Halftone Color: Diffusion of Light within Paper

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Abstract

The diffusion of light within paper can affect the color of a halftone print, an effect known as optical dot gain or the Yule-Neilson effect. A theory is presented that allows one to determine the tristimulus values of a halftone color in terms of the halftone microstructure including the effects of light diffusion. The formula presented is a generalization of the Neugebauer equations.

Introduction

The halftone color is a partitive mixture of the colors of the halftone microstructure – the colors of the inks, the various ink combinations, and the white paper.[1] The colors contributing to the partitive mixture, and the amount of each of these colors, can be affected by the diffusion of light within the paper. This article reports on a theory relating halftone color to the halftone micro-structure that includes the effects of light diffusion within the paper.[2]

A widely used model of the halftone print is the Neugebauer equations, which relate the colors of the micro-structure to the color of the region by assuming that the amount of each color contributing to the partitive mixture is equal to the fractional area covered by each color. This linear assumption, however, is not valid if there is significant migration of photons within the paper as a result of scattering.[3, 4] Because of scatter, a photon may exit the paper at a point different from the point at which it entered the paper. The effect this diffusion has on the halftone color is often called optical dot gain, or the Yule-Nielsen effect.

In the theory presented, the tristimulus values of the halftone color are expressed as the trace of the product of two matrices – one matrix that expresses the different colors of the micro-structure and is a function of the ink transmittances and the paper reflectance; and the other a matrix that expresses the amount of each of these colors to contribute to the partitive mixture. The relative amount of each color is equal to the probability for the scattering process that gives rise to that color. These probabilities are calculated in terms of the dot shapes and sizes and in terms of the photon migration within the paper. The theory is a generalization of the Neugebauer equations; in the absence of scattering these results reduce exactly to the Neugebauer equations.

The degree of optical dot gain is characterized by the the lateral scattering length $\bar{\rho}$, which is the average distance a photon diffuses within the paper. If $\bar{\rho}$ is comparable to the screen period or to the size of the dots, then optical dot gain can be quite significant. Typically, $\bar{\rho} \sim 0.1$ mm, so for a 150 lpi screen, the ratio of scattering length to screen period r is: $\bar{\rho}/r \sim 0.6$.

Colors of the Microstructure

For simplicity, an AM halftone consisting of two inks is modeled, although the theory can easily be extended to any number of inks.[2] The bichromatic halftone microstructure consists of four regions – areas covered only by ink1, areas covered only by ink2, areas covered by the overlap of ink1 and ink2, and areas where there is no ink. Figure 1 shows the halftone microstructure for two inks.

Without scattering, there are four colors contributing to the partitive mixture: the color associated with each region. With diffusion, new colors contribute to the mixture. The ink layer can be considered a filter, and the light is transmitted through it twice: on incidence and on reflection. With no diffusion, the light passes through the same region on both incidence and reflection. With diffusion, however, the light may exit from a region different than that through which it is incident. Each of the colors produced can be labeled by a pair of indices indicating the regions of the microstructure through which it is transmitted on incidence and on reflection: 0 = bare paper, white; 1 = ink1; 2 = ink2; 3 = overlap of ink1 and ink2. For example, the color 13 is the color that results if light is transmitted through ink1, diffuses, and is then transmitted through the overlap of ink1 and ink2. The labels are symmetric.

The different colors of the microstructure – the micro-colors – can be arranged in a 4×4 symmetric matrix, with the diagonal elements the colors that obtain when there is no diffusion, and the off-diagonal elements "new" colors, colors that arise due to diffusion. Figure 2 shows the scattering processes that give rise to the off-diagonal elements.

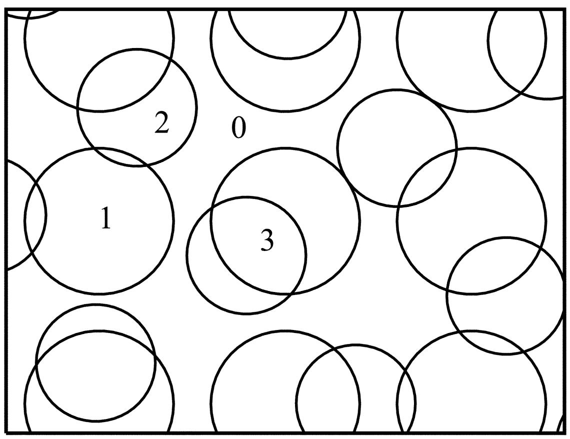

Figure 1. The halftone microstructure for two inks. There are four regions: ink1, ink2, overlap of ink1 and ink2, and white.

