC7000 with a BCI-61 ink cartridge was used with a poor quality coated paper. The paper was rather too yellow and suffers from variation of colour over a sheet. The variation has been crudely measured and found to be in the order $\Delta E \approx 2$. (In retrospect, a better quality paper should have been used. However, at the time the experiments were started, the publication of these results was not anticipated.)

The Canon HyperPhoto printer with standard HyperPhoto "paper" was also used for testing. No colour variation within a single sheet could be detected using the Minolta CR-321 Chroma Meter.

When paper colour variation and colorimeter accuracy are considered together, it is clear that it will not be possible to achieve an accuracy better than RMS ΔE of 2 ~ 3 for the Canon BJC7000 using this paper and about ΔE ≈ 1 for the Canon HyperPhoto printer.

Computer Programs

The author wrote all the programs used in these experiments using Microsoft Visual C++ 6.0.

Single Ink Characterisation

An inkjet printer places dots of approximately equal size on the page in a grid pattern. The location of dots is determined by the half-toning technique employed but, for most half-toning techniques, the intention is to place dots with some pseudo-random characteristics such that it does not produce any visual artefacts.

If the colour resolution of a single channel is n bits, the range is 0 to 2^n-1. This paper assumes that, for a value j, the half-toning algorithm will place j out of 2^n-1 dots. Where this is not the case, compensation must be made for the actual number of dots placed.

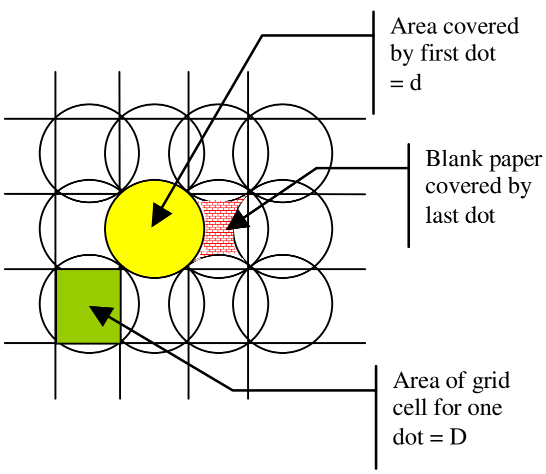

Fig 1 Representation of dots placed by an ideal inkjet printer

Let d be the area of a single dot, n be the number of bits of colour resolution for an ink and D be the grid area of a single dot. D will be $1/(2^n-1)$ of the area covered by 2^n-1 dots. On a reasonable inkjet printer, D must be less than d.

In XYZ colour space, the colour of a patch printed on this printer can be represented as a linear combination of the colour of the paper and the colour of a dot.

$$X = (1 - f(x))\, P_x + f(x)\, I_x \qquad (1)$$

where X is the X component of the XYZ colour space, $f(x)$ is the proportion of paper covered with ink (including any optical dot gain), P_x is the X component of the paper colour and I_x is the X component of the colour of the ink dot on the paper. (Similar formulae apply to the Y and Z components of XYZ.)

Let $f(x)$ have the domain [0-1] so that its definition is independent of the colour resolution of the printer. It must also have the range [0-1] on a reasonable inkjet printer. Based on figure 1, there are some observations we can make about the function $f(x)$.

$$f(0) = 0$$
$$f\left(\frac{1}{2^n - 1}\right) = \frac{d}{D(2^n - 1)}$$

Therefore its derivative at zero must be,

$$\begin{aligned} f'(0) &\approx \frac{d}{D(2^n-1)} \Big/ \frac{1}{2^n-1} \\ &= \frac{d}{D} \\ &= 1 + \frac{d-D}{D} \end{aligned}$$

Also, for a printer with ideal dot size,

$$f(1) = 1$$
$$f\left(\frac{2^n - 2}{2^n - 1}\right) = 1 - \frac{D - (d - D)}{D(2^n - 1)}$$

Similarly, its derivative at 1 must be,

$$\begin{aligned} f'(1) &\approx \frac{D-(d-D)}{D(2^n-1)} \Big/ \frac{1}{2^n-1} \\ &= \frac{D-(d-D)}{D} \\ &= 1 - \frac{d-D}{D} \end{aligned}$$

Provided that the printer has a dot size which is close to the ideal,

$$\frac{d - D}{D}$$

will be small and positive. Also, in general,

$$1 + \delta \approx \frac{1}{1 - \delta}$$

provided δ is small. Therefore, for an inkjet printer, the approximation

$$f'(0) \approx \frac{1}{f'(1)}$$

can be expected to hold.

