# C olorant Selection for Six-C olor Lithographic Printing 

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#### Abstract

Two criteria for selecting ink sets for six-color lithographic printing are discussed. The first is the volume, in CIELA B space, of an approximation of the gamut. The second is the number of pixels within a digital image (or images) whose colors are contained within the approximate gamut.

In order to obtain a balance between accuracy and efficiency, the different steps in the process are prioritized according to their impact on accuracy, their requirements for specialized measurements, and the computational load they create. Factors which have a first-order impact on accuracy are measured, while factors with higher-order, less controlling, impact are computed or estimated. $O$ verprints of two or more colorants are estimated using a spectral model, rather than measured from printed samples. C ombinations of inks on the gamut surface which contain intermediate amounts of inks are estimated using a spectrally-sharpened wideband model. The estimation is performed in a space in which subtractive colorants behave more linearly than density.

A convex hull of the gamut is constructed in the linear space, using Computer G raphics routines. Through repeated bisection, the surface of the gamut is sampled in the linear space. The coordinates of the vertices are then transformed into CIELA B color space, for display, comparison, and analysis.


## Introduction

Recent interest in so-called hi-fidelity printing, coupled with overcapacity on six-color printing presses, has resulted in a number of options for increasing the printing gamut. Several of these options are based on the use of col orants in addition to the traditional Cyan, Magenta, Yellow, and Black. Printers, print-buyers, and creative personnel are faced with the question, "W hich set of colorants is best?"

Because of the large number of inks ink manufacturers routinely produce, this is a difficult problem to solve using "cut and try" methods. For example, if we have 16 chromatic inks, plus Black, there are 12376 possible combinations of 6 ink sets, and 4378 combinations of five chromatic inks plus Black. The method suggested by Boll, [1] for example, assumes that a set of colorants has al ready been chosen. The
technique developed by Stollnitz is quite elegant, and may be used to select a colorant set, [2] but it requires special ized measurements (though these are few in number), and is compute-intensive. A technique based on efficient computation with minimal reliance on measurements of actual printed samples, is highly desirable.

## The Problem of Six Inks

M uch work has been done with seven-ink printing. [3, e.g.] To the traditional Cyan, M agenta, Yellow, and Black, one may add inks intermediate in hue, viz, Red, Green, and Blue. This is effective, because the grayness of the $C$ yan and (especially) M agenta inks makes it difficult to produce good $G$ reens with the traditional inks, and even more difficult to produce good Reds and Blues.

A nother approach for seven-ink printing involves a double-hit of Cyan, Magenta, and Yellow. This increases their density, and, usually the size of the gamut.

H owever, there are more six-color presses than there are seven. The solution for a six-ink set is less obvious, and does depend, to some extent, upon the image to be reproduced. To the traditional ink set we may add Red and Blue, or Red and $G$ reen, or $G$ reen and Blue, but which of these alternatives to choose is not obvious. Further, the six-ink case (if it includes Black) is more sensitive to interaction between the colorants because two-thirds of the spectrum shall be effectively covered twice, while one-third shall be covered only once.

## G amut Figure-of-M erit

This paper uses two criteria for answering the question of optimal ink set. The first is the volume, in the CIELA B color space, of the gamut. This criterion is image-independent. It is intended to provide a solution for a wide range of original images. The second criterion is the fraction of colors within a digital image which are contained in the gamut. This criterion is image-dependent. It has the advantage over the former criterion of counting only relevant colors within the gamut. However, it is intimately tied to a particular image.

## The $\mathbf{N}$ eed for A ccuracy versus the $\mathbf{N}$ eed for C onsistency

W hen comparing two or more gamuts, it should be kept in mind that the process is comparative. If some small, systematic errors are made, it should not affect the ultimate conclusion (which ink set is best) if the errors affect all ink set calculations in the same manner and magnitude. Therefore, we argue that it is more important in these cal-
culations to be consistent, rather than to be strictly accurate.

Some factors in the gamut size computations have a profound influence on the result. These factors include the color of the paper and the colors of the individual inks printed on this paper. We refer to these as first-order factors. Because of their profound influence on the results, care should be taken to see that they are measured accurately.

