Channel Reduction and Applications to Image Processing

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Abstract

A method of channel reduction is presented which reduces the amount of data to be processed during spatial image processing. Using a modified principal component analysis on non-overlapping blocks of an image, the spatial and spectral information in an image can be separated with very little loss. The processing need be applied only to the spatial information and the color information added after processing.

Introduction

Often times, when dealing with digital color images, it is desired to perform some sort of image processing on the spatial information. Current methods require that one process each of the channels (also called planes or colors) of an image separately [1]. Undoubtedly this increases the number of computations significantly. This paper presents a novel approach to reducing the number of channels in a color image. The channel reduction process is aimed at facilitating such color image processing situations by essentially separating the spectral information from the spatial information as in a paint-by-number. Then the image processing need be applied only to a single channel of data and the color added afterwards.

This paper begins with a description of a simple channel reduction method termed the "crayon" or MPP (maximum projected power) method. The next section discusses an improved method called the "crazy crayon" or MPV (maximum projected variance) method. The final two sections describe the mathematics behind each method and some applications of channel reduction.

The "Crayon" or MPP Channel Reduction Method

An image is broken up into non-overlapping blocks and a single "color", or crayon, is chosen for each of these blocks. We remove the "color" from each of the blocks and are left with a greyscale image that contains almost all of the spatial information. This greyscale image can then be processed with the desired filter and the "color" of each block reapplied afterward.

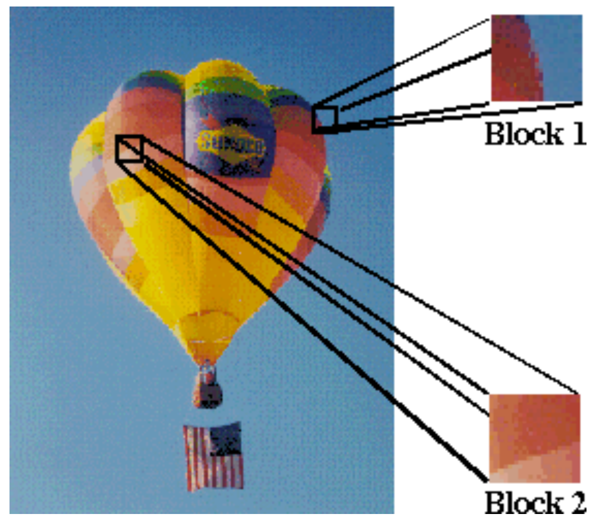
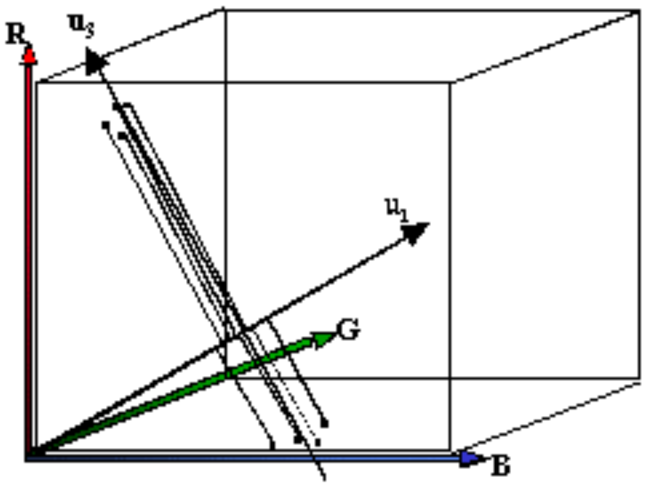
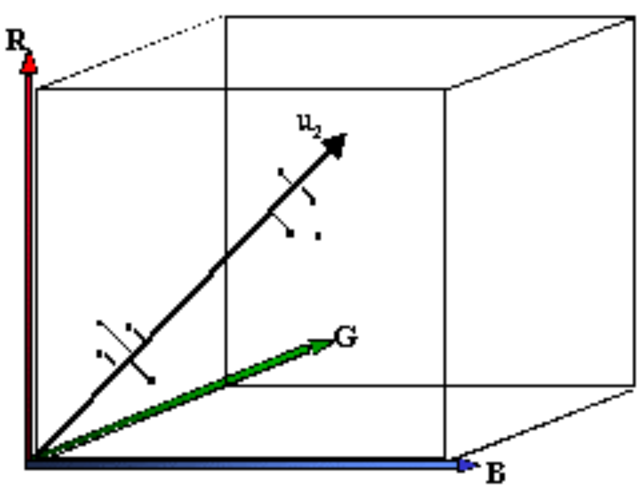


