 

# Color Mix Prediction for Printed Textiles

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## Abstract

A model is introduced which can predict tone and mix colors in textile printing. The factors which make this a more difficult process than match prediction for the textile dyeing and paint industries are highlighted. The model is capable of handling prints made with both reactive dyes and pigment colors, and is extended to deal with paste white colors. The Kubelka-Munk equations are solved for negative absorption coefficient, in order to treat colors whose reflectance slightly exceeds that of the substrate.

## 1. Introduction

The theory of color mix prediction in the textile dyeing and paint industries is well established, the first computer models having appeared in the early 1960's [1–3]. In contrast, virtually no work has been published on color mix prediction for printed textiles. The research described in the present paper aims to fill this gap. It is based on the solution of the Kubelka-Munk equations [4,5] for the reflectance $R$ of a layer of colorant applied to a substrate of reflectance $R_g$

$$R = \frac{1 - R_g(a - b \coth bSX)}{a - R_g + b \coth bSX} \quad (1)$$

where

$$a = 1 + K/S, \qquad b = \sqrt{a^2 - 1}$$

Here $K$ and $S$ are the absorption and scattering coefficients of the colorant and $X$ is the layer thickness. The colored layers in printed textiles are in general semi-transparent, so the full solution of the Kubelka-Munk equations must be used. Mix prediction for textile dyeing [6] and paints [7] is an easier problem, because the layers are opaque and the solution to the Kubelka-Munk equations assumes a simpler form, dependent only on the ratio $K/S$.

Designs are printed on textiles using a relatively small number of screens, each of which can only print one color. One way of increasing the range of colors available to the designer is to make use of mixes or fall-ons i.e. regions where a layer of one color is printed on top of a layer of another color. Another way is to use halftones, familiar from the prepress industry. The raster patterns used are rather coarse, the best resolution used being around 50 lines per inch. Thus the dot pattern becomes rather obvious for weak tones. An alternative method of producing light shades, more pleasing to the eye, is to print tones of a paste white over a darker color. The model described here is capable of simulating all three processes.

## 2. Colorants

The colorants used fall into two classes: reactive dyes and pigments. Knowledge of the optical properties of the colorants needs to be fed in to the model. Dyes are water soluble and the individual dye molecules are several orders of magnitude smaller than the wavelength of visible light [8]. Consequently they have no scattering power; their sole function is to selectively absorb light of certain wavelengths. The pigment pastes used in the textile printing industry consist of dispersions of insoluble pigment particles in water, to which an acrylic resin binder has been added. The pigment particles possess no natural affinity for textile fibers, so the binder is used to literally glue them in place in the finished product. Acrylic resin is totally transparent when dry, therefore the optical properties of the paste are determined solely by the pigment particles. Mie theory [9] tells us that the scattering power of pigment particles depends strongly on their size, and the difference between their refractive index and that of the binder. Maximum scattering occurs when the particle diameter is about half the wavelength of light. Exact data on the sizes of colored pigment particles used in textile printing has proved hard to find. However it appears that the median diameter is typically less than 100 nm, which is sufficiently small for scattering to be almost negligible [10]. This hypothesis was tested by printing colored patches onto a black substrate. The patches proved to be virtually invisible, confirming that the pigments used were nearly transparent. Thus a colored layer consists of colorant particles embedded in textile fibers, with the colorant particles acting as selective absorbers of light and the fibers contributing the scattering.

## 3. Model

We now outline our physical model of the textile printing production process, concentrating initially on solid color fall-ons. In the textile printing production process water based colorant is forced though holes in a screen and into the top surface of the textile substrate. The water is then absorbed by the cloth i.e. it spreads out in both horizontal and vertical directions. The colorant particles are transported by the water and may also undergo diffusion due to Brownian motion. Considerable spreading must take place, because although the holes in the screen have diameters rather small compared to their separation, the cloth ends up being covered with an apparently uniform layer of color. The newly printed cloth then undergoes a finishing process: prints made with reactive dyes are steamed, washed and dried, and pigment prints are baked. By the end of this process the cloth is completely dry and the positions of the colorant particles become frozen. The net result is a distribution of colorant particles in the top layers of the substrate; if the cloth is thick enough, the bottom layers of substrate will remain uncolored.