Some factors have a significant effect on the accuracy of the results, though not nearly as profound as the first order factors. These factors include the color of two-, three-, four-, and higher-order overprints. We refer to these as sec-ond-order factors. W hile it is desirable to handle them accurately, it should not be necessary to treat them with the same level of care as the first-order factors.

Finally, some factors have less influence on the accuracy than the second-order factors. These include the behavior of the colorants between the gamut-limiting colors, or the corners of the gamut. W hile this behavior does have a significant influence on the actual size of the gamut, our postulate is that it affects prints produced under similar conditions by a similar amount. Thus, for the purposes of comparing gamut size, these higher-order factors are much less important than the others. Because accounting for these factors empirically would require a large number of measurements, and they have little influence on the ultimate decision of optimal ink set, the effect of these factors shall be modeled, rather than measured.

We recognize that more accuracy is needed when actually performing color-to-colorant conversions, generating ICC profiles, etc. H owever, the goal here is to perform some efficient calculations to determine which colorant sets merit a more detailed look, including the preparation of calibration targets, special press runs, and more accurate calculations.

## A Linear Space (A pproximately)

If printing inks are assumed to be optically non-scattering and non-fluorescent, an appropriate correction is performed for surface reflections, and the inks do not penetrate significantly into the paper, the subtractive printing process will behave according to the Beer-Bourger-Lambert Iaw, a specialized case of the Kubelka-M unk model. Under this Iaw, the spectral densities of any combination of inks will be a linear combination of the spectral densities of the inks themselves. In other words, the behavior will be linear in density space.

The non-linearity of ink behavior in density space is well documented. Tollenaar and Ernst provide an alternative to the K ubelka-M unk model for printing inks on paper. They used the following relationship: [4]

$$
\begin{equation*}
D=D_{\infty}[1-\exp (-m y)] \tag{1}
\end{equation*}
$$

where $D_{\infty}$ is the saturation density; $m$ is the rate at which the saturation density is approached; y is the ink layer thickness; and $D$ is the density of the print.

This formula is more linear in colorant amount than is density. We may invert it to obtain a translation from density to this linear space:

$$
\begin{equation*}
y=\left[\ln \left(D_{\infty}\right)-\ln \left(D_{\infty}-D\right)\right] / m \tag{2}
\end{equation*}
$$

Colorant behavior will be just as linear in this space as it will be in a space which differs from it by a multiplicative constant. Taking the constant as $m \cdot D_{\infty}$, and using the nomenclature " $E$ " to refer to this ordinate, we have:

$$
\begin{equation*}
E=D_{\infty} \cdot\left[\ln \left(D_{\infty}\right)-\ln \left(D_{\infty}-D\right)\right] \tag{3}
\end{equation*}
$$

or, in terms of reflectance:

$$
\begin{equation*}
E=D_{\infty} \cdot\left\{\ln \left(D_{\infty}\right)-\ln \left[D_{\infty}-\log (R)\right]\right\} \tag{3a}
\end{equation*}
$$

(We selected the scaling so that the derivative of Equation (3) is unity at $D=0$. Thus, for small densities, E values will be only slightly larger than the corresponding densities.) We may convert from E back to reflectance by:

$$
\begin{equation*}
R=\operatorname{antilog}\left\{-D_{\infty} \cdot\left[1-\exp \left(-E / D_{\infty}\right)\right]\right\} \tag{4}
\end{equation*}
$$

Equations (3a) and (4) are applied to two different types of data: to spectral measurements, which are used to compute the N eugebauer Primaries, and to spectrally-sharpened tristimulus data, which are used to create a convex hull of points on the gamut's surface. We have used a $D_{\infty}$ of 2.5 throughout this paper.

## O verview of the Procedure

We describe, step by step, the process of evaluating a single combination of inks. These steps run under a script which exercises this process for each combination of inks. We discuss the creation of this script in a later section.

## Estimating the N eugebauer Primaries

Given $n$ inks printed in a given sequence, there are $2^{n}$ ways in which these inks may be combined at any given point in a binary printing process. (These $2^{n}$ combinations are referred to as the $N$ eugebauer Primaries.) O ne possibility is the plain paper; $n$ additional possibilities are the single inks on the paper. The remaining combinations are also required to compute the gamut.

