Spectral-Based Ink Selection for Multiple-Ink Printing II. Optimal Ink Selection

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Abstract

An algorithm is described to select a set of six inks from a given ink database for a spectral-based six-color printing system which minimizes metamerism. Since there are C(n, 6) ink combinations for an ink database containing n inks, the number of ink combinations is a geometric figure when n is large. There are obviously impossible ink combinations which can be removed analytically.

Vector correlation analysis is first employed to remove the infeasible combinations. Utilizing the statistical primaries⁸ of a given spectral image requiring reproduction as the basis information, the ink-selection algorithm then searches for similar inks through a given ink database to obtain a few highest correlated inks for each statistical primary. Second, the ink-selection algorithm estimates the colorimetric and spectral accuracy of candidate ink sets in an empirically derived color mixing space which approximates the color formation of a halftone printing process. The candidate ink set with the highest spectral accuracy will be designated as the optimal ink set for a six color printing process thereby achieving the least metameric color reproduction.

Introduction

A research and development program for a spectralbased color reproduction system which minimizes metamerism between originals and reproductions has been initiated at the Munsell Color Science Laboratory at Rochester Institute of Technology. This research includes multi-spectral image acquisition^{1.4} and spectral-based printing.⁵⁻⁷

The current research is concerned with bridging these analysis and synthesis stages of color reproduction. There are two components of the current research. The first component, designated as part I: colorant estimation of original objects, was published and presented at the sixth Color Imaging Conference, diagrammed in Figure 1.⁸ The second component, this submission, focuses on an algorithm for ink selection leading to the least metameric color reproduction.



*Figure 1: The general outline for colorant estimation of original objects.*⁸

Given an ink database with many candidate inks, the process of selecting an optimal ink subset can be very tedious because of the many combinations. There are C(n, 6) combinations of possible ink sets for n inks to be chosen for a six-color printing process. For example, if n = 100 then there are 119,205,240 combinations. As a practical example, even if n = 18 (the Pantone 14 basic colors in addition to the four process colors) then there still exist 18,564 combinations to choose from. To estimate the performance of each ink set in terms of colorimetric and spectral accuracy, spectral printing forward and backward models are needed for evaluating the spectral

reconstruction. This requires building 18,564 six-color printing models for the 18-inks database and estimating the total performance model by model. The amount of computation for trying 18,564 combinations is unreasonable. Clearly, testing each ink set is impractical and a robust ink-selection algorithm is required.

First Order Ink Selection by Vector Correlation

Although there are 18,564 combinations of ink by selecting six inks from an 18-ink database for a six-color printing process, a large number of combinations are obviously impossible (e.g., red, rubine red, rhodamine red, warm red, orange, and black). The question boils down to on what basis can these impossible ink sets be excluded analytically. For a printing system that performs colorimetric-based color reproduction, it can be achieved visually by determining all the colorimetric values of a given image inside a colorimetric gamut spanned by a set of ink combinations. Generally, the larger the colorimetric gamut a set of inks can provide, the more accurate will be the colorimetric reproduction.

The set of statistical primaries, derived by a colorant estimation algorithm, is utilized as a set of basis functions to remove impossible ink combinations. Once the statistical primaries are uncovered, the process requires correlating the statistical primaries to a set of physical inks which are the most likely to reproduce the image spectrally at the synthesis stage. This can avoid estimating the performance of irrational ink combinations. Theoretically, if an exact set of inks exist in current manufacture or in storage, then the use of the exact ink set will yield the ideal or closest spectral reproduction relative to the input spectral image. Since the statistical primaries are image dependent, the probability of an exact ink set existing in a given ink database is low. Hence, the exact spectral reproduction cannot be achieved. A compromise has to be made to balance between colorimetric and spectral accuracy. Since the colorimetric match is the first priority for any of color application, it is necessary to trade slight spectral accuracy for higher colorimetric accuracy. This compromise will be discussed in a later section.

