Color Separation for Four Color Printing

Werner Praefcke
Institut für Elektrische Nachrichtentechnik
Aachen University of Technology (RWTH)
52056 Aachen, Germany

Abstract

In most applications four color printing is used to enlarge the printable color gamut especially in the dark color area. In this work we give a motivation for applying four color printing processes in order to achieve better color accuracy over different illuminants. For this reason we introduce a hypothetical four-dimensional observer whose four-dimensional color space can be completely addressed with four color printing.

Finding the four colorant values of the printer for a desired four-dimensional color is done by color separation. We compare a color separation algorithm which is founded on the analytical inversion of the three-ink Neugebauer equations with a direct iterative inversion algorithm. Both approaches are shown to surely converge to the real valued colorant combination. The direct inversion requires no more than three iteration cycles.

1. Introduction

Four-ink printing is widely used to improve the quality of color reproductions. The most important aspect is the enlargement of the printable color gamut in regions of dark colors [1]. Furthermore by substituting the combination of cyan, magenta, and yellow ink in grey areas by black ink a reduction of ink costs can be achieved. However the aim is always the match of the print to a given color under a specific reference illuminant. Because of metamerism effects the match under different illuminants can not be guaranteed.

For a color match under arbitrary kinds of illuminants it would be necessary to reproduce the original reflectance spectrum. This is not possible, it would require a print process with an infinite number of colors to offer infinite degrees of freedom. Recently, there have been several approaches to introduce color imaging based on the spectral reflectance information. Although for the problems of image acquisition and storage there are proposals around, solutions addressing the practical reproduction of spectra are still demanded.

Nevertheless, the fourth color of a four-ink process can already be employed with the aim not only to enlarge the gamut but also to improve the spectral accuracy of the reproduction. This could be regarded as an intermediate but realizable step towards a multispectral solution for color reproduction.

1.1. Four-Dimensional Observer

In order to match the color of a given surface for N different illuminants there are N sets of illuminant weighted color matching functions for which a match is required. This would involve a printing technique with $3 \times N$ inks. To decrease this large number of inks to a number of four we have to replace the human visual system with its three cone sensitivity functions with an imaginary system of four properly chosen sensitivity functions.

A four-dimensional observer which catches the most information with regard to only the spectral characteristic of a spectra set can be derived by means of the principal component analysis. This approach disregards the varying sensitivity of the eye over the visible spectrum and is thus suboptimal. Another suggestion taking this into account can be found in [2]. Here the first three channels are identical to those of the human observer for a fixed illuminant, the fourth color channel is the main component in the remaining spectral space with regard to all other illuminants. An alternative approach is proposed in [3]. Following that method the four channels are oriented along the main axes of the $3 \times N$ dimensional spectral space with regard to a color space which is close to visual uniformity.

All approaches result in a set of four spectral sensitivity functions according to which a color match is required. This match can be achieved exactly with the four-ink process provided that the desired color falls inside the printable gamut. Furthermore it requires a *well-behaving* process to ensure that there are no multiple valid solutions in the colorant domain.

2. Inversion of a Four-Ink Neugebauer Process

The four-dimensional Neugebauer equations [1, 4] can be written in the following form.

$$
\begin{aligned}
x_1 &= k_0 + k_1c_1 + k_2c_2 + k_3c_3 + k_4c_4 + \\
&\quad k_5c_1c_2 + k_6c_1c_3 + k_7c_1c_4 + k_8c_2c_3 + k_9c_2c_4 + \\
&\qquad k_{10}c_3c_4 + \\
&\quad k_{11}c_1c_2c_3 + k_{12}c_1c_2c_4 + k_{13}c_1c_3c_4 + \\
&\qquad k_{14}c_2c_3c_4 + k_{15}c_1c_2c_3c_4 \\
&= f_1(k_i) \\
x_2 &= f_2(l_i), \quad x_3 = f_3(m_i), \quad x_4 = f_4(n_i) \qquad (1)
\end{aligned}
$$

The coefficients k_i, l_i, m_i, n_i are derived from the four-dimensional color coordinates of the process's primary colors and all combinations of their overprint. The four printer driving parameters c_1, c_2, c_3, c_4 are then converted to the color values x_1, x_2, x_3, x_4.