The N eugebauer Primaries were estimated by application of Equation (3a) on a spectral basis. The E-spectrum of the paper and the individual inks (on the same paper) may be computed through Equation (3a). The spectrum of the overprint of Ink 1 and Ink 2, for example, may be computed by adding the E-spectrum of Ink 1 to the E-spectrum of Ink 2, and subtracting the E-spectrum of the paper (because it is already accounted for in the spectra of both inks). The process may be extended to any number of inks. A fter the Espectra have all been estimated, they may be converted back into reflectance spectra through Equation (4).

## Computing the C onvex H ull

$N$ ot all of the $N$ eugebauer Primaries will be at the edge of the gamut. For example, the combination of an Orange and a C yan may produce a nearly-neutral dark color, but not as dark as the Black ink alone. Such an overprint would be already contained in the gamut producible by the other $N$ eugebauer Primaries.

Our premise is that color mixing of inks on paper is linear in E -space. We make the further simplifying assumption at this point of linear mixing in a three-dimensional E space, wherein the coordinates are computed from tristimuIus values. Rather than $X, Y$, and $Z$, however, we have used a spectrally-sharpened version, based on the following transformation matrix:

$$
\left[\begin{array}{l}
R  \tag{5}\\
G \\
B
\end{array}\right]=\left[\begin{array}{ccc}
1.1882 & 0 & -0.1701 \\
-0.5808 & 1.5705 & 0.0606 \\
0 & 0 & 1.2118
\end{array}\right] \cdot\left[\begin{array}{l}
X \\
Y \\
Z
\end{array}\right]
$$

(W e elected to use spectrally-sharpened tristimulus values because they tend to transform into E-coordinates which behave more linearly than those computed from XYZ tristimulus values, which are quite broadband.)

In order to discard primaries whose color could be produced with "legal" amounts of other primaries, we exercised a C onvex Hull routine [5] on the 64 points in $\mathrm{E}^{3}$ space. This resulted in a collection of points, in $E^{3}$ space, which were the vertices of the gamut in that space, and a list of point triplets which define the triangles which cover the gamut's surface. This structure is referred to in computer graphics as a "Triangular M esh."

At this point, it is possible to transform from $E^{3}$ to $X Y Z$, then to the CIELA B color space, yielding a gamut description in CIELA B. The triangular mesh in CIELA B will no Ionger necessarily be convex, because the transformation from $E^{3}$ to CIELA $B$ is non-linear.

## First Gamut Volume C alculation

A rough indication of the gamut volume may be computed on this minimal gamut description in CIELA B. A point is selected in the interior of the gamut which, in conjunction with each triangle in the mesh, defines a series of (normally) mutually exclusive and exhaustive sub-gamuts (each a simplex). The volume of each sub-gamut is computed, and the volumes summed to yield an approximate gamut volume in CIELA B.

This rough volume calculation may be used to cull poor combinations so further effort need not be expended on them.

## C ounting In-G amut Pixels

Besides the volume which a gamut occupies in the CIELA B color space, we have also used the fraction of pixels in a digital image which are also contained in a gamut as a figure of merit. This may be accomplished on the triangular mesh in $\mathrm{E}^{3}$.

Before computing any gamuts, the digital image is converted into $E^{3}$, first going into $X Y Z$ (using, perhaps, an ICC profile), thence to the spectrally-sharpened tristimulus values, and finally into $E^{3}$ via Equation (3a). Because it is not a visually uniform color space, it is necessary to retain more than 8 bits per component per pixel. Although we have used eight decimal digits of floating point, we believe that no more than 16 bits per component per channel should be required.

A fter a convex hull gamut in $\mathrm{E}^{3}$ is computed for each ink set, we may test each pixel in the digital image to see if it is within or outside of the gamut. The proportion of with-in-gamut pixels is recorded.

## Refining the $G$ amut in CIELAB

Because of the non-linear nature of the transformation from $E^{3}$ to $C I E L A B$, each edge in the triangular mesh in $E^{3}$ must, in general, map to a curved locus in CIELA B. However, we have not yet allowed for this.