Intuitively, the utility of the statistical primaries is to search for the exact or similar inks in a given ink database. Vector correlation can be used to compare the similarity among them. A similarity measurement for ink1 and ink2, shown as Eq. (1), is quantified by the correlation coefficient, ρ , which is the cosine angle between a statistical primary and an ink from a large ink database, where $\Psi_{\lambda_{ink1}}$ is the linear colorant vector of ink1 and λ is a wavelength within the visible spectrum. Hence, the closer to unity is the correlation coefficient, the higher the

similarity between a statistical primary and an ink in the database.

$$\rho = \frac{\sum_{\lambda=400}^{700} \Psi_{\lambda,ink1} \Psi_{\lambda,ink2}}{\sqrt{\sum_{\lambda=400}^{700} \Psi_{\lambda,ink1}^2} \sqrt{\sum_{\lambda=400}^{700} \Psi_{\lambda,ink2}^2}}$$
(1)

Since the chance of selected inks from a large ink database which are identical to the statistical primaries is low, candidate inks corresponding to each primary can be selected using a threshold of an acceptable correlation coefficient, say 0.90, or the highest twenty. Further filtration of the selected candidates is done by adopting from several candidates, that with the highest chroma for each statistical primary. A larger colorimetric gamut corresponding to a better possibility of colorimetric reproduction has been elucidated by various literature and experiential evidence.⁹⁻¹² The use of the highly chromatic primaries for colorant mixing yields a larger colorimetric gamut which is essentially desired when an exact spectral color reproduction cannot be accomplished. Compromises have to be made by trading slight spectral accuracy in exchange for colorimetric accuracy. This is the reason for choosing candidates with the highest chroma for balancing between colorimetric and spectral accuracy. Based on this selection method, there are 64 (2^6) possible combinations of candidate ink sets. This is a significant reduction from 18,564 combinations down to 64. Nevertheless, it is still necessary to pinpoint the exact ink set for the application of the least metameric reproduction.

Continuous tone Approximation

Having developed a method for choosing several possible inks for each statistical primary, we now require an analytical description for the printing process to facilitate the final ink selection. The intuitive choice is the six-color printing model based on the Yule-Nielsen modified spectral Neugebauer equation. In order to build 64 six-color printing models, it is required to make sample preparations for printing 64 sets of ramps of the Neugebauer primaries and 64 sets of verification targets for the 64 ink combinations. This amount of sample preparation is prohibitive. Even if the sample preparation could be replaced by computer simulation by employing certain assumptions, the model building effort is still computational costly and time inefficient. Hence, a more efficient mechanism for estimating the colorimetric and spectral performance of the 64 selected ink sets is desired.

The further removal of a number of combinations among the 64 is judged by scrutinizing the ink sets which are incapable of spanning the vector space of the given spectral image. We make an assumption that halftone reproduction can be approximated by a continuous-tone model for subtractive color mixing.^{12,14} The continuous-tone modeling techniques used by the research program^{15,16} at Munsell Color Science Laboratory are mostly based on Kubelka-Munk turbid media theory.^{17,18} However, for this application, Kubelka-Munk theory has insufficient accuracy. Alternatively, an empirical transformation, shown as Eq. (2), and its inverse transformation, shown as Eq. (3), is used for approximating the color mixing behavior of a halftone printing process.

$$\Psi_{\lambda} = \mathbf{R}_{\lambda,\text{paper}}^{\frac{1}{w}} - \mathbf{R}_{\lambda}^{\frac{1}{w}}$$
(2)

$$R_{\lambda} = (R_{\lambda, \text{paper}}^{\frac{1}{W}} - \Psi_{\lambda})^{W}$$
(3)

