Generation of Virtual Spectral Color Target and Its Application

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Abstract

The spectral sensitivities of image capture devices should be carefully designed to guarantee colorimetric color reproduction. As a standard color test chart, ISO/DIS 12641 is conveniently used for calibrating the scanners or printers. However, its color gamut is not wide enough to evaluate the devices such as digital camera & display and the color distributions are not uniform systems because the chart is a photographic print. This paper presents a virtual color target to estimate the spectral goodness of color devices. A virtual spectrum with given L*a*b* value is generated from the fundamental metamer uniquely obtained by inverse projection of XYZ tristimulus value and an addition of arbitrary metameric black spectrum. The goodness of typical color input sensors is measured and compared with actual color chips using this virtual spectral target.

Introduction

Various color targets are used to estimate the performance of color devices. Standard test chart ISO/DIS 12641 however, have not wide gamut enough to evaluate such device as digital camera and the color distributions are not uniform because the chart is a photographic print. We proposed a virtual color target 4 with sinusoidal spectral power distributions (sine SPDs) and applied it to the estimation of color filter sets 5,6. But it was difficult to generate the sine SPDs with uniform distributions in CIELAB space. This paper presents a method to synthesize a set of color spectra with wide gamut distributed uniformly in CIELAB space. A virtual spectrum with given L*a*b* value is generated from the fundamental metamer with an addition of metameric black. The spectral goodness of color sensors is measured by the fundamental spectral error $E[||\mathbf{e}||^2]$ after versus before passing through the sensors as introduced by Vora and Trussell 2. The reliable values of $E[||\mathbf{e}||^2]$ could be obtained for typical color sensors and compared with the results by actual chips and sine SPDs.

Sensing Error in Fundamental Spectra

An input SPD C(λ) is described as an n-dimensional vector

$$\mathbf{C}=[C_1, C_2, \ldots, C_n]^t \,;\; C_i = C(\lambda_i),\quad t = \text{transpose} \tag{1}$$

By the projection of **C** onto Human Visual Subspace(HVS), **C** is decomposed into the fundamental metamer **C*** and residual **B** by Cohen 3

$$\mathbf{C}=\mathbf{C^*}+\mathbf{B},\quad \mathbf{C^*}=\mathbf{P_v C},\quad \mathbf{B}=(\mathbf{I}-\mathbf{P_v})\mathbf{C} \tag{2}$$

where **I** is the identity matrix and

$$\mathbf{P_V}=\mathbf{A_L}(\mathbf{A_L}^t\mathbf{A_L})^{-1}\mathbf{A_L}^t,\quad \mathbf{A_L}=\mathbf{LA} \tag{3}$$

$\mathbf{P_v}$ denotes the orthogonal projection operator onto HVS from input color vector space and a residue **B** is called metameric black with zero stimulus not perceived to human vision. Where **A** and $\mathbf{A_L}$ denote the CIE color matching matrix and that by biased with illuminant **L**.

$\mathbf{P_v}$ extracts a fundamental metamer **C*** that carries the tristimulus vector **T** as

$$\mathbf{T}=\mathbf{A_L}^t\mathbf{C}=\mathbf{A_L}^t\mathbf{C^*} \tag{4}$$

The projection operator $\mathbf{P_F}$ for a set of tri-color sensors is obtained by

$$\mathbf{P_F}=\mathbf{F}(\mathbf{F}^t\mathbf{F})^{-1}\mathbf{F}^t \tag{5}$$

where, **F** is the n x 3 tri-color sensor sensitivity matrix with filter functions, red r(λ), green g(λ), and blue b(λ). Thus, the fundamental metamer $\mathbf{C_F^*}$ through a color sensor for input **C** is given by the projection operator $\mathbf{P_{FV}}$ onto HVS

$$\mathbf{C_F^*}=\mathbf{P_{FV}C},\quad \mathbf{P_{FV}}=\mathbf{P_v P_F} \tag{6}$$

Here, input spectrum **C** is decomposed into $\mathbf{C_F^*}$ and residual $\mathbf{B_F}$ through color sensor in HVS like as

$$\mathbf{C}=\mathbf{C_F^*}+\mathbf{B_F} \tag{7}$$

Tristimulus vector **T** for **C** under illuminant **L** is given by

$$\mathbf{T}=\mathbf{A_L}^t\mathbf{C}=\mathbf{A_L}^t(\mathbf{C^*}+\mathbf{B})=\mathbf{A_L}^t(\mathbf{C_F^*}+\mathbf{B_F})=\mathbf{T_F}-\Delta\mathbf{T_F} \tag{8}$$

$$\mathbf{T_F}=\mathbf{A_L}^t\mathbf{C_F^*},\quad \Delta\mathbf{T_F}=\mathbf{T_F}-\mathbf{T}=-\mathbf{A_L}^t\mathbf{B_F} \tag{9}$$

where, $\mathbf{T_F}$ and $\Delta\mathbf{T_F}$ denote the tristimulus vector captured by color sensor and its error caused by the mismatch between sensor matrix **F** and color matching function $\mathbf{A_L}$.