Now consider an overlay of blue on red. At time $t = 0$, a layer of red is laid down. At some later time $t = T$, a layer of blue is laid down. By this time the water concentration at the top surface of the substrate will be less than it was at $t = 0$, so some blue colorant will be absorbed, but not as much as if it had been laid down in isolation. This effect is known as *resist*. The extra water absorbed transports the blue particles as before, but it also contributes to transport of the red particles, as the cloth has effectively been rewetted. When the cloth is dry, a mixture of blue and red pigment particles will be distributed throughout its top layers. The distribution of blue particles with depth will in general differ from that of the red particles, quite apart from there being fewer blue particles due to resist. Owing to the extra water involved, one might expect the thickness of a two color layer to be rather greater than that of a single color layer.

### 3.1. Implementation for Solid Colors

Implementation of the model might appear a formidable task, because of the difficulty in calculating exactly the distribution of pigment particles with depth. However, the particles are now embedded in the textile fibers which scatter light significantly. Thus incoming light has little chance of reaching a deeply embedded particle; it is far more likely to be back scattered by a fiber. This suggests simple approximations to the depth distribution may be perfectly adequate. The approximation chosen is to have colorant particles distributed *uniformly* throughout the top layers of substrate.

Resist is modeled by reducing the number of particles from the second color printed by a constant resist factor $q = 0.45$. Two models have been constructed. In the first, the mixed layer has the same thickness as the single color layer. In the second, the mixed layer has thickness proportional to the total amount of water absorbed, so is a factor of $1 + q$ thicker than a single layer. For nearly transparent colorants, the dimensionless scattering coefficient $SX$ is a property of the fiber. $SX = 0.8$ gave good results for the prints on a cotton substrate under study and was consistent with attempts made to infer its value from spectrophotometric measurements.

The model was incorporated into a CAD software system, with the color produced by one base color falling on another being predicted as follows:

1. Given the spectral reflectance curves of the base colors and a (wavelength independent) value for the scattering coefficient $S$, the Kubelka-Munk equations are solved iteratively to give $K/S$ as a function of wavelength $\lambda$ for each base color.

2. The absorption coefficients $K_1$ and $K_2$ for the base colors are calculated.

3. The absorption coefficient for the fall-on is then $K_1 + qK_2$ and the scattering coefficient is $S$ or $(1 + q)S$, depending on model.

4. The fall-on $K/S$ values are converted back to reflectance values using Eq. (1).

Examination of the reverse side of samples supplied by a pigment printer showed that the colorants do penetrate significantly into the fabric, and overlays appeared to penetrate deeper than single color layers. As might then be expected, the second model gave rather better results than the first. Results for two solid color fall-ons are shown in Figure 1, the predictions being compared with measurements made with an X-Rite 968 spectrophotometer. The color difference between prediction and measurement is $\Delta E_{\mathrm{Lab}} = 5.46$ for blue on red and $\Delta E_{\mathrm{Lab}} = 3.62$ for red on blue. Because of resist, the fall-on color depends markedly on the order in which the base colors are printed.

Three color fall-ons are handled in a similar way, a resist factor of 0.15 being applied to the third color printed.

### 3.2. Tone prediction

The approach taken here is to assume that the screen engraver will try to produce a series of tonal steps which correspond to the principle of uniform visual increments. Consequently, the model does not need to consider effects such as dot gain [11], the nonlinear response of the human eye and dot overlap, as these will have been taken into account by the engraver, probably on a trial and error basis. The only exception is an additional dot gain which arises in certain tonal fall-ons. This is discussed in Section 4.

Examination of production samples showed that the layer thickness for a tone was less than for a solid color.

Even a 90% tone appeared much thinner. For the weakest tones individual dots in the raster pattern are clearly visible on the top surface of the cloth, when viewed with a magnifying glass, suggesting the layer thickness is nearly zero.

