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Illuminant Estimation: Beyond the Bases

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Abstract

We describe spectral estimation principles that are useful for color balancing, color conversion, and sensor design. The principles extend conventional estimation methods, which rely on linear models of the input data, by characterizing the distribution or structure of the linear model coefficients. When the linear model coefficients of the input data are highly structured, it is possible to improve the quality of a simple linear model by estimating coefficients that are invisible to the sensors. We illustrate these principles using the synthetic example of estimating blackbody radiator spectral power distributions. Then, we apply the principles to typical daylight illuminants that we measured over the course of twenty days in Stanford, California. We show that the distribution of the daylight linear model coefficients that approximate the daylight spectral power distributions are highly structured. We further show that from knowledge of the coefficient structure, nonlinear algorithms using N sensors estimate the data as well as linear algorithms using N+1 sensors.

Introduction

Linear models are an important first step in reducing the dimensionality of a data set. These models define a limit on the range of possible inputs by specifying a subspace that contains the original data.[1,2] Characterization of the data, however, need not end with specifying the linear model basis functions. The distribution of the linear model coefficients may also provide useful knowledge about the input data.[3-5] This knowledge can both increase the efficiency of the approximation and provide guidance when designing devices to measure the inputs.

In this paper we describe spectral estimation principles that are useful for color balancing, color conversion, and sensor design. These principles build upon the linear methods that have been used in color science and estimation for many years;[1,6-8] the principles extend current methods by offering a way to incorporate knowledge about likely data into the spectral estimation process.

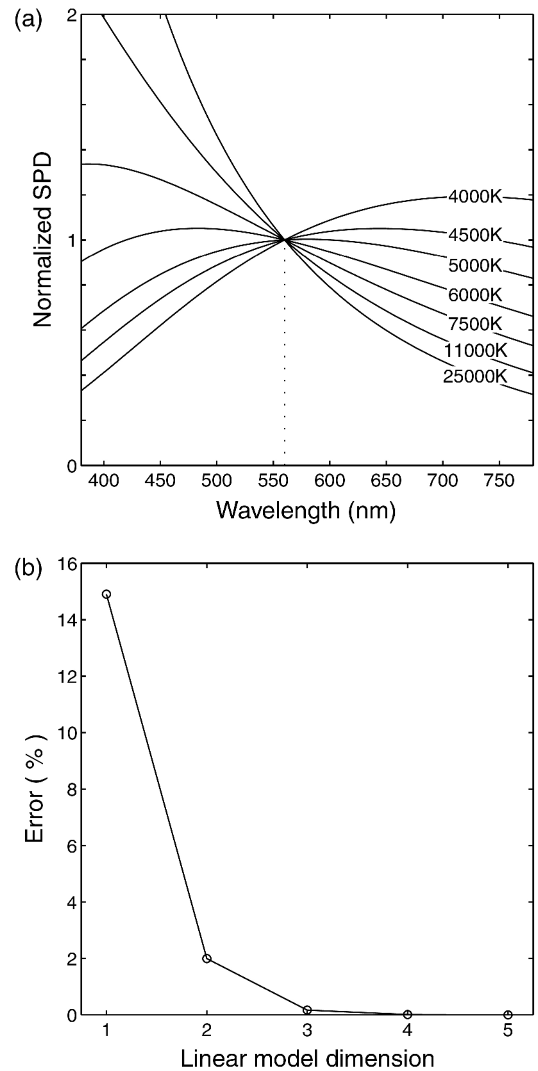

Figure 1. Blackbody radiators. (a) Spectral power distributions of a sample of blackbody radiators with temperatures between 4,000 K and 25,000 K. The curves have been normalized to unit value at 560 nm. (b) The percent error for the best linear model fit as a function of the linear model dimension.