Furthermore, $\mathbf{T_F}$ can be separated into two parts as follows.

$$\mathbf{T_F}=\mathbf{A_L}^t\mathbf{C_F}^*=\mathbf{A_L}^t\mathbf{P_{FV}C}=\mathbf{A_L}^t\mathbf{P_{FV}}(\mathbf{C}^*+\mathbf{B})=\mathbf{T_{FC}}+\Delta\mathbf{T_B} \tag{10}$$

$$\mathbf{T_{FC}}=\mathbf{A_L}^t\,\mathbf{P_{FV}C}^*, \qquad \Delta\mathbf{T_B}=\mathbf{P_{FV}B} \tag{11}$$

$\mathbf{T_{FC}}$ and $\Delta\mathbf{T_B}$ mean the sensor responses to fundamental metamer $\mathbf{C}^*$ and metameric black $\mathbf{B}$.

Here we should notice the sensor makes non-zero response $\Delta\mathbf{T_B}$ to $\mathbf{B}$, though human vision has zero stimulus $\mathbf{A_L}^t\mathbf{B}=0$. In short, the tristimulus error $\Delta\mathbf{T_F}$ is composed of $\Delta\mathbf{T_C}$ and $\Delta\mathbf{T_B}$

$$\Delta\mathbf{T_F}=\Delta\mathbf{T_C}+\Delta\mathbf{T_B}\,, \qquad \Delta\mathbf{T_C}=\mathbf{T_{FC}}-\mathbf{T} \tag{12}$$

Where, $\Delta\mathbf{T_C}$ corresponds to the fundamental error and $\Delta\mathbf{T_B}$ to the metameric black error. The error $\Delta\mathbf{T_B}$ caused by metameric black $\mathbf{B}$ should be taken into account.

The spectral goodness of sensor is measured by the mean square error $E[\|\mathbf{e}\|^2]$ between true fundamentals $\mathbf{C}_n^*$ and sensed fundamentals $\mathbf{C_{Fn}}^*$ for n=1~N test color chips by

$$E[\|\mathbf{e}\|^2]=[(1/N)\sum_{n=1}^{N}(\mathbf{C_{Fn}}-\mathbf{C}_n)^2] \tag{13}$$

Generation of Virtual Spectral Target

The following three different color targets are used for color sensors' quality test.
(1) ISO/DIS 12641 standard color target (real 264 chips)
(2) Sine SPDs (virtual chips)
(3) Synthesized SPDs (proposed virtual LAB chips)

Figure 1 (a) shows the color distributions for ISO/DIS 12641 and (b) for sine SPDs(1000 chips). The distributions of ISO/DIS 12641 are limited inside the gamut of photographic print and are not uniform. On the other hand, sine SPDs can cover wider gamut and may be effective test stimuli for digital cameras, but are not still distributed in uniform.

Here, we generated the virtual spectra with given CIELAB value $\mathbf{C_{LABn}}=[L_n^*, a_n^*, b_n^*]^t$ as the following steps.

[Step 1] Convert the CIELAB value $\mathbf{C_{LABn}}$ into CIE-XYZ tristimulus value $\mathbf{T}_n=[X_n, Y_n, Z_n]^t$.
[Step 2] Compute the fundamental spectrum $\mathbf{C}_n^*$ from $\mathbf{T}_n$.
[Step 3] Add a metameric black component $\mathbf{B}_m$ to $\mathbf{C}_n^*$. Thus, finally we get the color spectrum

$$\mathbf{C}_{m,n}=\mathbf{C}_n^*+\mathbf{B}_m \tag{14}$$

An input spectrum $\mathbf{C}_{m,n}$ is estimated by using pseudo-inverse transform from Eq. (4) as

$$\mathbf{C}_{m,n}=\mathbf{P_{INV}}\,\mathbf{T}_n+(\mathbf{I}-\mathbf{P_{INV}}\,\mathbf{A_L}^t)\mathbf{V} \tag{15}$$

where, $\mathbf{V}$ means an arbitrary vector. Here, we should notice the 1st term of Eq. (15) is equal to the fundamental metamer $\mathbf{C}_n^*$ and the 2nd term denotes a metameric black.