Three matrices express the micro-colors: $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$, the elements of which are the micro-color X, Y, Z tristimulus values. The elements of the matrix $\mathbf{X}$ are:

$$\mathbf{X}_{nm} = \int S(\lambda)\mathbf{t}_n(\lambda)\mathbf{t}_m(\lambda)R_p(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda,$$

where $S(\lambda)$ is the spectral power distribution of the illumination, R_p is the reflectance of the paper, $\bar{x}(\lambda)$ is a color matching function, and the transmittance vector $\mathbf{t}$ is:

$$\mathbf{t} = [\ 1, \quad \tau_1, \quad \tau_2, \quad \tau_1\tau_2\ ],$$

with τ_n the transmittance of ink n. There are similar expressions for $\mathbf{Y}$ and $\mathbf{Z}$.

Halftone Color

The halftone color is obtained by summing over the micro-colors, weighted by the relative amount of each micro-color. The amount of each of the micro-colors to contribute to the partitive mixture is given by scattering probability – the probability for the scattering process that gives rise to the micro-color. The scattering probabilities depend on the size and shape of the dots, on the statistics of dot locations, and on the degree of diffusion within the paper.

The scattering probability $\mathbf{P}_{nm}$ is the joint probability that a photon enters the paper through region n and exits the paper through region m. The scattering probabilities are elements of a 4×4 symmetric matrix $\mathbf{P}$. If there is no diffusion, the matrix is diagonal. The $\mathbf{P}_{nm}$ are expressed in terms of the fractional ink coverages, μ_n, and the dot-dot probability P_n: the joint probability that a photon enters through a dot of ink n and exits through a dot of ink n. Calculation of the $\mathbf{P}_{nm}$ are outlined in the next section.

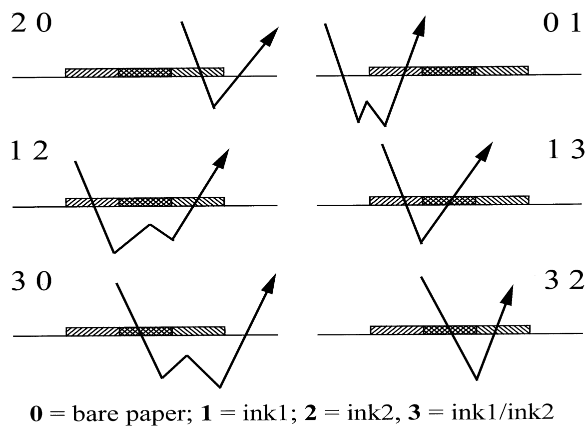

Figure 2. The scattering processes that give rise to the off-diagonal elements.

The tristimulus values of the halftone color are then given by:

$$X = \mathrm{Tr}\big[\mathbf{PX}\big], \quad Y = \mathrm{Tr}\big[\mathbf{PY}\big], \quad Z = \mathrm{Tr}\big[\mathbf{PZ}\big], \quad (1)$$

where Tr[] indicates the trace. These equations are a generalization of the Neugebauer equations; they reduce identically to the Neugebauer equations in the absence of diffusion. Figures 3 and 4 show the predicted differences in color between the cases with and without diffusion. Yellow and magenta inks are used, with the magenta held at a constant fractional coverage of 0.5, and the yellow coverage varied from 0 to 1. In both figures the scattering length is $\bar{\rho} = 0.15r$, with r the screen period. Figure 3 shows the Lab* (no diffusion) gamut for the two inks, and the locus of points as and μ_{yellow} varied from 0 to 1 ($\mu_{\mathrm{magenta}} = 0.5$). Figure 4 shows DE*, DL*, DC*, and DH* as a function of μ_{yellow}. Even though the scattering length is rather small, one sees that the degree of optical dot gain is significant.

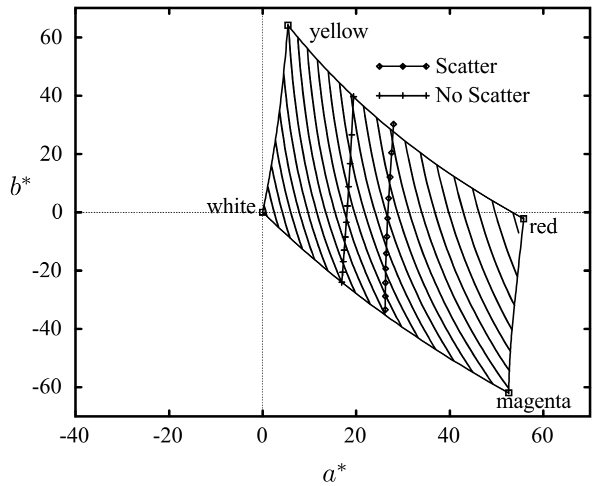

Figure 3. Lab gamut of the yellow and magenta inks and the colors obtained with* $\mu_{\text{magenta}} = 0.5$ *and* μ_{yellow} *varied from 0 to 1. Contour lines are lines of constant L* spaced 2.5 units apart, and* $\bar{\rho} = 0.15r$.