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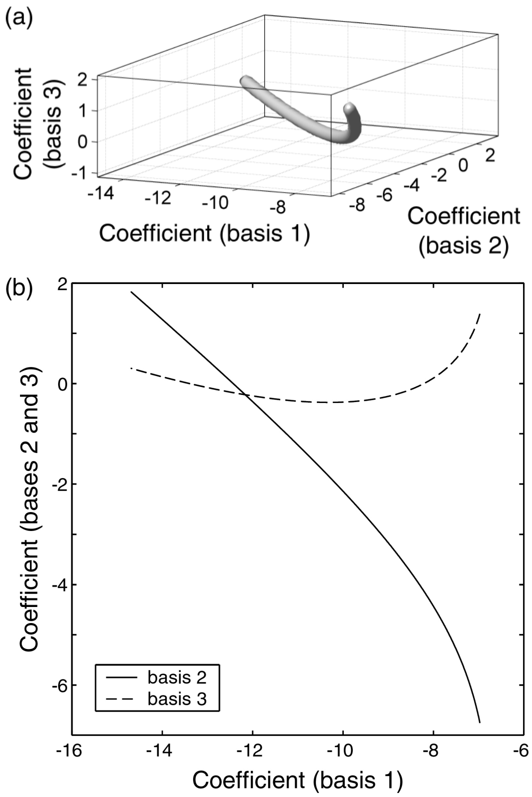

Figure 2. Coefficients of a three-dimensional linear model of blackbody radiators. (a) The curve of coefficients for the blackbody radiators. (b) The values of the second and third linear model coefficients are a function of the first coefficient, showing that the data are well-described by a one-dimensional nonlinear model.

Motivation

We begin with a synthetic calculation that illustrates a limitation of simple linear methods for spectral estimation. Suppose we wish to estimate the spectral power distribution of a blackbody radiator. Figure 1a shows the spectral power distributions of a collection of blackbody radiators with temperatures between 4,000 K and 25,000 K normalized to a common intensity at 560 nm. As the figure makes evident, a one-dimensional linear model cannot fit these curves. Figure 1b shows the percent error of the best-fitting linear model using one to five dimensions. At least three dimensions are needed to reduce the fitted error below 1% and four dimensions fit the data at roughly 0.01% error.

By construction, we know that variation in a single parameter (temperature) gave rise to the entire collection of spectral power distributions; yet, the dimensionality of an adequate linear model is at least three. Although these spectral power distributions were generated from a single parameter process (the blackbody radiator formula,[9]) an accurate linear model fit requires several dimensions. Where in the linear model is the information about the low dimensionality of the original process?

Figure 2a shows that the distribution of the linear model coefficients contains the key information about the original one-dimensional nonlinear process. This panel shows the coefficients of a three-dimensional linear model of the blackbody radiators: the coefficients sweep out a smooth curve. While the dimensionality of the linear model cannot be reduced, because the curve of coefficients is nonlinear, the low-dimensionality of the controlling parameter (temperature) is easily observed in the distribution of the model coefficients.

Figure 2b shows the relationship between the coefficients in another graphical format. The horizontal axis plots the first coefficient. The curves show how the values of the second and third coefficients covary with the first. Because the first coefficient specifies the others perfectly, it is apparent that a one-dimensional nonlinear model explains the data.

Had we approached the data using the conventional linear model analyses, it is likely that we would conclude that these blackbody radiators span a three-dimensional space. We might have missed the fact that the coefficients fall along a curve. Knowing this fact is important should we design sensors to estimate the temperature of a blackbody radiator. Specifically, if the data span three linear dimensions, then three linear sensors are necessary to estimate the temperature. Knowing instead that the coefficients are clustered on a curve, it is possible to estimate the temperature from the measurement of a single sensor: three sensors are two too many.

We offer this synthetic example to motivate the measurements described below. In these measurements, we analyze the distribution of coefficients of daylight illuminants. The purpose of the investigation was to decide whether simply stating the linear model basis functions, such as those provided by the CIE daylight model, captures the essential information about the illuminants. This would be the case if the measured coefficients for daylights are randomly distributed. Or, is there a great deal of structure in the coefficient distribution of real daylights? In that case, there is more to be learned by defining the distribution of these coefficients.

Methods

The methods we present can be applied to linear models of arbitrary dimension. We introduce the methods, however, using a simple example: the spectral power distribution of daylight incident on a building wall in Stanford, California. We begin with this example for two reasons. First, daylight illuminants play a very significant role in imaging;[9] hence, understanding the distribution of coefficients is important for many applications including color conversion, color balancing, and sensor design. Second, it is well-known that the variation in incident illumination is well-described by a low-dimensional linear model;[1,10,11] hence, the distribution of coefficients and computations can be illustrated using simple graphs.