Thus the fundamental spectrum $\mathbf{C}_n^*$ is simply recovered from $\mathbf{T}_n$ using colorimetric pseudo-inverse projection operator $\mathbf{P_{INV}}$ given by

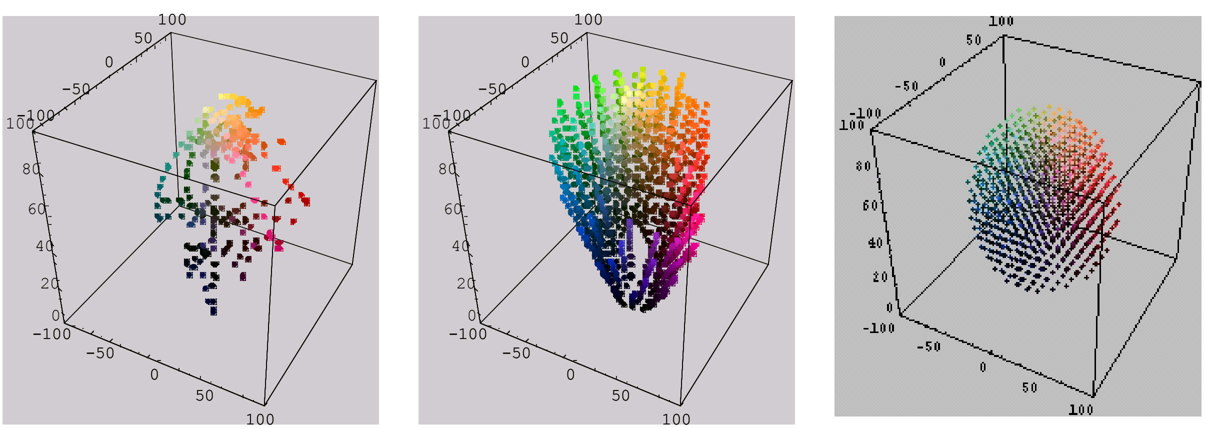
$$\mathbf{C}_n^*=\mathbf{P_{INV}}\mathbf{T}_n, \qquad \mathbf{P_{INV}}=\mathbf{A_L}(\mathbf{A_L}^t\,\mathbf{A_L})^{-1} \tag{16}$$

fundamental spectra $[\mathbf{C}_n^*]$; n=1~N are uniquely generated from a given set of $\mathbf{C_{LABn}}$.

First, $\mathbf{C_{LABn}}$; n=1~N fundamental chips are generated inside an ellipsoid limited by

$$\{(L_n^*-50)/L_0\}^2+\{(a_n^*)/a_0\}^2+\{(b_n^*)/b_0\}^2 \le 1 \tag{17}$$

For example, N=933 chips are generated in the range of $5 \le L_n^* \le 95$ and $-70 \le a_n^*, b_n^* \le 70$ by setting $L_0=45$, and $a_0=b_0=70$. This range will roughly correspond to the uppermost gamut of color hardcopies. The color distribution of these 933 fundamental LAB chips by the proposed method is shown in Fig.1 (c).