Since an analytical inversion of these four equations is not known we consider two iterative approaches. A first one reduces the equations to those of the three-ink Neugebauer process, which is analytically invertible. The second approach will invert the four nonlinear equations by means of a standard Newton technique.

2.1. Reduction to a Three-Ink Process

We fix one of the printer driving parameters, e.g. c_4, and derive a set of three-ink Neugebauer equations with coefficients K_i, L_i, M_i, N_i which depend on c_4.

$$
\begin{aligned}
x_1 &= K_0 + K_1c_1 + K_2c_2 + K_3c_3 + \\
&\quad K_4c_1c_2 + K_5c_1c_3 + K_6c_2c_3 + K_7c_1c_2c_3 \\
&= F_1(K_i) \\
x_2 &= F_2(L_i), \ x_3 = F_3(M_i), \ x_4 = F_4(N_i) \qquad (2)
\end{aligned}
$$

A set of three out of these four equations can be analytically inverted [5] leading to six solutions for the colorants c_1, c_2, c_3 as a function of given color values $x_{1,0}, x_{2,0}, x_{3,0}$. These can be either real or complex, the real solutions being inside or outside the colorant domain $c_i = [0 \ldots 1]$. Taking the colorant c_4 as a parameter for the three-ink Neugebauer equations the inversion renders six loci of possible solutions in the (c_1, c_2, c_3)-space. These move along as the parameter c_4 is varied, turning them into six curved lines. For each of the six solutions we take the parameter value of colorant c_4 to evaluate the fourth equation of eqs. (2). The squared error of this color coordinate $E = (\hat{x_4}(c_4) - x_{4,0})^2$ shows almost perfect quadratic dependency on the parameter c_4. Hence we observe a rather parabolic error function along each of the aforementioned six lines.

Each of these functions should be checked for its minimum (for its root) to find the solutions for the four color process. Each iteration step of the root search implies solving the three-ink Neugebauer equations which mainly requires a root search of a sixth-order polynomial. Since we aim for minimal computational cost for the whole algorithm and since only the real valued solution inside the colorant domain $c_i = [0 \ldots 1]$ is of interest (which we assume to be unique for a *well-behaving* process) we isolate the *most probable* of the six solutions and perform the root search only along its error function. The minimum can easily be found using a parabolic fit strategy like Brent's method [6] in very few iterations.

2.1.1. Selection of "Most Probable" Solution

During the optimization process and even for the final solution it is not impossible that individual colorant values c_i leave the colorant domain of $[0 \ldots 1]$. However it is essential to stay on the same of the six lines during the process in order to ensure a proper root search along its error function. Therefore we apply a simple strategy to find that colorant combination which is the *most probable* one even outside this range. It is distinguished by two characteristics. Firstly it is the real valued combination that has the smallest distance to the valid range. Furthermore those combinations are rejected that are all above or all below the valid range, in advantage for combinations that lie outside more symmetrically.

Although this algorithm is not as robust as the three-ink inversion method which securely finds all solutions it has proven to converge to the correct colorant values in all cases which were considered.

2.2. Direct Iterative Inversion of a Four-Ink Process

The second approach to invert the four-dimensional Neugebauer equations omits the reduction to the three-dimensional case thus avoiding the problem of deciding which is the *most probable* out of the six possible solutions. The Newton-Raphson method finds roots of nonlinear sets of equations by use of the function evaluation and evaluation of the function's derivative. The basic scheme is outlined as follows.

The four colorants are combined to a vector $\mathbf{c} = (c_1, c_2, c_3, c_4)^{\mathrm{T}}$, the four resulting color coordinates to a vector $\mathbf{x} = (x_1, x_2, x_3, x_4)^{\mathrm{T}}$. The Neugebauer equations describe the dependency $\mathbf{x} = \mathbf{x}(\mathbf{c})$. We can formulate the dependency of the target color vector $\mathbf{x}_t$ from any $\mathbf{c}_i$ as

$$
\mathbf{x}_t = \mathbf{x}(\mathbf{c}_i) + \mathbf{A}|_{\mathbf{c}_i} \cdot (\mathbf{c}_t - \mathbf{c}_i) + \mathbf{r}(\mathbf{c}_i) \qquad (3)
$$

with $\mathbf{A}|_{\mathbf{c}_i}$ being the Jacobian matrix of partial derivatives $a_{mn} = \frac{\partial x_m}{\partial c_n}$ at $\mathbf{c} = \mathbf{c}_i$ and with $\mathbf{r}(\mathbf{c}_i)$ being a term of higher polynomial order. Disregarding this term $\mathbf{r}(\mathbf{c}_i)$

leads to the linearization of the nonlinear equations and results in a simple iteration scheme for $\mathbf{c}_i$

$$\mathbf{c}_{i+1} = \mathbf{A}|_{\mathbf{c}_i}^{-1} \cdot (\mathbf{x}_t - \mathbf{x}(\mathbf{c}_i)) + \mathbf{c}_i. \qquad (4)$$