The $R_{\lambda_{\text{,paper}}}$ is the spectral reflectance factor of the paper substrate and $2 \le w \le \infty$. The use of $R_{\lambda,paper}^{\frac{1}{w}}$ as the offset vector has a significant meaning. Consider that transforming a spectrum, which is exactly $R_{\lambda_{\text{paper}}},$ to the linear color mixing space, the result is a zero vector. This corresponds to the fact that there is not any primary ink presented in the linear space. Furthermore, Eq. (3) transforms a zero in the linear space back to the exact reflectance spectrum of the paper, $R_{\lambda_{paper}}$. The justifications for the use of the proposed transformation for continuoustone approximation will be described in the verification section. Equation (2) transforms the spectral reflectance factor to the representation for a subtractive color-mixing process. Hence, synthesis, quantatively described by Eq. (4), is the linear combination of the primary colorants modulated by their corresponding concentrations.

$$\Psi_{\lambda,\text{mixture}} = \sum_{i=1}^{k} c_i \Psi_{\lambda,i}$$
(4)

The ψ_{λ} is the linear representation of a primary colorant normalized to its unit concentration, c is the corresponding concentration, and k is the number of the primary colorants.

Based on the assumption that a multiple-ink halftone printing process can be approximated by continuous modeling using Eq. (2) and (3), a direct constrained regression model using Eq. (4) was employed to estimate the performance of each candidate ink set. The estimated concentration for each primary is constrained to be positive. Positivity symbolizes the capability of a candidate ink set to span the entire colorant vector space of the target. If a negative concentration is reported using Eq. (4), then the corresponding ink set is only spanning the partial colorant vector space of the target. As a consequence, the spectral reconstruction by the ink set yields spectral error when the constraint of positivity is enforced. The final decision should favor the ink set(s) whose spectral reconstruction for a given target achieves the higher colorimetric and spectral accuracy. For this research project, the colorimetric accuracy was specified by the CIE color difference equation ΔE^*_{94} under standard illuminant D50 and the 1931 standard observer.¹⁹ The spectral accuracy was quantified by a metameric index which is calculated using ΔE^*_{94} under standard illuminant A and the 1931 standard observer based on a parameric correction.²⁰

Verifications and Results

Deriving a Linear Color Mixing Space for the Continuous Tone Approximation

The current research analysis is based on the validity for a linear colorant-mixing space which approximates the color-mixing behavior of a multiple-ink halftone printing process such that the vector correlation analysis and constrained regression can be performed. The verification of deriving an empirical transformation, Eq. (2), for a typical halftone printing process utilized the spectral reflectance factor of IT8/7.3 containing 928 samples printed by SWOP standard at 133 LPI screen frequency.²¹ The parameter in Eq. (2) to be optimized is w. The optimization process used a non-negative least square function, nnls(), built in MATLAB, to set up a constrained regression model, following Eq. (4), synthesizing every spectrum of the IT8/7.3 target. The nnls() performs a least squares match for each spectrum by constraining the corresponding concentration for each primary to be nonnegative. The w corresponding to the highest colorimetric and spectral accuracy will be adopted.

It was found that colorimetric and spectral errors monotonically decease as w approached infinity. Since the slope of decreasing average $\Delta E_{_{94}}^{*}$ and metameric indices were small when w is higher than three, the change of w did not increase the average performance significantly. It only reduced the maximum errors as w increased. The adopted w was 3.5. Colorimetric and spectral accuracy for the four SWOP primaries synthesizing 928 samples of the IT8/7.3 in the proposed linear colorant mixing space is shown in Table I. Low colorimetric and spectral error reveals that the four SWOP primaries approximately span the 928 samples of the IT8/7.3 target in the proposed linear color mixing space. That is, every sample is a linear combination of the CMYK primaries. The color formation for this halftone printing process is approximately described by mixing the CMYK in the linear colorantmixing space. The reconstructed spectra corresponding to the highest four colorimetric and spectral error estimations are shown in Figure 2.

Table I: The colorimetric and spectral accuracy for the four SWOP primaries synthesizing the 928 samples of IT8/7.3 target in the proposed linear color mixing space.