(a) ISO/DIS 12641 *(b) Sine SPDs* *(c) Proposed Uniform C_{LAB} chip*

Figure 1. Color distributions of test chips in CIELAB space

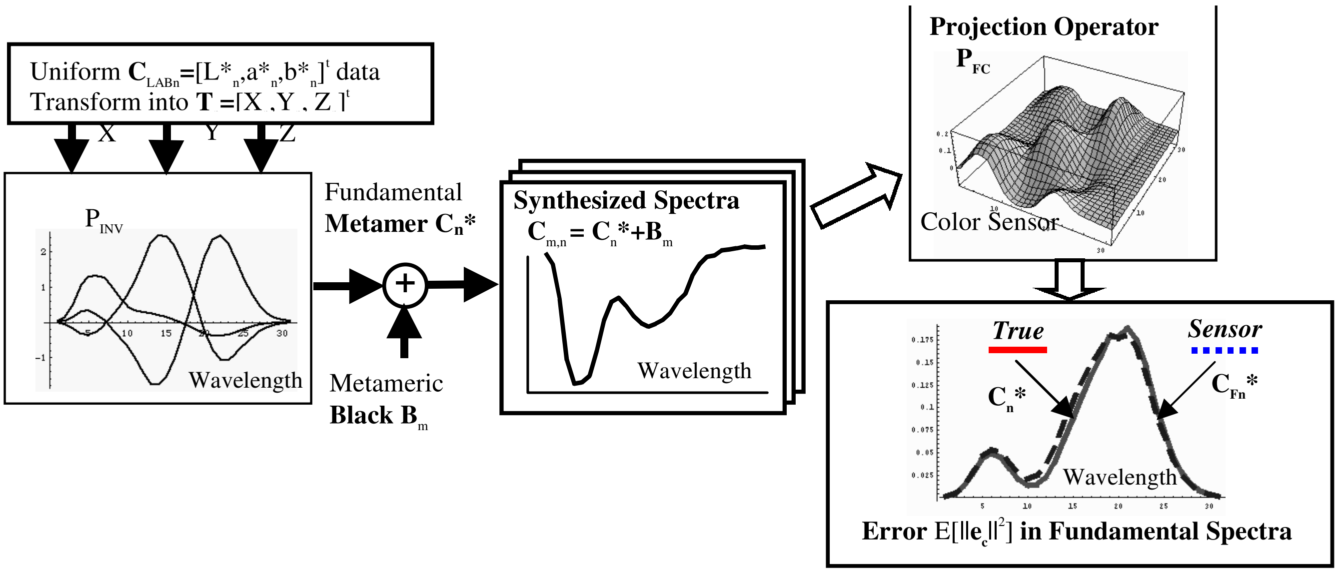


Figure 2. Flow diagram for testing the spectral response errors in color sensors

Figure 2 summarizes the flow diagram of proposed testing method.

On the other hand, a metameric black $\mathbf{B}_m$ is an arbitrary random vector with zero stimulus to human vision. However, the input devices make non-zero responses to $\mathbf{B}_m$ causing a colorimetric error. Then $\mathbf{C}_{m,n}$ including $\mathbf{B}_m$, should be used for testing. Although $\mathbf{B}_m$ can't be uniquely determined, we generated the following two types of metameric blacks $[\mathbf{B}_m]$; m=1~M.

[1] $\mathbf{B1}=[\mathbf{B1}_m]$ for gray inputs **:**

$$\mathbf{C1}_m=[g_i]^t\ ;\ g_i=0.1m,\ i=1\sim31,\ m=1\sim M1 \quad (18)$$

where, M1=10 **B1**s are generated by changing the level g_m by 0.1 step.

[2] $\mathbf{B2}=[\mathbf{B2}_m]$ for sine SPDs inputs **:**

$$\mathbf{C2}_m=[C_i^m]^t\ ;\ i=1\sim31,\ m=1\sim M2 \quad (19)$$

where, i-th element C_i^m of $\mathbf{C2}_m$ is given by λ_i component of sine SPDs with selected frequency f and phase ϕ as

$$C_i^m = \text{sine SPD}(\lambda_i, f, \phi)\ ;\ \lambda_i=400\sim700,\ i=1\sim31 \text{ by 10 nm step}$$
$$=(1/2)A[1+\sin\{2\pi f(\lambda_i-400)/10^3+(2\pi/20)\phi\}] \quad (20)$$

Here, M2=200 **B2**s are generated by changing (f, ϕ) as f=1,2,3,...,10(cycles/10^3nm), and ϕ=1,2,3,...,20, with the amplitude A=0.5. The chip index m is distinguished by the combinations of (f, ϕ).

The reason why sine SPD is selected as a metameric black source simply lies in the following features.

1. The raised sinusoidal wave gives positive and smoothed spectral shape and its change is limited by frequency f.
2. The input color hue is controlled by phase ϕ.

These features will be desirable as substitutes for metameric black source which real world spectra would have.

Because human vision has three color cone sensors, it is known to respond to the change of spectral shape at least up to f=1.5 cycles in 400~700nm wavelength range, that is f=5 cycles/10^3 nm for sine SPDs 8. Thus the maximum frequency f is limitted to 10 cycles/10^3nm, twice of 5 cycles/10^3. Figure 3 shows the plots of sine SPDs' loci in CIELAB color space for f=3, 5, 7, and 10 cycles/10^3nm as changing the phase ϕ=12,3,...,20. As clearly shown, each locus tracks a closed loop changing its color hues and becomes less sensitive for higher frequency than f=5 cycles/10^3nm.

Thus, two sets of uniform LAB chips are generated by adding the metameric black $\mathbf{B}_m$= $\mathbf{B1}_m$ or $\mathbf{B2}_m$ to $[\mathbf{C}_n*]$.