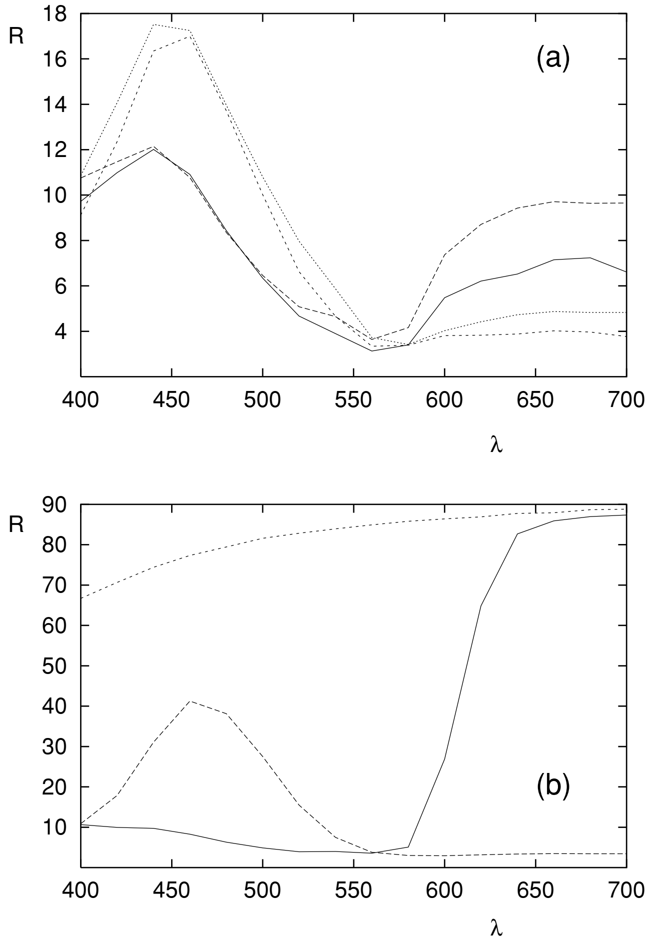


*Figure 1: (a) Percentage reflectance values for two color fall-ons. The solid line is for blue falling on red (measured); the long dashed line is blue on red (predicted); the short dashed line is red on blue (measured); and the dotted line is red on blue (predicted). (b) Spectral reflectance curves for the base colors involved in the fall on: red (solid lines); blue (long dashes) and the substrate (short dashes).*

Making the approximation that all tones are in fact surface colors allows the reflectance curves for tones of a base color to be predicted in a simple way. The base color reflectance curve is converted into an absorption coefficient curve using Beer's law [5] for transparent colors

$$KX = \frac{1}{2}\ln(R_g/R)$$

In the default model, the absorption coefficient for a tone of strength $p$ $(0 \leq p \leq 1)$ is simply $pKX$, which can be converted back into a reflectance using the inverse Beer's law relation

$$R = R_g \exp(-2pKX)$$

The output from a particular engraver is unlikely to follow this ideal form exactly. If such output is being simulated, a piecewise linear fit is used to predict the variation of the absorption coefficient with tonal strength i.e. two straight line fits are used, one for strong tones and one for weak tones, the fit being continuous at the crossover value.

Good agreement was seen between predicted and measured tones, both for simple test designs and real designs consisting almost entirely of complex tonal work.

### 3.3. Tonal fall-on prediction

The previous subsection provides a method for converting a tonal layer consisting of colored dots into an equivalent solid color. As less colorant is used in producing a tone than in producing a solid color, it follows that less water is being applied to the cloth. Thus any color printed over a tone should suffer less from resist. This effect has been incorporated into the model. The thickness $X$ of a tonal layer is made to vary in a piecewise linear way with tonal strength $p$, while the resist factor $q$ increases linearly from 0.45 at 100% tone to 1 at 45% tone and below. The extended model gave good results, although not as good as for solid colors, mixes of tones being typically somewhat lacking in chroma.

