Digital Color Quantization Patterns in CIE Spaces and Artifacts from Misaligned Color Planes

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Abstract

The colors in digital images are necessarily constrained to quantized values by the digitisation process. This quantization of the CIE color spaces produces fundamental geometric patterns. Pixels from real digital images must lie within these patterns, resulting in geometric artefacts. These artefacts are intensified by errors in the sampling such as misalignment of the color planes. The shape of the artefacts may indicate the processing history of the image.

Introduction

A color plot of a digital image is produced by projecting the pixels into a linear color space such the CIE (r,g) chromaticity coordinate system [1]. When this is done, the color plots of real images exhibit a variety of geometric artefacts or patterns [2]. At first sight, these patterns are attractive and striking because of their geometry. Their practical use is that they may give insight into the processing history of the image.

This paper demonstrates the patterns with the color plot of the standard test image *lena*, then considers two types of pattern that arise in a color plot: the fundamental quantization pattern in every digital color image; and the effects of misaligned color planes in the image.

Method of Projection

An image is projected into color space by transforming the color of each of its pixels into the coordinates of the desired space. Points are usually plotted as solid dots, giving a scatter plot. Alternatively they can be displayed in greyscale or in pseudocolor with different values according to the number of pixels having each color, giving a density plot.

In a digitial image, each pixel is a triplet of numbers representing the quantized levels of three primary colors. The triplet is written (R',G',B') with the primes indicating that the color components are non-linear in light intensity, having been gamma corrected by the image capture system prior to being quantized and stored [3]. Each component R', G' and B' is typically a byte in the range from 0 to 255, resulting in a gamut of 16,777,216 colors.

The CIE RGB tristimulus values are linear in light intensity [1,4]. They are obtained by dividing each component by 255 and performing the gamma law on each component:

$$(R,G,B)=\left(\left(\frac{R'}{255}\right)^{\gamma},\left(\frac{G'}{255}\right)^{\gamma},\left(\frac{B'}{255}\right)^{\gamma}\right) \tag{1}$$

where gamma γ is the value estimated to undo the preceding gamma correction. In this notation, each R, G and B is in the range [0,1]. They may be scaled to [0,100] or any other range but such scaling will make no difference because they are normalized in the next step.

The CIE chromaticity coordinates (r,g) are obtained by normalising each tristimulus value R, G and B with respect to their sum:

$$(r,g) = \left(\frac{R}{R+G+B}, \frac{G}{R+G+B}\right) \tag{2}$$

Each chromaticity coordinate r and g lies in the range [0,1] and from their definition r+g must also lie in the range [0,1] so the CIE (r,g) chromaticity space is a triangle with corners at coordinates (0,1), (1,0) and (0,0) corresponding to red, green and blue respectively. Points can be plotted in this triangle or transformed by a linear mapping into the primary equilateral color triangle at coordinates:

$$(x_{\text{tri}},y_{\text{tri}})=\left(r+\frac{g}{2},\frac{\sqrt{3}}{2}g\right) \tag{3}$$

As an example, the color plot is shown for the standard lena test image [5]. The color image and its color plot are shown in Figure 1(a) and Figure 1(b) respectively. The artefacts in the color plot are visible clearly in the enlargement in Figure 2.

Figure 1(a). Standard lena image, full color.

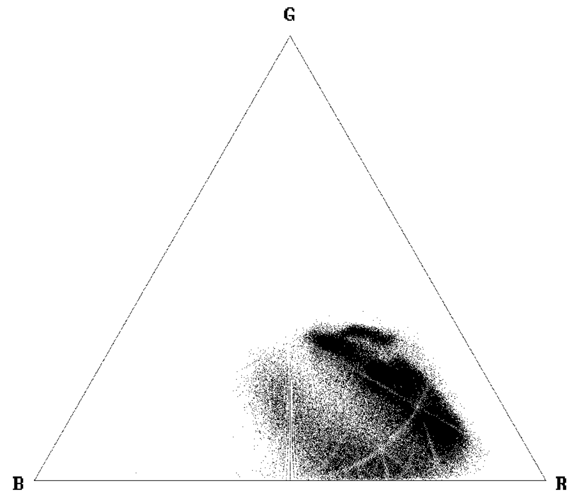


Figure 1(b). Color plot of lena.