[**Set 1:** M1 × N=9,330 **virtual chips**]

$$\mathbf{C}_{m,n}=\mathbf{C}_n*+\mathbf{B1}_m\ ;\ m=1\sim10,\ n=1\sim933 \quad (21)$$

[**Set 2:** M2 × N=186,600 **virtual chips**]

$$\mathbf{C}_{m,n}=\mathbf{C}_n*+\mathbf{B2}_m\ ;\ m=1\sim200,\ n=1\sim933 \quad (22)$$

In **Set 1**, M1=10 metameric blacks are added to each $\mathbf{C}_n$*, generating totally M1 × N=9,330 chips, while in **Set 2**, totally M2 × N=186,600 chips. These two sets of virtual chips, however, include the same fundamental spectra $[\mathbf{C}_n*]$ and only N=933 different colors are distinguished to human vision.

Figure 4 shows an example of three different metameric black spectra, one from a gray input, other two from sine SPDs.

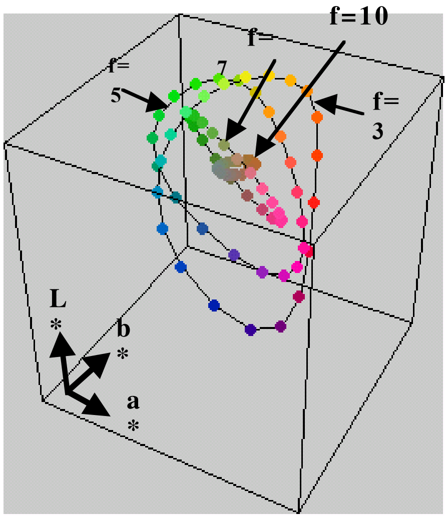
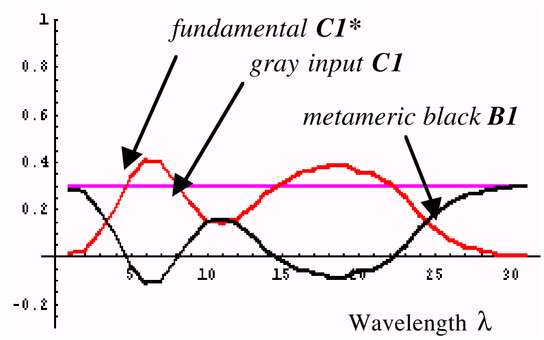
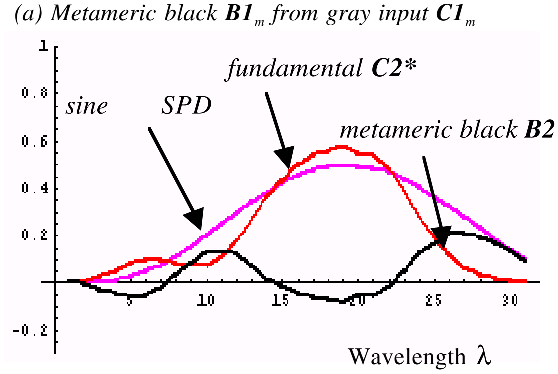


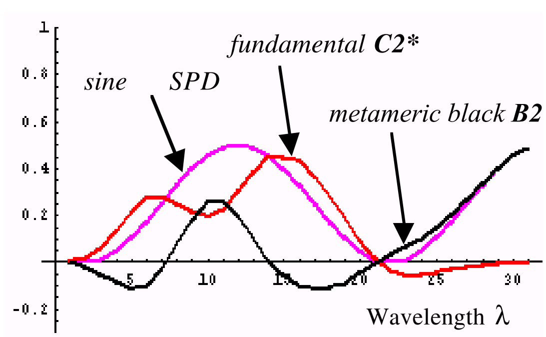
Figure 3. Spectral loci of sine SPDs vs. frequency f



