Modeling the Yule-Nielsen Effect on Color Halftones

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Abstract

The Neugebauer approach to modeling color cmy halftones generally has to be modified in order to correct for the Yule-Nielsen light scattering effect. The most common modification involves the Yule-Nielsen "n" factor. A less common, but more fundamentally correct modification of the Neugebauer model involves a convolution of the hal-tone geometry with the point spread function, PSF, of the paper. The probability model described in the current report is less complex than the PSF convolution approach but is still much less empirical than the Yule-Nielsen "n" model. The probability model assumes the Neugebauer equations are correct and that the Yule-Nielsen effect manifests itself in a variation in the XYZ tristimulus values of the eight Neugebauer primary colors as a function of the amounts of c, m, and y printed. The model describes these color shifts as a function of physical parameters of the ink and paper which can be measured independently. Experimentally the effect is easiest to see in the shift in the color of the paper between the halftone dots, and experimental micro-colorimetry is presented to verify the model.

Introduction

Recent publications have described a halftone model for tone reproduction based on the Murray-Davies equation modified to describe the changes in the reflectance of the halftone dot and the paper between the dots as the halftone dot area fraction is varied.[1,2] In this report the model is expanded to describe halftone color reproduction. The model describes the change in color of the halftone dots and the paper between the dots as the c, m, and y ink area fractions are varied. As will be shown, the model provides a convenient link between empirical models such as the Yule-Nielsen modified Neugebauer equation and *a priori* models based on the optical spread function of paper.[3]

There are two key parts of any model of color halftoning. These are Part (1), dot placement and overlap, and Part (2), the weighted sum of the individual colors of the dots in the halftone image. Often Part (1) is modeled by the Demichel equations (f_1 =(1-c)(1-m)(1-y), through f_8 = c m y) which assume a random placement of the c, m, and y ink dots to produce the eight Neugebauer primary colors; white, cyan, magenta, yellow, red, green, blue, and black.[4] Then for Part (2) one may assume the reflectance of the image is given by a weighted sum of the reflectances of the eight Neugebauer primary colors.

$$R(\lambda) = \sum_{i=1}^{8} f_i R_i(\lambda) \qquad (1)$$

From equation (1) and the definitions of the CIE tristimulus values, one may write the Neugebauer sum for the overall halftone tristimulus values,

$$T = \sum_{i=1}^{8} f_i T_i \qquad (2)$$

where T and T_i represent the CIE X, Y, or Z tristimulus value for the overall image and for the individual Neugebauer primary colors.

Variation from the predictions of the simple Neugebauer model of halftone color reproduction is typical-ly observed. Variation from the Demichel equations in Part (1) is often observed when halftone dots are not applied randomly. In addition, failure of Part (2), dot addition by equations (1) and (2), is commonly observed . Two causes of a variation from equations (1) and (2) are well known. The first is called physical dot gain and involves the physical increase in the size of printed dots. In this case the f_i in equation (1) refer to the intended color area fraction, not the area fraction actually printed. In the current report the c, m, and y terms and the eight f_i terms will be used to represent the actual dot and color areas and will be measured experimentally by image microdensitometry.[1,2] This avoids the problem of physical dot gain. Nevertheless, equations (1) and (2) still typically fail to describe the reflectance and color of printed halftones due to a second failure of Part (2) called the Yule-Nielsen effect, or optical dot gain.

The Yule-Nielsen Effect

When light enters the printed substrate it scatters laterally. This increases the probability the light will emerge under a halftone dot and be absorbed, thus decreasing the value of R(λ) at each wavelength. The result is a darker image than predicted by equation (1) and variation from the color predicted by equation (2). Various corrections to the Neugebauer equations have been suggested such as the application of an empirical factor called the "Yule-Nielsen n factor" either to equation (1) before calculation of the tristimulus values,

$$R(\lambda)^{1/n} = \sum_{i=1}^{8} f_i \cdot R(\lambda)^{1/n} \quad (3)$$

or directly to the Neugebauer equations.[5,6]

$$T^{1/n} = \sum_{i=1}^{8} f_i \cdot T_i^{1/n} \quad (4)$$

While application of the n factor can yield a much more accurate description of halftone color reproduction, the n factor is empirical and not easily related to the fundamental optical characteristics of the ink, the halftone geometry, and the optical properties of the paper. Moreover, equations (3) and (4) are no longer related. Indeed, they are mutually exclusive.