This observation lead the author to an attempt to model the colour of a single ink blend using a function derived from the simple reciprocal $y = 1/x$ because it is the simplest function which meets this criterion.

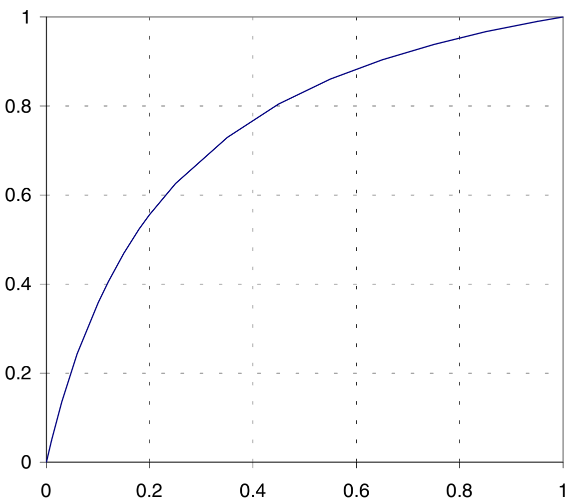

Figure 2 Graph of section of 1/x curve which has been inverted and scaled to the range [0-1]

Figure 2 shows a graph of a section of the 1/x curve. This symmetrical section has been inverted and scaled to have a range and domain of [0-1]. It can be represented by the function

$$f(x) = \frac{x}{x(1-k)+k} \tag{2}$$

which has all of the required properties. Figure 2 was computed for $k = 0.2$.

If the assumption is correct, equations (1) and (2) together provide a complete model of a single ink blend on an inkjet printer.

This model was fitted (by the use of iterative least squares, because the observation equations are not linear in k), to measurements of patches printed on an inkjet printer. The model was found to fit well for light coloured inks but did not have adequate accuracy for dark coloured inks.

In order to extend the model to better fit the observed blends, the simplest extension was attempted first. That is, each dot was considered to consist of two sections: a major section, I, which is close to the average ink colour; and a minor section, H, of different colour.

However, even this extension needed further simplification to make it computable. It was decided to assume that the placement of the major and minor sections was random and not related to one another. While this assumption is false at face value, there is some justification for it as the placement of the minor (or major) section of one dot is uncorrelated with the placement of the minor and major sections of all other dots.

Based on this premise, the new estimator for the colour of a patch becomes

$$\begin{aligned} X = \; & P_X(1-f(x))(1-g(x)) \\ & + I_X f(x)(1-g(x)) \\ & + H_X(1-f(x))g(x) \\ & + HI_X f(x)g(x) \end{aligned}$$

where $f(x)$ represents the proportion of the paper covered with major sections, $g(x)$ represents the proportion of the paper covered with minor sections, P_x represents the X component of the paper colour (as before), I_x represents the X component of the colour of the major section (which will be close to the overall ink colour), H_x represents the X component of the minor colour and HI_x represents the X component of the colour of overprinted major and minor sections.

This is somewhat like mixing two totally independent inks where one is the major section of the ink dot and the other is the minor section. The mixing is modelled using a bilinear interpolation between the “Neugebauer primaries” after the non-linearities of the ink responses have been removed using the $f(x)$ and $g(x)$ functions.

For this composite dot model, the function $f(x)$ is unchanged from the simple model. However, the same function cannot be used for $g(x)$. The form of the function $f(x)$ arises because the dots overlap. However, the minor sections are assumed to be sufficiently small that they do not significantly overlap. Therefore the $g(x)$ function was selected to be a very simple linear function.

$$g(x) = k'x$$

The k' is different from the k of the $f(x)$ function.

It was also discovered that it is not be possible to solve for both of the H_x and HI_x values because they are not independent and nor are they independent from k'. Therefore, it is necessary to add two further constraints to the system. This can be done without loss of generality. The constraints

$$\begin{aligned} I_X &= HI_X \\ I_X &= H_X \end{aligned} \tag{3}$$

were chosen because they make the subsequent formulae simpler. At first sight, this appears to negate the assumptions of the composite dot model entirely but, with careful consideration, it can be seen that it is still a valid result of the composite dot assumption.