*(a) Metameric black **B1**$_m$ from gray input **C1**$_m$*

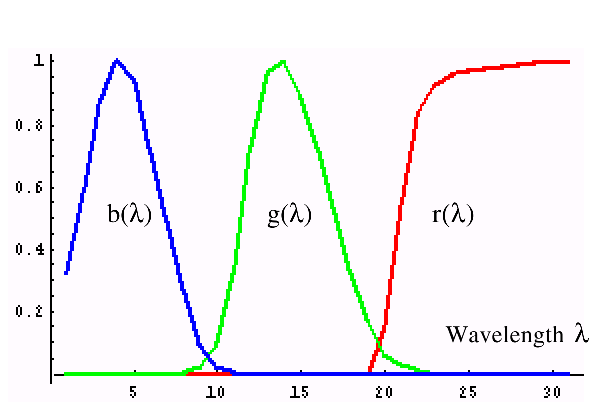


*(b) Metameric black **B2**$_m$ from sine SPD **C2**$_m$(f=3)*

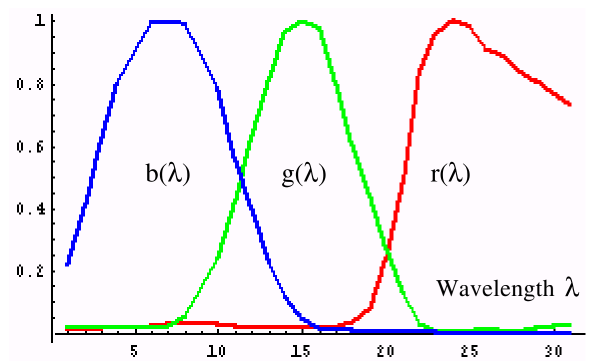


*(c) Metameric black **B2**$_m$ from sine SPD **C2**$_m$(f=5)*

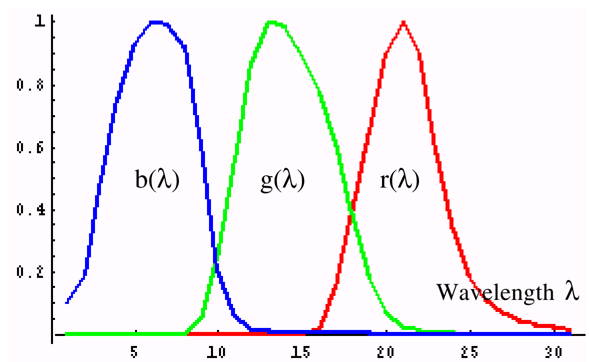
Figure 4. Examples of metameric black spectra used for testing



(a) Wratten filter set



(b) CCD RGB sensor



(c) RGB color scanner

Figure 5. Spectral sensitivities of tested color sensors

Results

The following three devices are tested in the experiments.

[1] Wratten filters (#25, #58, #47B)
[2] RGB 3-CCD camera
[3] Color Scanner

The spectral sensitivities are shown in Figure 5.

In general, the spectral sensitivities are different from ideal color matching functions (Luther condition). These differences can be reduced by applying a linear matrix operation (filter correction) on the tricolor signals. The linear matrix calculation methods for digital photography are discussed under the different illumination conditions and the color errors are evaluated using Macbeth Color Checker 7.

The sensor goodness should be measured after filter correction, because any linear combinations of color matching functions are also color matching functions.

Here the correction matrix $\mathbf{M_C}$ has been determined by the method of least squares as

$$\mathbf{M}_C=(\mathbf{A}_L{}^t\mathbf{RF})(\mathbf{F}^t\mathbf{RF})^{-1}, \quad \mathbf{R}=E[\mathbf{C}_{m,n}\,\mathbf{C}_{m,n}{}^t] \tag{23}$$

where $\mathbf{R}$ is the correlation matrix for $\mathbf{C}_{m,n}$. Matrix $\mathbf{R}$ reflects the statistical characteristics of the color chips to be matched by the linear transformation.

The projection operator $\mathbf{P_{Fc}}$ after filter correction in Eq. (5) is calculated by

$$\mathbf{P}_{Fc}=\mathbf{RF}(\mathbf{F}^t\mathbf{RF})^{-1}\,\mathbf{F}^t \tag{24}$$

Figure 6 shows how the tested sensors extract the fundamental metamers from an example of virtual spectrum with $\mathbf{C}_{LABn}$= [L_n*, a_n*, b_n*]^t =[70, -30, 20]^t and where the spectral error happens. CCD sensor causes larger spectral error than Wratten for this sample, though it works almost better than Wratten for ISO/DIS 12641 chips. This means the error comes from unwanted response to metameric black spectrum and virtual chips give more severe test than the real chips. While, color scanner works almost perfect after filter correction, because the spectral shape $\mathbf{F}$ is carefully designed as compared with that of CCD.

The spectral mean square errors $E[\|\mathbf{e}_c\|^2]$ after filter correction are analyzed for all of **Set 1** and **Set 2** virtual chips. The errors are graphically analyzed for metameric blacks by gray sources in Figure 7 (a) and by sine SPD sources in Figure 7 (b).