A Priori Models of Color Halftones

A more fundamental approach to modeling halftone color reproduction is to describe the lateral spread of light in the paper with a Point Spread Function, PSF.[3,7,8] The PSF is a function which describes the probability of a photon emerging from the substrate at a distance r from its point of entry into the substrate. By performing appropriate convolution calculations between the PSF and the geometry of the halftone dots, one can calculate the color of the halftone image. These convolution calculations are often carried out in the Fourier domain, and the PSF is often described in terms of the modulus of its Fourier transform, called the MTF of the substrate. On an even more fundamental level, one might attempt to derive the PSF/MTF from first principles of optics.[9] However, it is more common to begin with an empirical description of the PSF or MTF function. The authors have found the following expression to be a useful model of the MTF of common papers, [1]

$$MTF = \frac{1}{1 + (k_p \omega)^2} \quad (5)$$

where ω is spatial frequency in cycles/mm and k_p is an empirical constant proportional to the mean distance light travels in the paper before re-emerging as reflected light.

By starting with the PSF/MTF function and the geometry of the halftone dots, one can derive a mean level probability, P_{ij}, for light to emerge from the substrate under Neugebauer dot i after having entered the substrate under Neugebauer dot j. Rather than begin with an empirical expression for PSF/MTF, we have explored empirical expressions for the P_{ij} functions. Probability functions for black and white halftones have been described elsewhere. In the current report we describe probability functions for c m y color halftones.

The Probability Model

For traditional clustered dot halftones it has been shown experimentally that the probability function for light emerging from the same dot through which it entered the paper, P_{jj}, is given as follows,

$$P_{jj} = 1 - (1 - f_j)\left[1 - (1 - f_j)^w + (1 - f_j{}^w)\right] \quad (6)$$

where f_j is the Neugebauer area fraction of the dot and w is an empirical constant previously shown experimentally to relate to the PSF/MTF constant, k_p.

$$w = 1 - e^{-Ak_p\nu} \quad (7)$$

The frequency term, ν, is the dots per inch of the halftone pattern, and A is a constant characteristic of the particular halftone pattern used. [1] For the random placement of stochastic halftone dots, such as those produced by error diffusion, the following probability function has been shown to apply.[2]

$$P_{jj} = 1 - w(1 - f_j{}^{1.2}) \quad (8)$$

In addition, the probability of light emerging from the substrate under dot i after having entered through dot j can be shown to be the following.

$$P_{ij} = (1 - P_{jj})\left(\frac{f_i}{1 - f_j}\right) \quad (9)$$

For a c m y color halftone, this provides a total of 64 probability terms corresponding to all combinations of light entering and exiting the substrate under Neugebauer dots 1 through 8.

If the spectral transmittance of the three inks are known, then the transmittance of the eight Neugebauer dots are known also. For example, for the white (f_1) the transmittance is t_1=1, and for the black (f_8) the transmittance is $t_8 = (t_2\, t_3\, t_4)$, the product of the c, m, and y transmittance. With these transmittance, the Neugebauer fractions, f_i, and the probabilities, P_{ij}, one can calculate the reflectance of each of the eight Neugebauer dots.

$$R_i(\lambda) = R_g(\lambda) t_i(\lambda) \sum_{ji=1}^{8} \left(P_{ij} t_j(\lambda) \frac{f_j}{f_i} \right) \quad (10)$$

Then the overall spectral reflectance of the image may be calculated using equation (1). By integrating R(λ) and the eight $R_i(\lambda)$ with the CIE color matching functions, the tristimulus values, XYZ, for the overall image and for the individual Neugebauer primary colors can be calculated. It should be noted that equation (2) does not follow from equation (10) and does not apply in this model.

The Recipe

To apply the probability model of color halftone reproduction the following recipe can be followed. (A) Apply the Demichel equations (or other model) to determine the color area fractions, f_1 through f_8 from the ink area fractions c, m, and y. (B) Measure the spectral transmittance of the three inks and calculate the transmittance of the eight Neugebauer colors, $t_1(\lambda)$ through $t_8(\lambda)$. (C) Calculate the probability functions P_{jj} using equation (6) for a rotated screen, clustered dot halftone, or using equation (8) for a stochastic halftone. The values of w and A may be used to fit the model to data or may be determined independently. (D) Calculate the remaining probability functions, P_{ij}, with equation (9). (E) Determine the reflectance of the eight Neugebauer colors with equation (10) and the reflectance of the overall image with equation (1). These reflectance functions are then integrated to produce tristimulus values.