	ΔE_{94}^{*}	Metameric Index		
Mean	0.7	0.1		
Stdev	0.7	0.2		
Max	5.1	1.4		
Min	0.0	0.0		
RMS	0.005			



Figure 2: The four spectra reconstructed with the highest colorimetric and spectral errors based on the linear colorant mixing space where the solid line is the measured spectrum and the dashed line is the reconstructed spectrum.

Although the four spectra are reconstructed with the highest colorimetric and spectral errors, the reconstructed curves are well tracing the originally measured spectra. This implies that the halftone printing process can be well described by continuous tone approximation based on the proposed transformation. Hence, the derived new Ψ space can be used for estimating the performance of the 64 ink sets without heavy halftone modeling efforts.

Vector Correlation Analysis in \PsiSpace

A set of 105 color patches generated by hand mixing six opaque poster paints (Sakura cerulean blue No. 25, Sakura rose violet No. 22, Pentel yellow No. 5, Pentel sap green No. 63, Pentel ultramarine No. 25, and Pentel black No. 28) was employed as the presumed reproduction target representing an arbitrary image. Its six statistical primaries, shown in Figure 3, were estimated by the algorithm of colorant estimation pictorially described in Figure 1.

The 14 Pantone basic and four process primaries printed on coated paper were utilized as an ink database to test the ink selection algorithm. (Color names and their abbreviates are yellow (Y), yellow 012 (Y 12), orange 021 (O 21), warm red (Wr), red 032 (R), rubine red (Rr), rhodamine red (Rh), purple (Pu), violet (V), blue 072 (B 72), reflex blue (Rb), process blue (Prs B), green (G), black (K), process yellow (Prs Y), process magenta (Prs M), process cyan (Prs C), and process black (Prs K).)



Figure 3: The six statistical primaries derived from the 105 mixtures by the colorant estimation algorithm.

All the spectral reflectance factors for the statistical primaries and the ink database were first transformed by Eq. (2) to a linear colorant-mixing space. Then, correlation coefficients of all 18 inks in the database with the statistical primaries were calculated using Eq. (1). Up to two candidate inks for each statistical primary with the highest chroma among those highly correlated inks were chosen. Their correlation coefficients and chroma are tabulated in Table II. Since this research aims at using one black and five chromatic inks to perform spectral reproduction, the two black inks in the ink data based are the certain candidate inks for the continuous tone Hence, estimation. their chroma and correlation coefficients with the sixth primary were not shown in Table II.

Table II: The correction coefficients and the chroma of the 18 inks with the five chromatic statistical primaries.

Candidate Inks	Primary 1	Primary 2	Primary 3	Primary 4	Primary 5	Chroma
Process Blue	0.98					68.6
Process Cyan	0.98					64.1
Rhodamine Red		0.94				80.1
Purple		0.88				86.8
Yellow			0.98			112.6
Process Yellow			0.97			106.3
Green				0.36		82.3
Blue 072					0.66	88.8
Reflex Blue					0.65	75.4

There are two candidates for each statistical primary except for primary 4 due to the lack of a similar ink existing in the 18-ink database. It was forced to choose only one with the highest correlation coefficient among the 18 inks with respect to primary 4. Another situation happens when an ink in the database is simultaneously chosen as the candidate for two or more primaries; that is, an ink is selected more than twice. Then this ink is a sure candidate for forming the optimal ink set. Thus, the candidacy of this ink should be removed from the other primaries. This ensures that when forming ink combinations by all candidates, there are no double or triple selected inks for an ink combination, consequently, leading to a combination of smaller colorimetric and spectral gamut.