Table 1 summarizes the ensemble of these spectral errors for the sensors tested by ISO/DIS 12641, sine-SPDs and proposed $\mathbf{C}_{LAB}$ targets. $E[\|\mathbf{e}_c\|^2]$ and $E[\|\mathbf{e}\|^2]$ denote the results with and without filter correction.

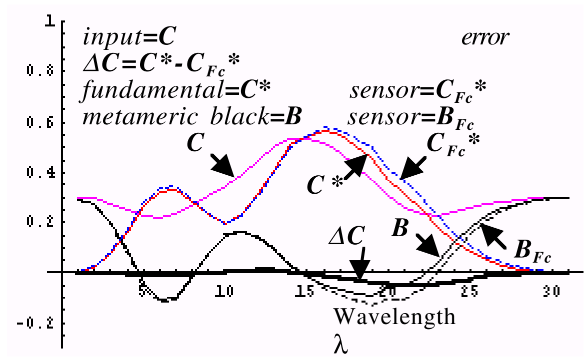
As clearly shown in Table 1 and Figure 7, the goodness of the tested three sensors is evaluated as

Color scanner > CCD sensor > Wratten filter

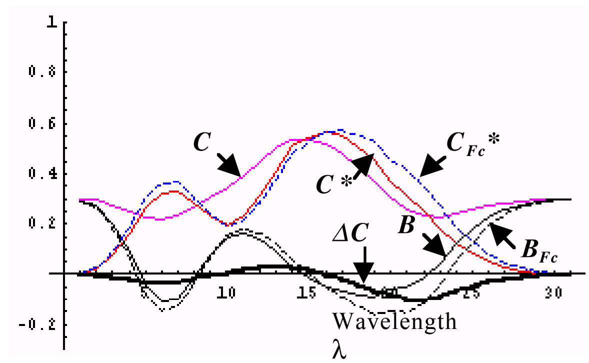
All the estimated values are consistent with the results by real chips and sine SPDs. The spectral sensing errors for both sine SPDs and proposed SPDs are almost the same order, but larger than that of real ISO/DIS 12641 target.

Discussion and Conclusion

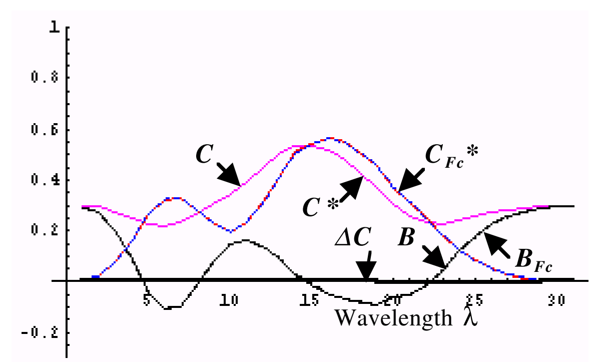
The design of spectral sensitivities of color image sensors is very important to guarantee the high definition image quality in total system. Here we discussed on the spectral errors in the input color sensor from a point of how it can extract the correct fundamental spectra. The spectral sensitivities of Wratten and CCD sensor seem to be intuitively mismatched to color matching functions, especially in reddish wavelength region. These undesirable spectral shapes make non-zero responses to metameric black spectra and increase the colorimetric sensing errors. Thus, the color sensors' error depends on the spectral shapes of input colors with same XYZ values but different metameric black spectra. However, ISO/DIS 12641 color chips, made of photographic material, have the smoothed spectral shapes without higher frequency components in their Fourier transform. This will be the reason why ISO/DIS 12641 chips bring the smaller metameric black errors than that of sine SPDs or proposed virtual SPDs.



(a) Wratten filter set



(b) RGB CCD camera



(c) RGB color scanner

Figure 6. Spectral response errors in tested color sensors

Furthermore, the color gamut of photographic chips are limited inside the small region, while the proposed virtual chips can be distributed in any desirable wider ranges. In

conclusion, the use of computer generated virtual color targets has the following advantages.
[1] Uniform distributions
[2] Tunable and wider color gamut
[3] Quick estimation without real chips.

However, the way to generate the metameric black spectra is not uniquely decided. Future works should be continued to find the better virtual test targets available for designing the high definition color sensors in real world.