Figure 1. An example of the two different types of blocks typically encountered in a color image

The process of choosing the best "color" for each block is the heart of the channel reduction process. Figure 1 shows a very colorful image of a hot air balloon and two typical blocks of this image. The distribution of pixels within the RGB cube for each of the blocks is shown in Figure 2. In the block labeled Block 2, it is easy to see that we might approximate all of these pixel colors as a single ratio of red, green and blue (shown as the vector $\mathbf{u}_2$ in Figure 2.b). In the HSV (hue, saturation and value) space this fixed ratio of red, green and blue corresponds to a fixed hue and saturation. Thus, the value, or intensity, of the "color" is allowed to vary throughout the block. It is useful to think of this vector as a single crayon that has been chosen for the block and one can vary the value, or intensity, by varying the pressure applied to the crayon. If this process is applied to each block in the image one can understand the paint-by-number analogy mentioned above. A greyscale image exists which contains almost all of the spatial information (that is, the content of the image is obviously a hot air balloon) and we have a collection of a few crayons which we will use to color the image after image processing. Here, the object is to pick the vector that minimizes the mean squared error of the pixels when projected onto the vector. That is, the projected power of the pixels is maximized. The method of choosing this vector is described later.



(a)



(b)

Figure 2. The distribution of pixels within the RGB cube for (a) Block 1 and (b) Block 2. Notice that in Block 2 the spatial information is contained in the intensity of the pixels and projecting these pixels onto vector u_2 is satisfactory. Using the vector u_1 in Block one the spatial information is completely lost and the color is misrepresented. However, the spatial information is preserved if the vector u_3 is used instead.

The "Crazy Crayon" or MPV Channel Reduction Method

Referring again to Figure 2, it is easy to see that this method of channel reduction will fail for blocks like the one labeled *Block 1*. In this case if a single ratio of *red*, *green* and *blue* is chosen, such as the vector labeled $\mathbf{u}_1$ in Figure 2.a, and all pixels within the block are approximated by this "color", two problems will occur. First, all of the pixels will be the wrong color (they will be purple instead of red or blue). Second, the spatial information within the block is completely lost (the channel-reduced block will be a fairly constant gray).

The reason that the crayon approach fails is because we are assuming that all of the spatial information in an image is contained in the *value*, or intensity, of an image. While this assumption is a good one for "smooth" or "natural" images, it falls short when the image has sharp color transitions as in the hot air balloon image. For blocks in which the spatial information is not contained strictly in the intensity a modification to our crayon is necessary. Consider a crayon with a rather crazy behavior. As you vary the pressure applied to the crayon the "color" changes rather than the intensity. The vector $\mathbf{u}_3$ in Figure 2.a represents the crazy crayon. Now, with this so-called crazy crayon, it is possible to separate the spatial and spectral information with very little error. The vector $\mathbf{u}_3$ maximizes the projected variance of the pixels and the choosing of this vector is described later.

Figure 3 shows a sharp image where there exist drastic color transitions. Applying the crayon method of channel reduction in this case fails miserably as can be seen in Figure 4.a. Notice how the edge of the balloon is now very jagged and those blocks on the edge are more of a gray color than the proper colors. However, Figure 4.b shows that after applying the crazy crayon method of channel reduction the edge is preserved and contains the proper colors. NOTE: Please view the color images in Figure 4 on the Proceedings CD. Due to the nature of the method, the greyscale images are misrepresentative.

Figure 3. A sharp image with drastic color transitions.

(a)

(b)

Figure 4. A sharp image after channel reduction via the crayon method (a), and the crazy crayon method (b). Image size: 350 by 250. Block size: 15 by 15. NOTE: Please view these color images on the Proceedings CD. Due to the nature of the method, the greyscale images are misrepresentative.

The Channel Reduction Process

An [N X M] pixel image which has multiple channels corresponding to the basis of the color space in use is broken into blocks of size [n X m]. A data matrix, X_i, is created for each block. The data matrix has dimension [nm X L] (where L is the number of channels) and is created through a lexicographical ordering of each channel block. For example, if an RGB image is to be channel reduced, then within each block the 2-D red (R_i) green (G_i) and blue (B_i) channels are lexicographically ordered to make three, nm length column vectors. That is, R_i -> $\mathbf{r}_i$, G_i -> $\mathbf{g}_i$ and B_i -> $\mathbf{b}_i$. These vectors are then arranged into the data matrix

$$X_i = \begin{bmatrix} \mathbf{r}_i & \mathbf{g}_i & \mathbf{b}_i \end{bmatrix} \qquad (1)$$