As a result of the new constraints, I_x replaces HI_x and H_x in equation (3). If $x = 1.0$ is substituted in equation (4) below, it can be seen that the I_x value still represents the colour of the ink.

The new predictor for the colour becomes

$$\begin{aligned} X = \; & P_X[(1-f(x))(1-g(x))] \\ & + I_X[f(x)(1-g(x))+g(x)] \end{aligned} \tag{4}$$

This is fortuitous because it suggests that we can introduce an assumption of non-uniform dot colour with addition of only a single new parameter, k'. This means that we do not need to significantly add to the number of observations made to characterise a single ink blend.

Table 1 RMS residuals of fit of composite dot model to Canon HyperPhoto single ink blends

Channel number	Colour	RMS ΔE
0	Cyan +	0.85
1	Cyan –	0.10
2	Magenta +	1.12
3	Magenta –	0.21
4	Yellow	0.52
5	Black	1.08

Table 2 RMS residuals of fit of composite dot model to Canon BJC 7000 single ink blends

Channel number	Colour	RMS ΔE
0	Cyan	1.17
1	Magenta	1.93
2	Yellow	1.27
3	Black	1.38

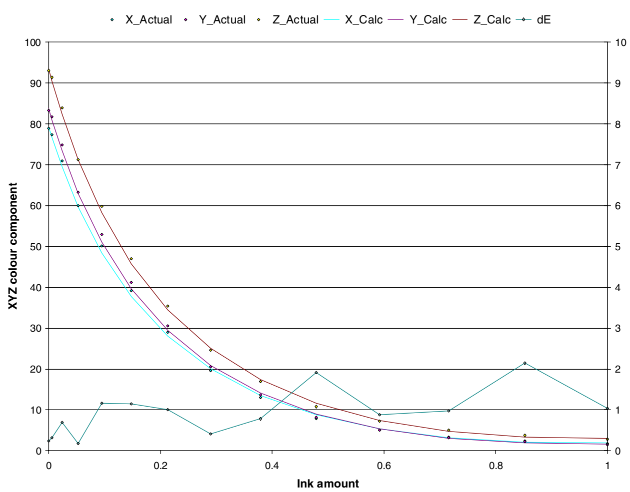

Figure 3 Result of fit of composite dot model to Canon HyperPhoto black ink blend

The same technique of iterative least squares was employed to fit this model to the observed data. The above results were obtained from simultaneously fitting all inks for a printer so that only a single value for paper colour was computed. While the fitting was done in CIE XYZ, the observations were weighted using the CIE L*a*b* colour space so that the use of an RMS ΔE metric was meaningful. The least squares computation was done independently for the X, Y and Z components and the ΔE values computed afterwards from the three sets of results.

The results in tables 1 and 2 were computed from the measurement of 16 patches for each ink. They are as good as can be expected from the equipment used in this experiment, which indicates that the model is as good as can be developed using this equipment. In particular, the graph for the Canon HyperPhoto black ink presented in figure 4 is one of the worst case inks (given the lower error expectation for the HyperPhoto printer). The residuals are measured in ΔE and relate to the right hand axis of the graph.

The author suspects that the Minolta CR-321 colorimeter used for these measurements may have worse accuracy closer to black and that this may partly explain the larger residuals in the near black colours in Figure 3. However, this theory has not been tested.

The residuals are within what is required to make a useful printer characterisation.

Multi Ink Characterisation

Multi-ink characterisation can be modelled by a logical extension of the single-ink composite dot method.

The single-ink method assumes a mix of two “inks”, being the ink's major and minor sections. For more than one ink, a “double Neugebauer set” is used. This is defined similarly to the usual Neugebauer set as shown below.

Let **S** be the set of all inks. Then the usual Neugebauer set is the set of colours of all combinations of the elements of set **S**. The double Neugebauer set is similarly defined but is based on a set **S'** which contains the inks’ minor and major sections. This basis set **S'** contains double the number of elements. Therefore, the number of elements in the double Neugebauer set is the square of the number in the usual Neugebauer set.