References

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Table 1. Evaluation Results in Fundamental Spectral Errors

Sensors / Test Targets	Wratten Filter Set		RGB 3-CCD Camera		Color Scanner	
	$E[\lVert\mathbf{e}\rVert^2]$	$E[\lVert\mathbf{e}_c\rVert^2]$	$E[\lVert\mathbf{e}\rVert^2]$	$E[\lVert\mathbf{e}_c\rVert^2]$	$E[\lVert\mathbf{e}\rVert^2]$	$E[\lVert\mathbf{e}_c\rVert^2]$
ISO/DIS 12641	0.2795	0.0063	0.0639	0.0036	0.0418	0.0008
1000 Sine SPDs	0.3477	0.1790	0.1453	0.0920	0.0263	0.0039
Set 1 : 9330 LAB virtual chips	0.4847	0.2173	0.4075	0.1806	0.0467	0.0159
Set 2 :186600 LAB virtual chips	0.4511	0.1351	0.2265	0.1038	0.0263	0.0088

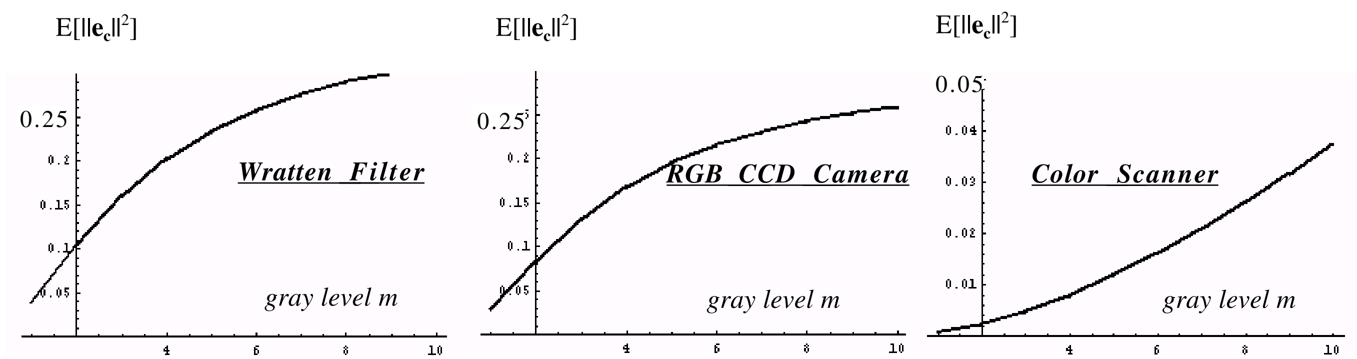

Figure 7 (a). Changes in spectral response errors vs. gray levels of metameric black sources ***B1***

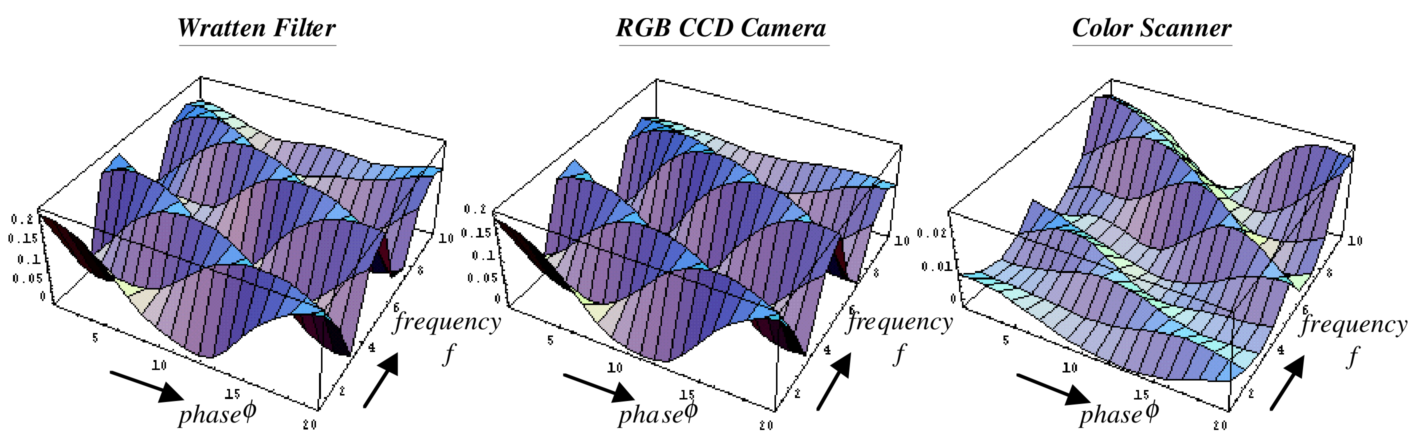

Figure 7 (b). Changes in spectral response errors vs. (f, ϕ) of metameric black sources ***B2(*** *sine SPDs)*