### 3.4. Modifications for Reactive Dyes

Reactive dyes react with hydroxyl groups in the fiber, so tend to get bound to specific sites [12]. This provides another mechanism for resist, since the first color printed is favored, as it has first choice of sites. As the dyes are applied in aqueous solution, the original resist mechanism is also present. Dye molecules can diffuse more easily than pigment particles, owing to their smaller size. The exact distribution of colorant with layer depth will therefore be different, but the qualitative situation will be much the same, so the model should still work, possibly with some adjustment to the parameters. Good matches were indeed obtained with production samples printed using reactive dyes.

## 4. Paste White Colors

Scandinavian textile printers in particular use paste white as a means of producing lighter shades while avoiding the coarse grained effect evident when printing weak tones of a base color. The paste white can be printed as either the bottom layer or the top layer in the design. Various strengths of paste white are used.

 

If the paste white is the last color down, then colors printed previously will appear lighter due to the additional scattering from the white pigment particles. The effect of the white pigment is reduced by the resist phenomenon. On the other hand, if the paste white is printed first, the strength of subsequent colors is diminished by resist as well as by scattering from the white pigment. Thus printing the white as the top layer gives a much more subtle effect than printing white as the bottom layer.

The importance of resist in determining the strength of the mix color can be seen by contrasting what would happen if there was no resist and no physical intermixing between layers. Then printing the white first would have virtually no effect; one would effectively just be substituting one white substrate (cloth) by another (cloth coated with white pigment).

The mix model described above was extended to cater for paste white colors. Essentially the model handles whites in the same way as ordinary colors, except that the dimensionless scattering coefficient $SX$ varies with the strength of the white paste. The mix color predicted by the model agreed very well with the test samples when the paste white was applied as the top color. Good agreement was also found when solid colors were printed over the paste white. However rather surprising results were obtained when weak tones of blue or red were printed over white. As one increases the tonal strength of the white from 20% to 100%, one would expect the blue/red to become lighter and lighter, and indeed this is what is predicted by the model. The samples showed precisely the opposite effect — printing 20% blue over 100% white gives a stronger color than printing 20% blue over 20% white. This striking phenomenon can be explained in terms of *variable dot gain*. Applying 100% white to the cloth makes it much wetter than applying 20% white. Consequently the halftone dots which constitute the 20% blue spread out to cover a much larger area. This was confirmed by examining the samples under a magnifying glass. The increase in color strength caused by the increased area coverage more than compensates for the loss of strength caused by the dots becoming thinner.

The following simple model serves to illustrate what is happening. Suppose the tonal strength is chosen so that the colored dots which make up the tone cover precisely 50% of the area of the cloth. Suppose also that when the tone is printed over paste white, the dots spread to such an extent that they now cover 100% of the area of the cloth. Assume, for simplicity, that the colorant used is transparent and that the cloth and paste white are perfect reflectors. If the reflectance of a spread out dot is $x$, then the reflectance of an original, unspread dot must be $x^2$, as it is twice as thick. The mean reflectance of the print on cloth is $(1 + x^2)/2$, while the mean reflectance of the print over paste white on cloth is $x$. Since $(x - 1)^2 \geq 0$ it follows that

$$(1 + x^2)/2 \geq x \tag{2}$$

i.e. the print on paste white always appears darker than the print direct on to cloth. Moreover, the darker the dot color, the greater is the difference between the two prints. Thus the effect will be more noticeable for strong blues and greens than it will for strong yellows and reds. An extreme case is for perfect black dots ($x = 0$). Then the print on paste white will appear perfectly black while the print on cloth, when viewed from a distance sufficiently great for the eye to be unable to resolve individual dots, will appear mid-gray (50% reflectance).

A similar phenomenon occurs when printing weak tones of a dark color over a light color. The original test samples produced for this research using blue, red and yellow colors show the effect well. Printing 20% blue over 100% yellow, gives a far stronger green than would otherwise be expected.

Work is currently in hand to incorporate the variable dot gain effect into the model. Note that the effect acts antagonistically to resist. Models which vary the resist factor for a particular layer by an amount which depends on the tonal strength of the layer and the ones below, do not seem to give a strong enough color change. Models which renormalise the tonal strength appear more promising.