The X component of the patch colour is then predicted by the formula

$$X = \sum_{p \in K} \left\{ P_X \left[\prod_{i=0}^{n} Maj_i \in p \,?\, f_{X_i}(a_i) : 1 - f_{X_i}(a_i) \right] \left[\prod_{i=0}^{n} Min_i \in p \,?\, g_{X_i}(a_i) : 1 - g_{X_i}(a_i) \right] \right\}$$

where a_i is the proportion of dots of ink i, K is the set of double Neugebauer primaries, P_X is the X component of the XYZ colour of the primary p, $Maj_i \in p$ is true if the i^{th} ink’s major section is included in the double Neugebauer primary p, $Min_i \in p$ is true if the i^{th} ink’s minor section is included in the double Neugebauer primary p,

$$f_{X_i}(a_i) = \frac{a_i}{a_i(1 - k_{X_i}) + k_{X_i}}$$

$$g_{X_i}(a_i) = k'_{X_i} a_i$$

and the expression $q \,?\, a : b$ takes the value a if the logical predicate q is true, or b if q is false.

The above formula applies independently to the X, Y and Z components of the XYZ colour space.

This model was named the **Double Neugebauer Model** by the author.

For n inks, the set of double Neugebauer primaries contains 2^{2n} members. This can be very large as table 3 indicates. Computation of every member of the double Neugebauer set would result in a prohibitive number of measurements and amount of computation. Fortunately, it is not necessary to compute all of these values.

Just as it is not possible to know the values for H_X and HI_X in the single-ink composite dot model, when computing the parameters for the Double Neugebauer Model it is not possible to determine all the elements in the double Neugebauer set. For example, the X components of primaries $H_1I_1I_2$ and I_1I_2 cannot both be determined. In addition, they are not independent from the k'_{Xi} parameter. As we did in the single ink case, without loss of generality we can assume that these two colours are equal. We must also find some constraint on the k'_{Xi} parameters so that we get a non-singular least squares matrix.

The least squares observation equations for the multi-ink problem are non-linear because the k_{Xi} terms are in the denominator. Attempting to use an iterative procedure for simultaneously computing, say, 128 values would result in inverting a matrix of size 128x128 many times. This would be uncomfortably slow. In addition, it would be reasonable to expect that the process would be unstable.

These three issues can be addressed simultaneously by separating the computation into two phases.

Phase I Using data for single inks only, solve simultaneously for P_X and all the I_{Xi}, k_{Xi} and k'_{Xi}. This is already known to be stable and results in good predictions for these values. It also fixes the k'_{Xi} so that we can use them in the next phase of the solution.

Phase II To make the least squares matrix for this phase non-singular, it was necessary to add further constraints. It is sufficient to constrain the colours of all primaries that involve both minor and major sections for a given ink to be the same as the primary that does not involve the minor section. This is similar to the additional constraints used in phase I. For Phase II, using observation data spread over the printer's gamut, solve for the remaining elements of the set of double Neugebauer primaries.

Phase II of the solution is not only independent from this first phase, but is linear and so can be solved directly, thereby reducing computation time and increasing the likelihood of a stable solution. Phases I and II must be repeated independently for each of X, Y and Z.

For Phase II, we can further reduce the amount of computation by observing that the probabilities associated with some of the double Neugebauer primaries are so small that approximating these primaries with others that are similar in colour will have little affect on the final result. There were two components to the strategy used. Firstly, all primaries that involved two or more minor sections were replaced with the primary that used the same major sections but without any minor sections. Secondly, those primaries that would flood if printed do not need to be known accurately. They can be replaced by primaries of similar colour that do not flood without affecting the usefulness of the model.

Using this technique means that, while we can compute a useful model, the k'_{Xi} and many of the double Neugebauer set which result from the computation will have little physical meaning. In practice, it was not possible to assign any meaning to the numbers that resulted. This does not take away from the rigour or usefulness of the resulting model, but does indicate that the initial assumptions are neither proved nor disproved by the result.

Employing the above scheme will result in the numbers in the final column of table 3.

Table 3 Numbers of elements in sets of double Neugebauer primaries

No. of inks	Normal Neugebauer set	Total Double Neugebauer set	Double Neugebauer set elements needed for Phase II
3	8	64	13
4	16	256	34
6	64	4096	125

By way of example, in the 3 ink case above, the 13 remaining members of the Double Neugebauer Set are $\{I_CH_M, I_CH_Y, I_MH_C, I_MH_Y, I_YH_C, I_YH_M, I_CI_M, I_CI_Y, I_MI_Y, I_CI_MH_Y, I_CH_MI_Y, H_CI_MI_Y, I_CI_MI_Y\}$

To make best use of the resulting characterisation procedure, three distinct sample sets were used.