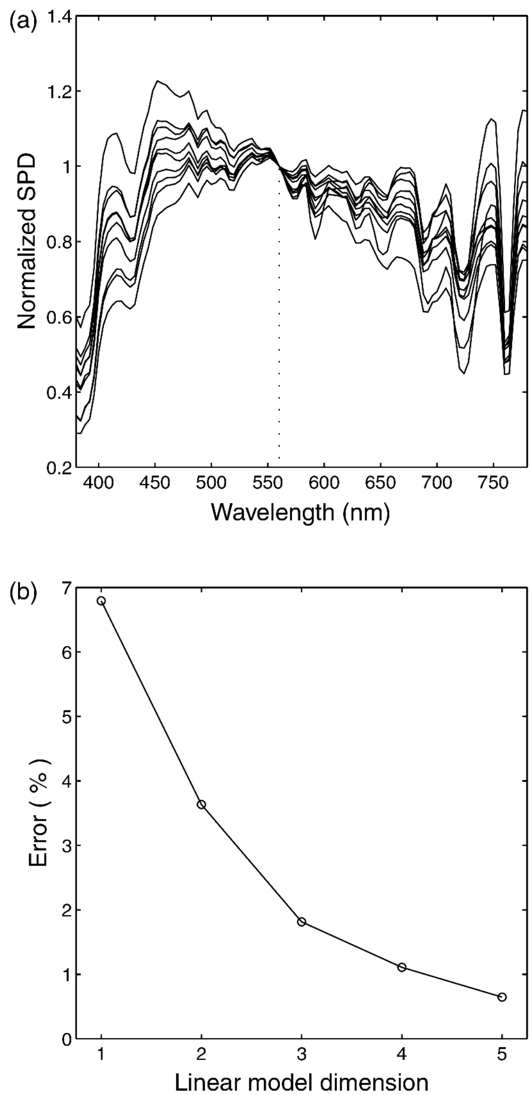

Figure 3. Daylight illuminants. (a) Spectral power distributions of daylights from the collected data set. The curves have been normalized to unit value at 560 nm for display purposes only. (b) The percent error of the best linear model fit as a function of linear model dimension.

Data Collection

We used a PhotoResearch SpectraScan 650 spectroradiometer to collect the daylight spectral power distributions. This instrument measures spectral radiance between 380 and 780 nm with 4 nm spectral bands. The instrument was positioned next to a window and daylight illuminants reflected from an outside building wall were measured. Over a period of twenty days, a computer controlled the spectroradiometer and acquired measurements once a minute from dawn to dusk. During the acquisition period, weather conditions spanned hard rain, light rain, overcast, partly cloudy and clear skies. All measurements were dated and time-stamped using the internal clock of the computer. A total of 11,990 spectral power distributions were measured.

At the end of the acquisition period, the spectroradiometer precision was measured. We made repeated measurements of a stable light source set to a variety of source intensity levels. We found that if the peak radiance of the light source in any spectral band was greater than or equal to a threshold level ($3x10^{-3}$ $W/m^2/sr/4nm$), the spectroradiometer was reliable to within 0.1% of the peak radiance. Below that threshold, the spectroradiometer reliability began to deteriorate. We excluded 1,234 spectral power distributions with a peak response less than the threshold value.

For the remaining measurements, we calculated the illuminant spectral power distributions by dividing the measurements by the spectral surface reflectance function of the building wall. The surface reflectance function was measured by comparing the light reflected from the building wall with the light reflected from a magnesium oxide block in the same position. A total of 10,756 daylight spectral power distributions comprised our data set. Figure 3a shows a few examples of collected daylight spectral power distributions normalized to unit value at 560 nm.

We use the term "daylight" here to refer to the light incident on the wall, and not in its more precise meaning of direct solar radiation through the atmosphere. In fact, some of the light incident at the wall is likely to have come from secondary reflections from other nearby objects (trees, other walls, the ground, etc.)

Linear Model

A linear model of the daylight spectral power distributions expresses the spectral power distributions as a weighted sum of basis functions:

$$E_e = B_e w_e \qquad (1)$$

where E_e (Nx1) is a column vector containing a daylight illuminant sampled at N wavelengths, B_e (NxP) is a matrix containing P basis functions as column vectors, and w_e (Px1) is a column vector containing the daylight illuminant model coefficients.

To evaluate the quality of linear model fits, we use a percent error measure:

$$Error = \frac{\| A - B \|}{\| A \|} \times 100\ \% \qquad (2)$$

where A is the original spectral power distribution, B is the linear model approximation of the spectral power distribution, and $\| x \|$ denotes the vector length of x.