Test of the Model

A three CCD chip color camera was used with a microscope, frame grabber, and software to carry out microdensitometric measurements of halftone test patterns printed with an HP-1600C color ink jet printer on HP-Gloss ink jet paper. Software was written to print error diffusion dots at 300 dpi. The dots were printed at a selected level of yellow ink (y=0.35) and a ramp of magenta (0<m<1). The microdensitometer was statistically calibrated to the dye set of the printer and the instrument light source (illuminant A) to provide CIEXYZ tristimulus values at each pixel location in the image of the halftone pattern. From histogram analysis of the CIEXYZ images the mean values of XYZ of the paper between the halftone dots (fraction f_1) were measured and plotted as a function of the amount of magenta dye printed, as shown in Figure 1. Figure 2 shows the change in chromaticity of the data between the dots, and also of the entire image, measured by the microdensitometer. The actual magenta and yellow ink area fractions, c and y, were measured by threshold segmentation of the original RGB images captured with the microdensitometer.

The model recipe described above was used to compare the data in Figures 1 and 2 to the model. (A) The ink fraction m and y (c=0) were used with the Demichel equations to calculate the Neugebauer fractions, f_1 through f_8. The measured values of m were used, but y was allowed to vary as an independent parameter. (B) The transmittance spectra of the magenta and yellow dyes were estimated experimentally from reflectance spectra measured on printed samples of 100% magenta and 100% yellow. The transmittance spectra of the individual dyes were calculated as $t_m(\lambda)=[R_m(\lambda)/R_g(\lambda)]^{1/2}$ and $t_y(\lambda)=[R_y(\lambda)/R_g(\lambda)]^{1/2}$. Steps (C) through (E) of the model recipe were then carried out. The value of Yule-Nielsen constant, w, and of the yellow dye fraction, y, were adjusted to achieve a fit between the model and the data, as shown by the solid lines in Figures 1 and 2. The fit was judged visually in this experiment rather than by statistical analysis.

Conclusion

The data in the figures is reasonably rationalized by the probability model with w = 0.9 and y = 0.3. This value of y was found to be close to the measured value of y = 0.35. An independent measurement of w was not carried out. However, a value of w = 0.9 (or n ≅ 1+w = 1.9) is quite reasonable for a 300 dpi error diffusion halftone.[2] Given the intrinsically low accuracy of a densitometric measure of color, especially applied with the typical artifacts of reflection microdensitometry, it appears the data and the model agree well. The model appears quite able to rationalize one manifestation of the Yule-Nielsen effect in which the color of the paper between the halftone dots varies as the ink fractions vary.

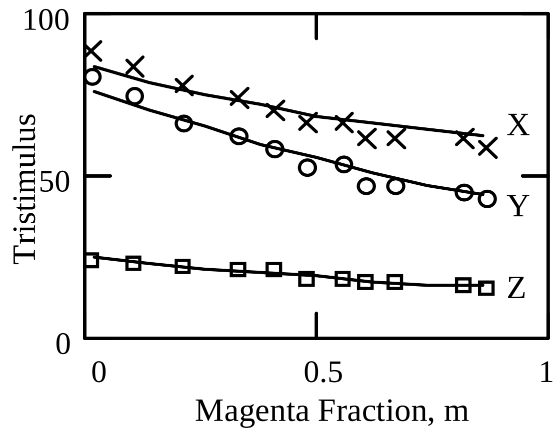

Figure 1: CIE Tristimulus Values versus Magenta Dye Fraction, m, at Yellow y=0.3 for the Paper Between the Halftone Dots.

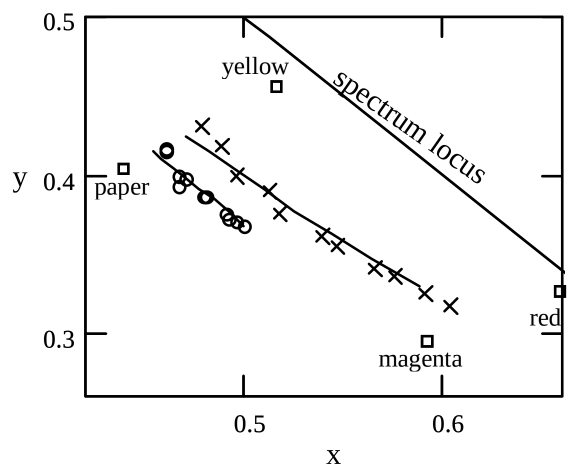

Figure 2: Trajectory of the Mean Value Image, X, and the Paper Between the Dots, O, for 0<m<0.86 at y=0.3. Also shown are the spectrum locus and the coordinates for the 100% magenta, yellow, red overprint, and unprinted paper.

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