Set A consisted of patches of single ink only and was used in Phase I only. It is necessary to determine the paper colour plus three parameters for each ink. If a redundancy of 100% is required to ensure reliable determination, it is necessary to print at least six patches for each ink. They should be roughly visually spread between paper and full ink. Paper colour must also be measured.

For most printers, there is no real necessity to accurately model the whole domain because it is not used in production images. **Set B** consists of patches spread over the part of the multi-ink domain which does not flood. This set is used to provide information to keep the overall model stable. When used in the least squares process the observations from Set B are weighted with a very low weight. A weight of 0.1 was used for the these experiments.

Set B should not contain any patches printed with one ink as these do not contribute to the second part of the characterisation. Similarly, it should not contain the paper colour.

Set B should contain all multiple ink Neugebauer primaries which do not flood. For a six-ink printer there are 64 primaries. Depending on the printer, about a third of these will flood and seven are not relevant because they consist of one ink or no ink at all. The remaining set will contain about 35 primary patches.

For printers with four inks, a more stable characterisation was achieved when Set B was augmented with some mid range samples.

For a three-ink printer, Set B is not required because it will not be different from Set C below.

This choice of the samples in Set B and the choice of the rules used to reduce the double Neugebauer set are intimately related. A bad combination can result in an unstable characterisation. While a good combination has be found, the nature of this relationship has not been fully investigated. It may be possible to find a better combination that allows the use of a smaller Set B.

The region of the model domain in which we are most interested is that which results from the black channel generation (BG), under colour removal (UCR) and ink split methods that will be used in the final printer characterisation. **Set C** is chosen to consist of colours that are visually evenly spaced throughout this region. It should contain at least double the number of samples as the number of elements from the double Neugebauer set that are required to be determined. It should not contain any patches printed with just one ink as they will not contribute to the second part of the model. Likewise paper colour is not relevant.

Experimental Results

Various choices of sample set and elimination strategy worked but the results varied in accuracy and usability. In general, for a good quality printer such as the Canon HyperPhoto printer, it is easy to achieve an RMS $\Delta E = 0.64$ for sample Set C using the strategy outlined above. As the sample set is about twice the size of the number of set elements that need to be determined, a reasonable error estimator is around $\sqrt{2}$ times this. That is, a reasonable estimator of error for this procedure is RMS $\Delta E \approx 0.9$. This is consistent with predictions of experimental error and indicates that it is not possible to achieve a better result with this equipment.

The Canon BJC-7000 printer was tested with CMYK inks and similar results were obtained with an error estimator of RMS $\Delta E \approx 2.0$ Again, this is within expected experimental error.

For the BJC7000 with four inks, the resulting characterisation was inverted to produce printer characterisation LUTs. These LUTs were then used to print various test images with excellent results.

Conclusions

The physical assumptions of the mathematical models proposed in this paper are neither proved nor disproved by the results. However, the models have been demonstrated to be sufficiently accurate to be useful in practice on the printers that were tested. The computation on a modern PC takes only seconds and the results can be numerically inverted to produce LUTs for inclusion in printer characterisation profiles.

Further tests are required on other printers. There might also be gains from further experimentation with the choice of sample sets and the strategy for reduction of the set of double Neugebauer primaries. Also, no experimentation has been done on the simultaneous characterisation of multiple UCR/BG strategies but the author expects that this will work by the inclusion of multiple sample sets of the Set C type.

The strategy, as described, works only for inkjet printers with a single dot size. Expansion of the model to multiple dot sizes would also be useful.

References

1 Press, William H.; Flannery, Brian P.; Teukilsky Saul A.; Vetterling, William T. "Numerical Recipes in C, The Art of Scientific Computing", *Cambridge University Press*, (1988).

2 Balasubramanian, Raja; "Optimization of the spectral Neugebauer model for printer characterization", *Journal of Electronic Imaging* **8**(2), 156-166, (1999)

3 Hickey, Bruce; "Fitting Data of Arbitrary Dimension with B-Splines and Applications to Colour Calibration", *Masters Thesis, University of Waterloo, Ontario Canada*, (1994)

4 Drew M. S., Finlayson G. D., Luo M. R. and Morovic J.; "Colour Printer Characterization by Regression with Greyspace Constraint", *Proceedings of the 6th IS&T/SID Color Imaging Conference,* 193-196 (1998).