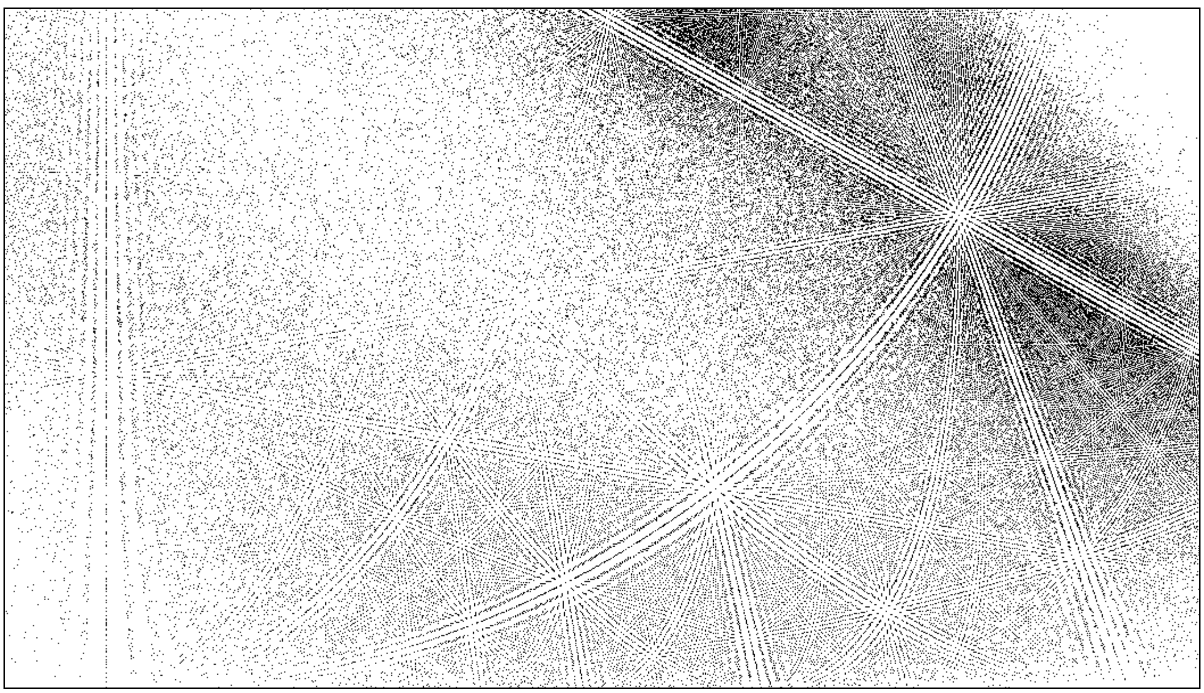


Figure 2. Enlargement of the color plot of lena.

The Fundamental Quantization Pattern

Every color plot will exhibit a fundamental pattern of points arising from the quantization of the three color components. In the process of digitizing an image, each component is quantized into a finite number of quantization levels. The type of quantization may be linear in intensity or non-linear in intensity, but either way, the number of levels will be finite. It follows that there will be a finite number of possible chromaticity coordinates that can be obtained by equation (2). This set forms the underlying quantization pattern in any color plot.

The values of chromaticity coordinates r and g are interrelated by equation (2) and the normalization by the denominator $R+G+B$ is non-linear. A change to the blue tristiumulus B will shift both the r and g chromaticities towards or away from their origins by a non-linear amount. Similarly R will affect g and G will affect r in non-linear ways. The quantization of R, G and B results in a sequence of lines or curves in r and g corresponding to each level.

It is impractical to draw the full plot for 256 quantization levels in this paper because there are too many points in the color gamut. With 16,777,216 colors, their coordinates are too close to be distinguished if printed here. Instead the shapes are seen by plotting fewer colors, such as the gamut of 32,768 colors from 32 quantization levels. Two such plots are shown. In Figure 3(a), the coordinates are plotted with no gamma correction, γ = 1.0. In Figure

3(b) the points are plotted for $\gamma = 2.0$. Their artefacts follow lines and circles respectively.