Necessary condition for the success of the Newton-Raphson method is a print process with a Jacobi matrix $\mathbf{A}|_{\mathbf{c}_i}$ which is non-singular at any valid colorant combination $\mathbf{c}_i$. Here the valid range for $\mathbf{c}_i$ must be regarded to be somewhat larger than the four-dimensional unit cube in order to obtain reasonable results even for colors which lie outside the physically printable gamut of the printer. In these cases the algorithm converges to negative colorant values or values above one which can be clipped or treated in another way afterwards. A further requirement is the presence of a unique solution in or nearby the valid colorant domain to ensure global convergence.

The Neugebauer print process that we used for our simulations is *well-behaving* in a sense that these necessary conditions are met and the algorithm converges to a unique solution independently from the initial colorant values $\mathbf{c}_0$.

In comparison there are multi color printers which apply only a subset of four colors out of a larger number of primaries at a time. Here it can be expected that the conditions are not guaranteed. This kind of approach is intended to enlarge the gamut of a printer by joining the gamuts of all the subset print processes. In our application we are not primarily concerned with the enlargement of gamuts but with a better spectral accuracy of the printed color. Therefore we always use all available colors in order to exactly match an equally dimensional color vector. The spectral characteristic of our primaries will be of rather regular kind in a sense that they will have one stop band in the visual range the center wavelengths being about equally spaced (see Figure 1). In this environment the printer model is expected to be *well-behaving* and to match input color coordinates with unique solutions at which the algorithm converges globally. Our experiments have always confirmed this assumption. The only effect of the initial values is the number of iterations needed for convergence.

3. Results

We tested both algorithms with a four-ink printer model based on reflectance spectra for its primaries which were derived in the context of [7]. There a simulated four-color printer based on the Neugebauer equations was provided with optimal reflectance functions regarding the reproduction under different illuminants. The shape of the paper white and the four primary reflectances for this process is depicted in Figure 1.

The four-dimensional observer was defined by the sensitivity functions derived as the first four analysis functions

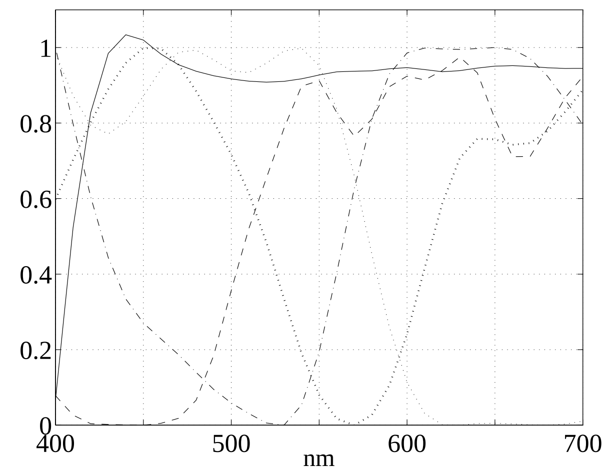

Figure 1: *Primaries of four-ink process. Solid: paper white; dashed/dotted: four inks.*

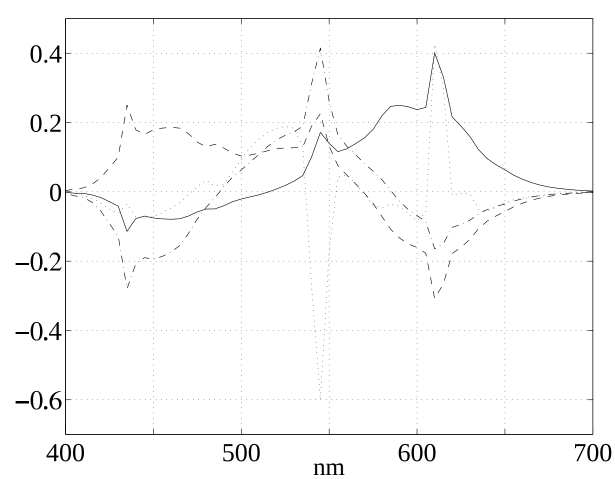

Figure 2: *Sensitivity functions of four-dimensional observer.*

	Iterations		
Spectra set	min	max	mean
Printer	4	11	7.84
Munsell	6	11	7.70
DuPont	6	11	7.93
Objects	4	11	7.85