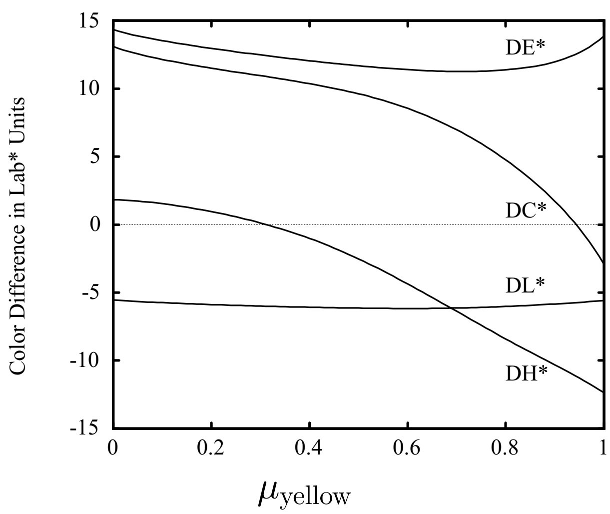

Figure 4. Color differences between scatter and no scatter. $\mu_{\text{magenta}} = 0.5$ *and* μ_{yellow} *varied from 0 to 1, and* $\bar{\rho} = 0.15r$.

Scattering Probabilities

The scattering probabilities depend on the size and shape of the dots, the statistics of dot positions, and on the degree of diffusion of photons within the paper. In the following, it is assumed that the screen angle is such that the locations of the dots of ink1 are effectively random with respect to the dots of ink2.[1]

At the core of the calculation of the $\mathbf{P}_{nm}$ are the bracket integrals – complete integrals of the array functions and the point-spread function (PSF). The array function, $\mathcal{C}_n(x,y)$, depends on the dot shape, size and location: it is equal to 1 if there is ink n at x, y, and equal to zero otherwise. The normalized PSF, $H(x-x', y-y')$, is the conditional probability density that if a photon enters the paper at position x', y', it will exit the paper at x, y. The matrix of bracket integrals is:

$$\mathbf{H} = \begin{bmatrix} \langle H \rangle & \langle H\,\mathcal{C}_1 \rangle & \langle H\,\mathcal{C}_2 \rangle & \langle H\,\mathcal{C}_1\mathcal{C}_2 \rangle \\ \langle \mathcal{C}_1\,H \rangle & \langle \mathcal{C}_1\,H\,\mathcal{C}_1 \rangle & \langle \mathcal{C}_1\,H\,\mathcal{C}_2 \rangle & \langle \mathcal{C}_1\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle \\ \langle \mathcal{C}_2\,H \rangle & \langle \mathcal{C}_2\,H\,\mathcal{C}_1 \rangle & \langle \mathcal{C}_2\,H\,\mathcal{C}_2 \rangle & \langle \mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle \\ \langle \mathcal{C}_1\mathcal{C}_2\,H \rangle & \langle \mathcal{C}_1\mathcal{C}_2\,H\,\mathcal{C}_1 \rangle & \langle \mathcal{C}_1\mathcal{C}_2\,H\,\mathcal{C}_2 \rangle & \langle \mathcal{C}_1\mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle \end{bmatrix},$$

where the bracket notation is:

$$\langle\, U\,H\,V\,\rangle = \frac{1}{Area}\int\!\!\int U(x,y)\,H(x-x', y-y')\,V(x',y')\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}x'\,\mathrm{d}y'.$$

The bracket integrals give the probabilities that a photon enters and/or exits the dots: for example, the bracket integral $\langle \mathcal{C}_1\,H\,\mathcal{C}_1 \rangle$ is the probability that a photon both enters and exits a dot of ink1, which is the dot-dot probability P_1. The bracket integral $\langle H\,\mathcal{C}_1 \rangle$ is the probability that a photon enters through a dot of ink1.