The principal component analysis (PCA) [2] consists of finding the Grammian, or zero lag cross-channel covariance, of the matrix that is computed as

$$K = X^T X \qquad (2)$$

(where the subscript i has been dropped) and has dimension [L X L]. In the case of an RGB image, K takes the form,

$$K = \begin{bmatrix} \mathbf{r}^T\mathbf{r} & \mathbf{r}^T\mathbf{g} & \mathbf{r}^T\mathbf{b} \\ \mathbf{g}^T\mathbf{r} & \mathbf{g}^T\mathbf{g} & \mathbf{g}^T\mathbf{b} \\ \mathbf{b}^T\mathbf{r} & \mathbf{b}^T\mathbf{g} & \mathbf{b}^T\mathbf{b} \end{bmatrix} \qquad (3)$$

Only six of the nine values in K must be calculated due to the symmetry of the matrix (ie. $\mathbf{r}^T\mathbf{g}=\mathbf{g}^T\mathbf{r}$). Further reduction in computation can be achieved by subsampling the blocks during the lexicographical ordering.

The desired principal components are the eigenvectors of K and can be found through a number of ways, presently a singular value decomposition (SVD). There are faster PCA methods which are adaptive and find only the first d (out of L) principal components. Performing an SVD on K yields

$$K = U\Sigma U^T \qquad (4)$$

where the columns of U are the eigenvectors and the eigenvalues reside in non-increasing order, in the diagonal matrix S. The major principal axis, which will maximize energy, is then $\mathbf{u}_1$, the first column of U.

Before storing $\mathbf{u}_1$ in the mapping matrix, a projection is made giving the energies of all of the pixels along $\mathbf{u}_1$. These energies will be stored in the column vector,

$$\mathbf{s} = X\mathbf{u}_1 \qquad (5)$$

The vector may be rearranged into a matrix to resemble the original image block and all blocks are combined to create a single channel, S. This matrix may now be transformed by any linear transformation, T (ie. smoothing filter, noise reduction filter, etc.) giving a new matrix, or image, S_f=TS.

Once the desired processing is complete, the new filtered color image may be found by scaling the major axis for each block by the energies now contained in S_f. In this way, each block of scalars, S_f, is transformed into a block of 3-vectors, all of which lie along the major axis, $\mathbf{u}_1$.

In the crazy crayon method, where a major axis is to be found which will maximize variance, the Grammian, or the zero lag autocorrelation, of X-μ_X is found. This new matrix may be written as

$$K_{zm} = (X - \mu_x)^T (X - \mu_x) \ . \qquad (6)$$

That is, maximizing the energy in zero mean data is equivalent to maximizing variance in non-zero mean data. The remainder of the method is identical to the first method except the eigenvectors and eigenvalues are found for K_{zm} and after processing and expansion to three channels, the mean pixel value of the block must be added back in.

Applications

While this channel reduction method is a form of lossy image compression, its usefulness is more that it affords a means to digitally process a compressed image. However, it might be noted that this method can be used as a pre-compression to other compression methods thereby giving another factor of three (in the case of a three color image) in compression.

One particular use of the method is in hybrid optical/digital imaging systems. These imaging systems employ a special optical element called a cubic phase modulation (CPM) plate that code a wavefront as its entering the optical system [3]. After detection of the image by a CCD or CMOS detector the captured image may be digitally processed to decode the image. The coding and decoding are done to ensure that the entire signal (image of a 3-D object with depth) makes it through the channel (optical system) with very little loss. The CPM plate and associated digital processing are designed to eliminate signal loss due to misfocus of those objects outside the depth of focus (DOF) of the optical system. A reduction in computational burden due to the signal processing can be realized through the use of channel reduction. This is especially important in real-time, color, extended DOF video applications.

Conclusion

A method of channel reduction was presented which reduces the amount of data to be processed during spatial image processing of color images. Essentially a modified principal component analysis is preformed on non-overlapping blocks of an image. The major principal component is used to remove and store the color information and leave behind a single channel of data containing the spatial information. The image processing can be preformed on this single channel of data and the color added afterwards.

References

1. Arthur R. Weeks, Jr., *Fundamentals of Electronic Image Processing* , SPIE Optical Engineering Press, Washington, 1996, pp. 228-293.
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3. Edward R. Dowski, Jr. and W. Thomas Cathey, *Applied Optics*, **34**, 1859 (1995).