To calculate linear model basis functions that are designed to minimize percent error, we used the following procedure. First, we normalized each daylight measurement by its vector length. This normalization is similar to the procedure used to build the CIE daylight model. (In that case the daylights were normalized to a common value at 560 nm.) Then using the singular-value decomposition on a matrix containing the normalized spectral power distributions, we calculated the left singular matrix. The columns of this matrix serve as the linear model basis functions, B_e.

The absolute intensity of daylight contains important information that is useful in estimation.[12] For example, if the daylight intensity is high, the daylight is more likely to be direct sunlight than atmospheric scattering. When the intensity is low, the reverse is probably true. An estimation process should not discard this information. To calculate the linear model coefficients of each illuminant, w_e, without losing intensity information, we multiplied the transpose of B_e with the original unnormalized daylight spectral power distributions. This linear model, unlike the usual CIE daylight model, contains absolute intensity information about the illuminants.

Figure 3b shows the decreasing percent error as a function of linear model dimension. For three basis functions the average error is 1.81%. Hence, three linear dimensions approximate the full spectral power distributions of our daylight data reasonably well.

The daylights we have collected are consistent with the CIE daylight model in two ways. First, the linear model dimensionality needed to achieve each quality level is similar. To predict both relative spectral power distribution and intensity information to within a 5% error, the CIE daylight model requires three dimensions. Second, the CIE daylight model provides a reasonably good fit for our data. We did not use the model mainly because the basis functions used by the CIE are not orthogonal and this makes certain computations inefficient.

Estimation Algorithm

We developed a nonlinear algorithm for estimating the unknown/undetectable linear model coefficients from known/detectable coefficients. The specifics of the algorithm, which are beyond the scope of this paper, will be presented in a future paper; a general description of the algorithmic steps is offered here.

The algorithm transforms the linear model basis functions so that the linear model coefficients can be divided into two groups: (A) the model coefficients that are measured by the camera sensors, and (B) the model coefficients that are not measured by the sensors because they are orthogonal to the sensor responsivity. The procedure for determining this transformation is described elsewhere.[13] Based on this transformation, the sensor responses are used to measure the linear model coefficients in group (A). The values of these measured coefficients are compared with coefficients in the original data set. Samples in the original set that are similar to the measured coefficients are found. Finally, group (B) coefficients of these similar samples are used to estimate the group (B) coefficients of the measured sample.

Results

The goal of our measurements was to evaluate whether the acquired daylights fall within a small subregion of a higher dimensional linear model. Figure 4 shows the distribution of daylight coefficients of a three-dimensional linear model. The distribution is shown from two different points-of-view. As can be seen in the different panels, the majority of the daylight coefficients follow a curved surface very closely. This can be seen most clearly in panel b. The daylight coefficients do not fill the entire three-dimensional space; they are highly structured.

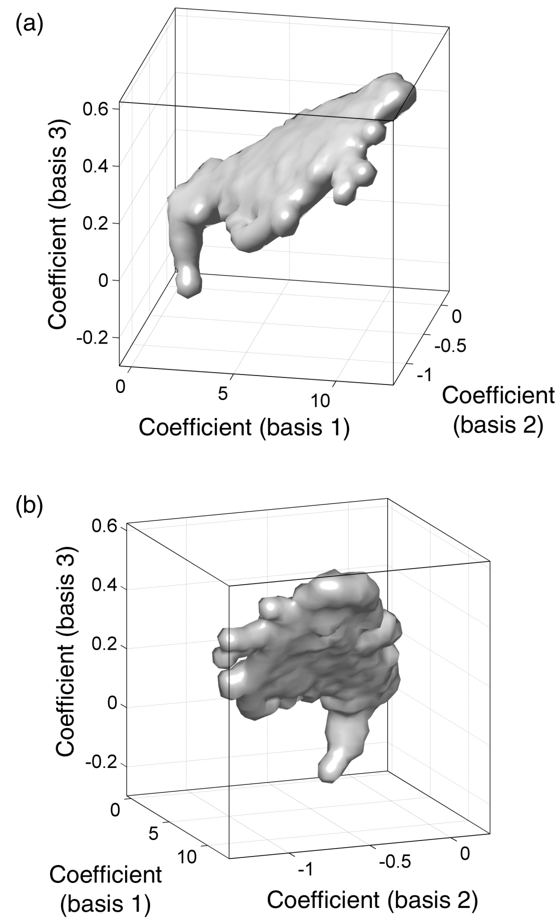

Figure 4. Coefficient distribution of a three-dimensional linear model of daylights. Two different points-of-view are shown. The distribution is a curved surface in a three-dimensional Euclidean space; it is not a space-filling distribution. Hence, the first and second bases coefficients can be used to estimate the third basis coefficient.