It is important to distinguish between *dots* and *regions*. In the bichromatic print, there are two dots and four regions. The probability that a photon enters a dot n and exits a dot n is not the same as the probability that it enters and exits a region covered by ink n alone. The matrix of bracket integrals gives the probabilities for entering and exiting *dots*; the matrix must be linearly transformed to obtain the scattering probabilities – the probabilities for entering and exiting *regions*. The transformation matrix is:

$$\mathbf{u} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

and the $\mathbf{P}_{nm}$ are given by:

$$\mathbf{P} = \mathbf{u}^T\,\mathbf{H}\,\mathbf{u},$$

with $\mathbf{u}^T$ the transpose of $\mathbf{u}$.

$\langle H \rangle$	1
$\langle H\,\mathcal{C}_1 \rangle$	μ_1
$\langle H\,\mathcal{C}_2 \rangle$	μ_2
$\langle H\,\mathcal{C}_1\mathcal{C}_2 \rangle$	$\mu_1\,\mu_2$
$\langle \mathcal{C}_1\,H\,\mathcal{C}_2 \rangle$	$\mu_1\,\mu_2$
$\langle \mathcal{C}_1\,H\,\mathcal{C}_1 \rangle$	P_1
$\langle \mathcal{C}_2\,H\,\mathcal{C}_2 \rangle$	P_2
$\langle \mathcal{C}_1\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle$	$\mu_2\,P_1$
$\langle \mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle$	$\mu_1\,P_2$
$\langle \mathcal{C}_1\mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle$	$P_1\,P_2$

Table 1: Bracket Integrals for Two Inks

Consider for example the matrix element $\mathbf{P}_{23}$, which using the above formula is given by:

$$\mathbf{P}_{23} = \langle \mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle - \langle \mathcal{C}_1\mathcal{C}_2\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle$$
$$= \langle \mathcal{C}_2(1-\mathcal{C}_1)\,H\,\mathcal{C}_1\mathcal{C}_2 \rangle.$$

$\mathcal{C}_1\mathcal{C}_2$ is equal to 1 for regions covered by both inks, and zero everywhere else. $\mathcal{C}_2(1-\mathcal{C}_1)$ is equal to 1 for regions covered by ink2 and *not* covered by ink1, and zero everywhere else. The right side of the above equation gives the probability that a photon enter a region covered by both ink2 and ink1, and exit a region covered only by ink2. This is precisely $\mathbf{P}_{23}$.

It should be noted that since the PSF is normalized:

$$\sum_{n,m} \mathbf{P}_{nm} = 1.$$

The array functions are given by a convolution of the dot distribution functions, $g_n(x,y)$, and the dot shape function, $\omega_n(x,y)$:

$$\mathcal{C}_n(x,y) = g_n(x,y) * w_n.$$

The distribution functions are sums of delta-functions that pick out the cell centers, and the shape functions, defined within a cell, are equal to 1 over portions of the cell covered with ink n, and zero otherwise.

The bracket integrals have been calculated[2] for the case of random dots, i.e. the dots of ink1 effectively random with respect to the dots of ink2.[1] The results are given in Table 1, where μ_1, μ_2 are the fractional coverage of ink 1 and ink 2, and P_1, P_2 are the dot-dot probabilities. The dot-dot probabilities are given by:

$$P_n = \mu_n^2 \sum_{ij} |\mathcal{J}_n^{ij}|^2\,\tilde{H}_{ij}$$

where $\mathcal{J}_n^{ij}$ and $\tilde{H}_{ij}$ are the Fourier transforms of the dot shape function and the PSF respectively, evaluated at $\sqrt{i^2+j^2}/r$. Other expressions for the dot-dot probabilities have been obtained.[5, 6]

Conclusion

A theory has been presented that generalizes the Neugebauer equations to include the effects of light diffusion within paper. Due to light diffusing within the paper, there are more colors of the micro-structure contributing to the partitive mixture than there are without diffusion. The amount of each color contributing is equal to the probability for the scattering process that gives rise to that color.

References

[1] G.L. Rogers, "Neugebauer revisited: random dots in halftone screening," *Color Res. Appl.* **23**, 104-113 (1998)

[2] G.L. Rogers, "Effect of light scatter on halftone color," *J. Opt. Soc. Am. A* **15**, 1813-1821 (1998)

[3] J.S. Arney, C.D. Arney, and P.G. Engeldrum, "Modeling the Yule-Nielsen halftone effect," *J. Imaging Sci. Technol.* **40** 233-238 (1996)

[4] G.L Rogers, "Optical dot gain in a halftone print," *J. Imaging Sci. Technol.* **45**, 643-656 (1997)

[5] J.S. Arney, "A probability description of the Yule-Nielsen effect," *J. Imaging Sci. Technol.* **41**, 633-640 (1997)

[6] G.L. Rogers, "Optical dot gain: lateral scattering probabilities," *J. Imaging Sci. Technol.* **42**(4) (1998)