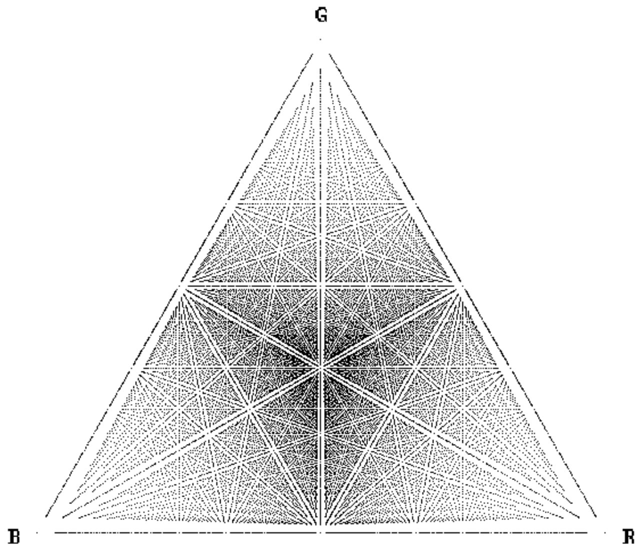


Figure 3(a). Fundamental quantization pattern for $\gamma = 1.0$ and 32 quantizer levels.

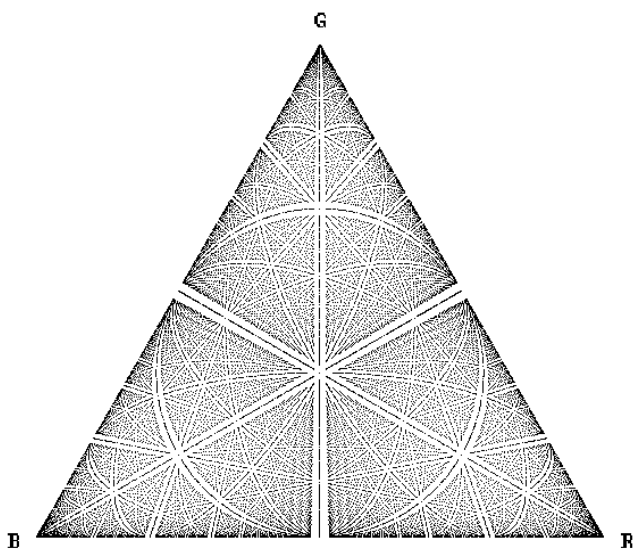


Figure 3(b). Fundamental quantization pattern for $\gamma = 2.0$ and 32 quantizer levels.

Patterns from Misaligned Color Planes

When a color image is scanned or captured, there is an assumption that the three primary color planes are exactly aligned. In practice this may not be the case. The effect of misaligned planes is to change the colors at the edges of objects. If the red plane is shifted left relative to the green and blue planes, each white object will have a red leading edge and cyan trailing edge.

The color errors are localized on the image because they occur at the boundaries of objects, highlighting their edges. When plotted in color space, the points may migrate a noticeable distance from their intended colors, in a clear geometric pattern. It would be interesting to investigate whether the type of misalignment can be determined from the geometry of the pattern.

To demonstrate the effect of misaligned planes, two images of colored spheres have been generated artificially. Both images contain four spheres, one green, one blue, one red and one gray. In the first image, the spheres exhibit diffuse reflection, so each point on each sphere reflects the same chromaticity with different luminance. In the second image, the spheres exhibit specular reflection, so each point on the sphere reflects a color mix of the sphere chromaticity and the white illuminant. These images are shown in Figures 4(a) and 4(b) respectively.

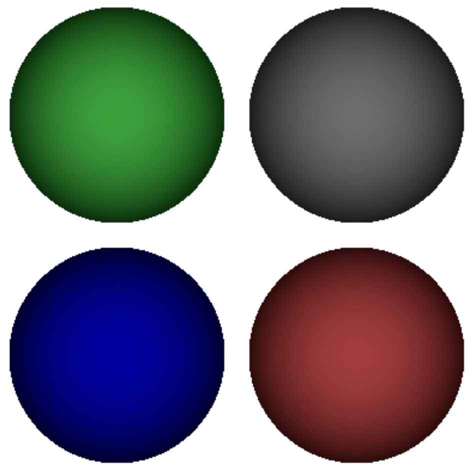


Figure 4(a). Coloured spheres with diffuse reflection.