Table 1: *Number of iterations needed for inversion using the reduction to a three-ink process.*

	Iterations		
Spectra set	min	max	mean
Printer	0	3	2.25
Munsell	1	3	2.31
DuPont	1	3	2.47
Objects	1	3	2.24

Table 2: *Number of iterations needed for inversion using the direct iterative inversion.*

for coding of reflectance spectra in [3]. They were constructed with respect to illuminants A, C, D65, Xe, and F11 and are shown in Figure 2.

The first experiment was carried out with the aim to match the colors again which were produced by the same considered printing process. We calculated the four-dimensional color values for colorant combinations of $c_i = 0.05, 0.5, 0.95$. The resulting 3^4 = 81 colors were then inverted to colorants again. A second experiment was based on the spectra set provided by Vrhel et al. [8] which is taken from 64 Munsell chips, 120 DuPont chips, and 170 natural objects.

Both the color separation methods were initialized with a colorant value of $c_4 = 0.5$ for the method using the reduction to a three-ink process and $c_1 = c_2 = c_3 = c_4 = 0.5$ for the direct inversion respectively. The numbers of iterations for each of the spectra sets regarding the two color separation methods are given in Tables 1 and 2.

For the first strategy even without any sophisticated initialization the average number of iterations is around 8. Therefore the computational expense for one color separation is due to only around 8 root evaluations of a polynomial of sixth order. In [5] the “Laguerre” algorithm is suggested for this purpose.

The direct inversion method requires three or less iterations for convergence, each one mainly consisting of a computation of the Jacobian matrix and a 4 by 4 matrix inversion. The number of iterations is very insensitive to changes of the initial value, it does not exceed the number of 3 for starting values of $c_1 = c_2 = c_3 = c_4$ in the interval $[0.45, 0.65]$. For a hardware application it is possible to implement the algorithm especially designed for three iterations thus leading to a constant computation time for the separation.

4. Conclusion

Four-ink printers can be employed to improve the spectral accuracy of a reproduction with respect to changing viewing illuminants. For this reason a four-dimensional observer was introduced whose color sensation can be exactly matched with a four-ink printer following the Neugebauer model. A color separation strategy was suggested in form of a combination of analytical and iterative inversion methods. This was compared to a direct inversion of the four-ink process using a Newton strategy.

As far as *well-behaving* printer processes are concerned which are distinguished by unique colorant combinations for given colors the proposed color separation strategy proved its robustness. It could be shown that with the present four-ink technology a spectrally orientated color reproduction can be pursued with only small computational complexity for the color separation.

References

[1] H. R. Kang. *Color Technology for Electronic Imaging Devices*. SPIE, Bellingham, Washington, 1997.

[2] T. Keusen. Multispectral color system with an encoding format compatible to the conventional tristimulus model. *J. Imag. Sci. and Techn.*, 40(6):510–515, November 1996.

[3] W. Praefcke. Analysis-synthesis transforms versus orthogonal transforms for coding reflectance spectra. In *Proc. Color Imaging Conference*, pages 177–181, Scottsdale, Arizona, USA, November 1997.

[4] A. C. Hardy and F. L. Wurzburg. Color correction in color printing. *J. Opt. Soc. Am.*, 38(1):300–307, April 1948.

[5] M. Mahy and P. Delabastita. Inversion of the Neugebauer equations. *Color Res. Appl.*, 21(6):404–411, December 1996.

[6] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. *Numerical Recipes in C*. Cambridge University Press, Cambridge, MA, 2nd edition, 1992.

[7] W. Praefcke. Quality limits in color printing with more than three primary colors. In *Proc. EUSIPCO*, Rhodes, Greece, September 1998.

[8] M. J. Vrhel, R. Gershon, and L. S. Iwan. Measurement and analysis of object reflectance spectra. *Color Res. Appl.*, 19(1):4–9, February 1994.