## 5. Negative Absorption Coefficients

Normally, the absorption coefficient $KX$ is positive and the maximum reflectance of a colored layer is then given by setting $KX = 0$ in Eq. (1)

$$R_{\max} = \frac{1 - R_g(1 - 1/SX)}{1 - R_g + 1/SX} \tag{3}$$

This exceeds $R_g$ by an amount which increases as the scattering coefficient and hence the opacity of the colorant increases. In the course of this work certain bright reds and yellows were encountered whose reflectance exceeded this maximum by 1–2%. For these colors the algorithm outlined at the end of Section 3.1 failed to converge.

These problems were avoided by allowing the absorption coefficient $K$ to go negative. At first sight, Eq. (1) is only valid for $K \geq 0$, because if $K < 0$, the parameter $b$ becomes pure imaginary. However, introducing $\beta$ defined by

$$\beta = -ib = \sqrt{1 - a^2}$$

allows the expression for $R$ to be recast in the following form

$$R = \frac{1 - R_g(a - \beta \cot \beta SX)}{a - R_g + \beta \cot \beta SX} \tag{4}$$

which uses trigonometric functions rather than hyperbolic functions. Equation (4) is valid for $-2 < K/S < 0$ whereas equation (1) applies for all $K/S$ values outside this range. The two formulas are plotted in Figure 2, showing that they join smoothly at $K/S = 0$.

Physically, negative $K$ corresponds to emission rather than absorption, which is possible if the colorant is fluorescent. A more sophisticated treatment of fluorescence would require consideration of reflectances at wavelengths outside the visual range, and use of a dual monochromator spectrophotometer.

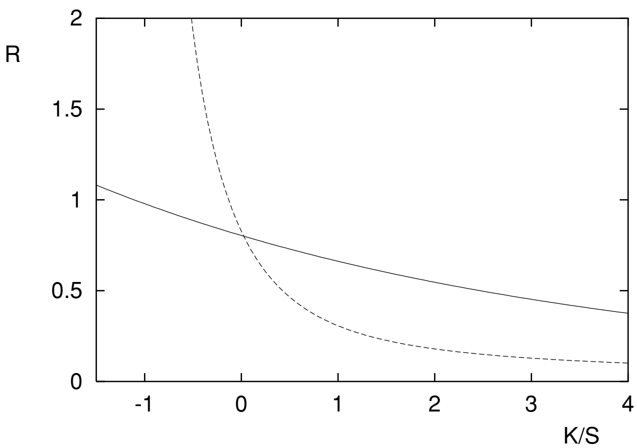


*Figure 2: Reflectance as a function of $K/S$ for $R_g = 0.8$. The solid line is for $SX = 0.1$ and the dashed line for $SX = 0.8$.*

Even if the colorants are not fluorescent, $K$ values slightly less than zero may arise due to the finite accuracy of spectrophotometer reflectance measurements and rounding errors in the calculations which convert $R$ to $K/S$.

## 6. Conclusions

In textile printing, the color of a fall-on depends strongly on the order in which the constituent colors are printed. The first color to be printed has the greatest influence on the fall-on color, due to the dominance of the resist effect. Variable dot gain may be important in certain situations, such as when a weak tone is printed over a solid color.

The model described here is capable of predicting fall-on colors to within a $\Delta E_{\text{Lab}}$ of around 5. While this is greater than the minimum color difference detectable by the human eye, the overall color balance appears correct when the model is applied to real designs. When incorporated in a CAD system, the model therefore provides a powerful tool which enables a textile designer to visualise how tone and fall-on colors will appear in the final production process, thus drastically reducing time and expenditure on producing sample prints.

## 7. Acknowledgements

This work was supported by the United Kingdom Department of Trade and Industry under the Eureka scheme. I thank the Eureka project partners Almedahl-Kinna and Mönsterateliern of Sweden, and Finlayson of Finland, for their assistance in providing production samples.

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