Given that the coefficients fall close to a surface, the coefficient positions can be specified using only two (not three) values. Moreover, it should be possible to estimate the spectral power distribution of these daylights using only two linear sensor measurements. We have developed a simple nonlinear algorithm, briefly described in the methods

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section, to estimate the higher order coefficients (3, 4, …, 7) from the coefficients measured by two sensors. The estimates of the third basis coefficients are shown in Figure 5. Notice that the surface has the same shape as the coefficient distribution in Figure 4a. The average percent error of the resulting daylight spectral power distribution using the nonlinear estimation algorithm was 1.89%. When using all three basis functions of the linear model, the percent error was 1.81%. It is apparent that two coefficients and a nonlinear estimation algorithm perform as well as three sensors and a standard linear estimation algorithm.

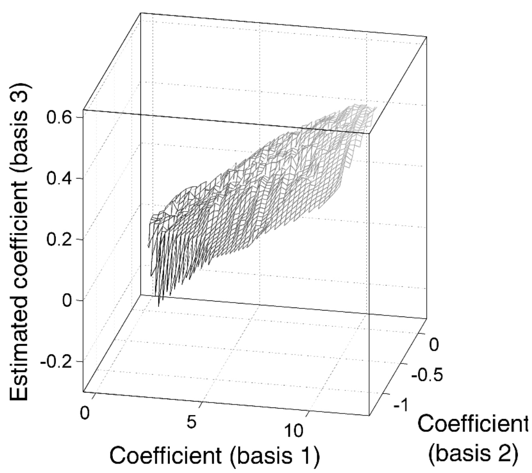

Figure 5. Estimation surface of the third basis coefficients of daylights using the first two. The surface is nonlinear and matches the shape of the coefficient distribution shown in Figure 4a. Using the surface and a two-dimensional linear model, a similar quality fit can be obtained as using a three-dimensional linear model.

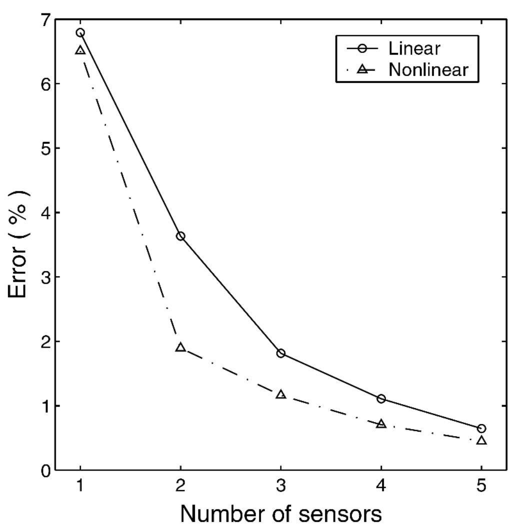

Figure 6. Comparison of the percent error for two illuminant estimation algorithms. The solid line shows the percent error for a simple linear estimation algorithm. The dashed line shows the percent error for a nonlinear estimation algorithm in which invisible coefficients are estimated from measured coefficients. The nonlinear estimation algorithm achieves the same percent error using one less sensor than the linear algorithm.

We have explored how well the nonlinear estimation algorithm performs with one through five sensors compared with a standard linear model using the same number of sensors. The results are shown in Figure 6. As a rough approximation, knowing the structure of daylight coefficients is equal to having one additional sensor. Hence, one can obtain the same error with fewer sensors and less cost.

Conclusion

Linear models are a good initial step for efficiently representing large data sets. Moreover, linear models work smoothly with classic linear mathematics so that these models are helpful for estimation algorithms. Complete characterization of a data set, however, need not end with calculating the linear model basis functions. Additional insight can be found in the distribution of the linear model coefficients.[3] In this paper, we have shown that significant structure exists in the linear model coefficients of typical daylights, and that it is possible to estimate values of one or more coefficients from knowledge of the others. We have shown that it is possible to use the coefficient structure to increase the accuracy of the spectral power distribution estimates, or allow system designs to obtain the same accuracy with fewer sensors and less cost.

Acknowledgements

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