One way of accounting for this curvature is to simply use many intermediate points. Because the color mixing is assumed to be linear in $E^{3}$, we may bisect each edge in the triangular mesh while it is still in $\mathrm{E}^{3}$. The collection of points may then be re-triangularized, to produce a finer triangular mesh in $\mathrm{E}^{3}$.

This process of edge bisection and re-triangularization may be repeated an arbitrary number of times to produce an arbitrary degree of compensation. We have found that three or four normally show the curvature quite well, and additional iterations add little.

A fter completing the bisection and re-triangularization process, the coordinates may be transformed into CIELA B, and imported into visualization tools for analysis or inspection. A refined volume in the CIELA B color space may also be calculated at this stage.

## A pplication of the Technique

In order to illustrate the technique, we performed calculations using a set of Pantone® Basic Colors and Pantone Basic Process C olors. O ur motivation for this restriction was to focus on a universe of readily-available, somewhat standardized inks, whose colors are familiar to many in the $G$ raphic A rts industry.

## First Example: Four Process C olors Plus Two Others

In the first example, seventy-eight six color combinations are chosen by starting with a CM YK ink-set (Pantone Process Yellow, Pantone Process M agenta, Pantone Process Cyan, and Pantone Process Black), and adding two colors from the list of 13 Pantone Basic Colors (Pantone Yellow, Pantone Yellow 012, Pantone $O$ range 021, Pantone W arm Red, Pantone Red 032, Pantone Rubine Red, Pantone Rhodamine Red, Pantone Purple, Pantone Violet, Pantone 072 Blue, Pantone Reflex Blue, Pantone Process Blue, and Pantone G reen) to make six-color sets.

A program was written which contained two loops which selected the two inks which were added to the Basic Process Colors to make up the ink set. The output of this program was a script which exercised all steps, including two


The projection of the four process-color gamut onto the ( $a^{*}, b^{*}$ ) plane, after two iterations of bisection-and-retriangularization.
iterations of bisection and re-triangularization, except the counting of in-gamut pixels. This consumed approximately 20 seconds on a 266 M Hz Pentium II-based Linux workstation. (We re-ran the process, without any bisection and retriangularization, and it required approximately 14 seconds.)

The combination of Pantone Green and Pantone Yellow 012 in conjunction with the four Pantone Basic Process Colors yielded the largest volume: 560160 cubic CIELAB units. (This is in comparison to the volume of 313282 for the four Process colors al one.)

We also computed the proportion of pixels within a pair of digital images which were contained within the gamuts. The pictures were $256 \times 295$ pixels and $256 \times 234$ pixels. The larger picture required approximately 20 seconds per picture, per ink combination; the smaller, approximately 15 seconds. The addition of Blue 072 and Yellow 012 to the four process inks increased the proportion of in-gamut pixels from $71.1 \%$ to $80.9 \%$ for the larger picture; the addition of 0 range 021 and $R$ hodamine Red to the four process inks increased the proportion from 51.4 \% to $65.7 \%$ for the smaller picture.

## Second Example: Black Plus Five 0 thers

In the second example, 4368 colorant combinations were considered by using Pantone Process Black in all combinations, and selecting the five other colors from the list of sixteen chromatic inks mentioned above. A program was written which generated a script to perform the first steps in the procedure outlined above, but it contained five loops, rather than two in the previous example, one for each to be selected.

To go through the same gamut volume calculation steps, including two bisection-and-retriangularization iterations, required approximately 20 minutes.


The corresponding projection of the gamut calculated for Process Black, Orange 021, Process Blue, Purple, Rhodamine Red, and Pantone Yellow.

The combination which produced the largest calculated volume included Orange 021, Process Blue, Purple, Rhodamine Red, and Yellow, in addition to the Process Black. N ote that this does contain a C yan (Process Blue), a M agenta (Rhodamine Red), and a Yellow (though not the Yellow in the set of Pantone Basic Process colors). The calculated volume was 689753 cubic CIELA B units.

## C onclusions

A computationally- and measurement-efficient technique for approximating gamut size has been disclosed. It is based on a transformation of density in which subtractive colorant behavior is believed to be more linear.

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