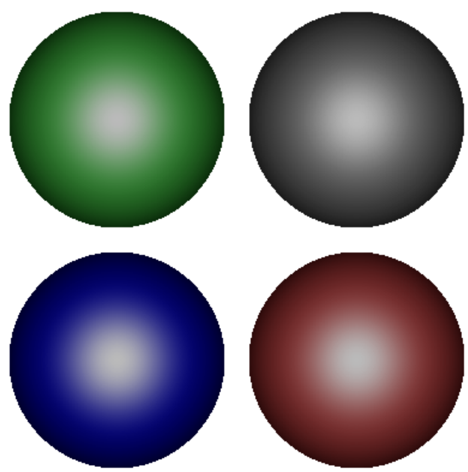


Figure 4(b). Coloured spheres with specular reflection.

The color plots of each set of 4 spheres, with perfect color plane alignment, are shown in Figures 5(a) and 5(b). The diffuse spheres each exhibit a small set of colors and the specular spheres exhibit a larger range of colors arising from mixing the illuminant with each sphere's color.

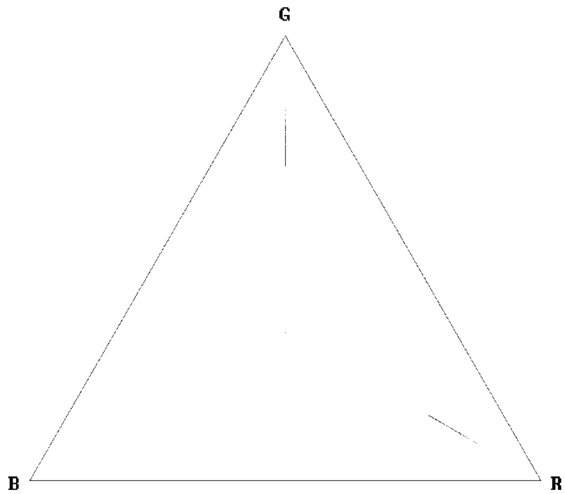


Figure 5(a). Color projection of diffuse spheres.

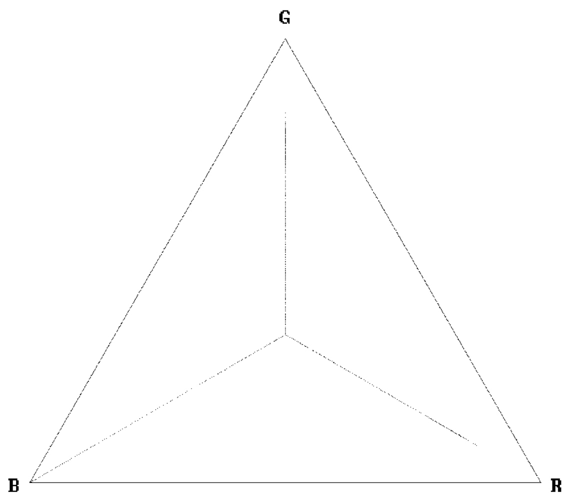


Figure 5(b). Color projection of specular spheres.

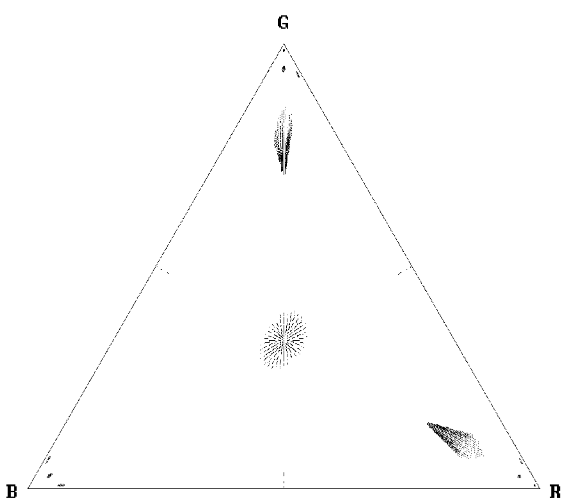
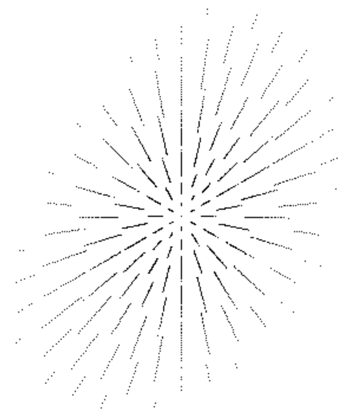
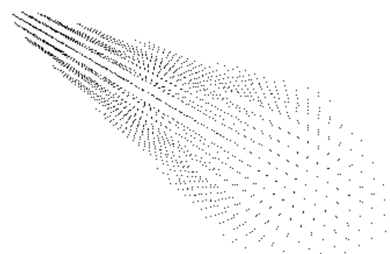


Figure 6(a). Color projection of diffuse spheres after the green plane components were shifted left and the blue plane components were shifted up.