Colorimetric and Spectral Performance by the Continuous-Tone Approximation

Thirty two (2x2x2x1x2x2) ink sets were formed by 11 candidate inks. Their colorimetric and spectral accuracy were estimated based on the constrained regression model using Ψ space. Since the validity of continuous tone approximation has been verified for the IT8/7.3 printed by SWOP standard, it is generalized to any of printing process meeting SWOP specification. Three ink sets were designated as the optimal ink sets for reproducing the 105 mixtures based on their highest spectral accuracy specified by the metameric index. Their spectral and colorimetric accuracy are listed in Table III and their ink combinations are described in Table IV.

Table III: The spectral and colorimetric accuracy of the three optimal ink sets.

	Metameric Index			$\Delta \mathrm{E}^{*}_{_{94}}$					
Ink Set	Mean	Stdev	Max	Min	Mean	Stdev	Max	Min	RMS
23	0.7	0.5	1.9	0.1	2.3	1.0	4.4	0.4	0.028
24	0.7	0.5	1.9	0.1	2.4	1.0	4.2	0.4	0.028
19	0.7	0.5	1.9	0.0	2.4	1.2	4.8	0.2	0.028

 Table IV: The Pantone color names of the three optimal ink sets.

Ink Set	Primary 1	Primary 2	Primary 3	Primary 4	Primary 5	Primary 6
23	Prs C	Rh	Y	G	B 72	Prs K
24	Prs C	Rh	Y	G	B 72	K
19	Prs C	Rh	Prs Y	G	B 72	Prs K

The performances among the three ink sets are not significantly different. Set 23 and set 24 are only different at the use of the sixth primary. It is concluded that the use of process black and the black is approximately invariant with the resultant performance. In addition, set 23 and set 19 are different by the use of the third primary. It is concluded that the use of yellow and process yellow is also approximately invariant with performance. Three reconstructed spectra for a sample set of 105 mixtures corresponding to the maximum error predicted by the three sets are plotted in Figure 4. The three reconstructed spectra are merely identical. Based on this observation, the three optimal ink sets approximately span the same color mixing space.



Figure 4: The three spectral reconstructions of a sample corresponding to the maximum prediction error by the three optimal ink set.

Three poorest performing ink sets were also inspected whose colorimetric accuracy is about 3.7 and 7.0 units of ΔE_{M}^{*} for average and maximum, respectively, and spectral accuracy of about 0.9 and 2.7 units of metameric index for average and maximum, respectively. Set 26, one of the worst performing ink sets, whose ink combination is process cyan, purple, process yellow, green, reflex blue, and process black, is utilized as an example in contrast to the capability of spanning the 105 mixtures by set 23, one of the optimal ink sets. Two reconstructed spectra for the sample by set 23 and set 26, plotted in Figure 5, are shown for visual comparison. By the numerical results, the colorimetric and spectral accuracy of the set 23 is higher than that of the set 26. By Figure 5, the shape of the spectrum reconstructed by set 23 whose RMS error is 0.041 is closer to the measured spectrum than that of the reconstruction by set 26 whose RMS error is 0.050. The implication is that the color space spanned by set 23 is a better approximation to the color space of the 105 mixtures than the color space spanned by set 26. Since the decision of choosing optimal ink sets is based minimizing metamerism, the above justification was made to correlate the effectiveness of a metameric index to a set of ink combinations which is the optimal selection.



Figure 5: The two reconstructed spectra by set 23 and set 26 for the sample used as the example in Figure 4.

Conclusions

An optimal ink set selection algorithm was proposed and justified. Its primary goal is to bride between multispectral acquisition systems and multiple-ink output systems for the least metameric color reproduction. It serves the purpose of removing the redundancy or the deficiency of large ink combinations of a given ink database. This research also proposes a linear color mixing space for a continuous-tone approximation to a halftone printing process. It dramatically reduces the large scale of halftone modeling efforts in estimating the validity of optimal-ink selection. The ink-selection algorithm is comprised of vector correlation analysis follow by a constrained regression analysis. The proposed approach is able to remove a large number of ink combinations from an ink database. It pinpoints the optimal ink combination for a spectral-based halftone color reproduction system minimizing metamerism.

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