Enlargement of central region of 6(a)



Enlargement of red region of 6(a)

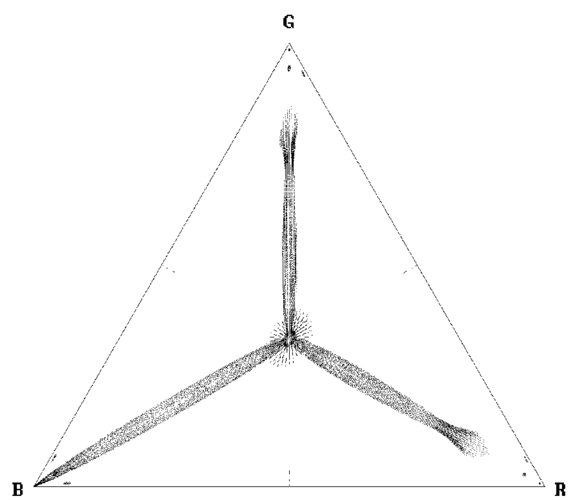
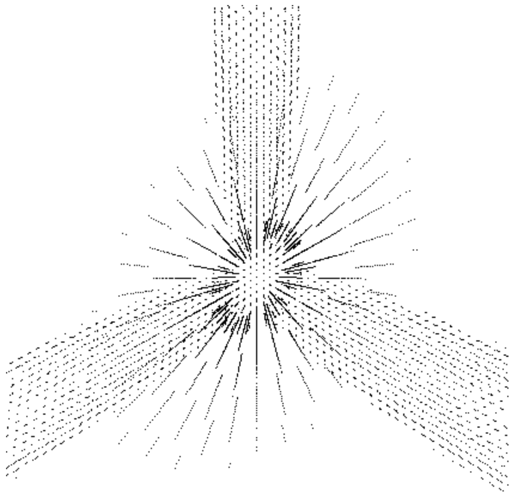
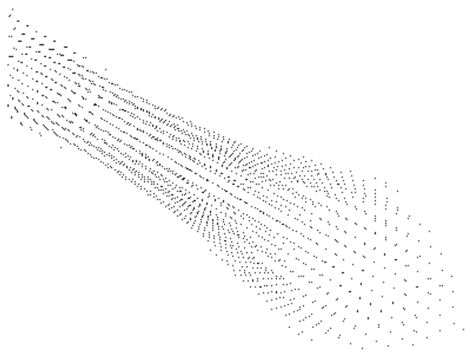


Figure 6(b). Color projection of specular spheres after the green plane components were shifted left and the blue plane components were shifted up.

When the color planes of the spheres are shifted, the points in the color plot migrate from each original point. The full plots are shown in Figures 6(a) and 6(b). In both cases, the green plane was shifted left and the blue plane was shifted up. Enlargments of the central and red portions of these plots display beautiful geometric shapes. The changes from the original plots appear similar to dilation operations in mathematical morphology [6] but the mask varies across the plot.



Enlargement of central region of 6(b)



Enlargement of red region of 6(b)

Conclusions

The quantization of colors in a digital image results in a geometric pattern in the CIE color spaces. The pattern from the standard test image *lena* has been shown. The plot contains an underlying or fundamental quantization pattern caused by the non-linear computation of the chromaticity coordinates.

The shape of the fundamental quantization pattern varies according to the gamma γ correction factor. Patterns have been shown for gamma $\gamma = 1.0$ and $\gamma = 2.0$.

The pattern from a digital image is plotted is permuted by image processing operations such as the digitisation errors introduced by misaligned color planes. Patterns arising from misaligned color planes were demonstrated using two artificial images of spheres, one with diffuse reflections and one with specular reflections. The effect of shifting the color planes is to spread out the points in the color space in a geometric way.

It would be interesting to investigate whether the type of misalignment or the processing history of the image can be determined from the geometry of the colour pattern.

References

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2. Keith R. L. Godfrey and Yiannis Attikiouzel, “Non-linear Quantisation Effects in Digital Colour Systems,” *First International Workshop on Wireless Image/Video Communications*, 63-7 (1996).
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