The Maximum Ignorance Assumption with Positivity

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Abstract

Color devices are not colorimetric. It follows then that some color correction must be done to map device RGBs to XYZ's. One common correction method involves finding the linear transform which takes device spectral sensitivities as close to the XYZ color matching curves as possible (in the least-squares sense). Thereafter, this transform is used to map device RGBs to XYZs. It is well known that this procedure is statistically justified so long as one assumes that all spectra with positive and negative power are equally likely to occur i.e., so long as one is *maximally ignorant* about the world. In this paper we point out that the maximally ignorant stance is unjustified since spectra with negative power cannot physically occur. This leads us to develop the notion of maximal ignorance with positivity i.e., we assume that all spectra which are everywhere all positive are equally likely. We demonstrate that this new maximal ignorance stance delivers considerable benefits in terms of improved color correction.

Introduction

XYZ tristimuli values are needed for accurate color reproduction. Unfortunately color devices are rarely colorimetric. That is to say the colors a device sees (e.g. RGBs) are not equal to XYZ tristimulus values. Getting a color device to see tristimuli is called color correction. Typically the correction procedure involves measuring the device response for some calibration set of spectra. A mapping scheme is then derived which takes device RGBs to XYZs. The scheme might involve a look up table with interpolation[9] or alternately (and the focus of this paper) RGBs might be mapped to XYZs using a single linear transform[12] However, in both cases good correction incurs an associated calibration cost.

The *maximum ignorance* approach to color correction is a method which operates without an explicit calibration data set and so without calibration cost. Instead the transform used for color correction is defined to be the mapping which best takes the device response functions onto the XYZ matching curves. This maximum ignorance approach to color correction is justified on two counts. First, Horn[4] (and more recently Vora and Trussell[12]) has shown that perfect color correction for any color stimulus is possible if and only if the device sensitivities are a linear transform from the color matching functions. Second, it is well known that when the world of color stimuli is populated with all possible spectra, with both positive and **negative** power at each wavelength, all occuring with equal likelihood—the so called *maximum ignorance* conditions—then the best least squares mapping which takes measured RGBs to XYZs is precisely the mapping which best takes device sensitivities to the color matching functions.

Unfortunately, the maximum ignorance assumption though practically useful (it **is** used for color correction) does not make physical sense—spectra with negative power do not ever occur, so assuming that they do might impact negatively on color correction. In this paper we consider the maximum ignorance assumption with positivity: the assumption that all everywhere positive spectra occur with equal likelihood. Our hope is that by removing spectra with negative power from consideration we will significantly improve the color correction afforded. Indeed, simulation experiments, reported later in this paper, indicate that this is the case.

Linear Color Correction

Let $\underline{X}(\lambda)$ denote the vector of standard observer color matching functions: $x(\lambda)$, $y(\lambda)$ and $z(\lambda)$. The XYZ tristimulus vector $\underline{x}$ corresponding to a reflectance $S(\lambda)$ illuminated by a spectral power distribution $E(\lambda)$ is equal to,

$$\underline{x} = \int_\omega E(\lambda)S(\lambda)\underline{X}(\lambda)d\lambda \tag{1}$$

where the integral is taken over the visible spectrum ω. Let us denote the m (where m is typically 3) spectral sensitivities of a color device (e.g. color scanner or color camera) as $\underline{D}(\lambda)$. The m-vector device response to $S(\lambda)$ illuminated by $E(\lambda)$ is equal to:

$$\underline{d} = \int_\omega E(\lambda)S(\lambda)\underline{D}(\lambda)d\lambda \tag{2}$$

Let us assume that the visible spectrum can be represented adequately by samples taken 10nm apart over the range 400-700nm (this assumption is routine and forms the basis for the linear systems approach to color vision). Adopting this convention will allow the integrals in equations (1) and (2) to be replaced by summations. It follows that $\underline{X}(\lambda)$ and $\underline{D}(\lambda)$ can be represented as 31×3 matrices $\mathcal{X}$ and $\mathcal{D}$:

$$\lambda_i = 390 + 10i \ (i = 1 \ldots 31) \tag{3a}$$

$$\mathcal{X}_{ik} = X_k(\lambda_i) \tag{3b}$$

$$\mathcal{D}_{ik} = D_k(\lambda_i) \tag{3c}$$

The double subscript $_{ik}$ denotes the ith row and kth column of a matrix.

Further let $C(\lambda)$ (the color signal) denote the product function $E(\lambda)S(\lambda)$ and $\underline{c}$ its vector approximation:

$$c_i = E(\lambda_i)\, S(\lambda_i) \tag{3d}$$

the single subscript $_i$ indexes the ith element of $\underline{c}$. It follows that we can rewrite equations (1) and (2) as:

$$\underline{x} = \mathcal{X}^t \underline{c} \tag{4a}$$

$$\underline{d} = \mathcal{D}^t \underline{c} \tag{4b}$$

where ^{t} is the the transpose operation.

Let the $31 \times n$ matrix $\mathcal{C}$ denote a set of n calibration color signal spectra. Each column of $\mathcal{C}$ contains a single color signal spectrum corresponding to the product of some spectral power distribution with some reflectance spectrum. The human observer and color device response to the entire calibration set are captured by the $3 \times n$ and $m \times n$ matrices $\mathcal{P}$ and $\mathcal{Q}$:

$$\mathcal{P} = \mathcal{X}^t \mathcal{C} \tag{5a}$$

$$\mathcal{Q} = \mathcal{D}^t \mathcal{C} \tag{5b}$$

Color correction is all about mapping the device responses $\mathcal{Q}$ to the corresponding tristimuli $\mathcal{P}$. The least-squares approach to color correction sets out to determine the $3 \times m$ matrix $\mathcal{T}$ which best maps $\mathcal{P}$ to $\mathcal{Q}$. Specifically, $\mathcal{T}$ is chosen to minimize:

$$\|\mathcal{T}\,\mathcal{Q} - \mathcal{P}\|_F \tag{6}$$

$\|.\|_F$ above denotes the Frobenius norm (the square root of the sum of squared differences between $\mathcal{T}\mathcal{Q}$ and $\mathcal{P}$). It is well known[3] that the matrix $\mathcal{T}$ which minimizes (6) is equal to:

$$\mathcal{T} = \mathcal{P}\mathcal{Q}^T\, [\mathcal{Q}\mathcal{Q}^t\,]^{-1} \tag{7}$$

In mathematical parlance $\mathcal{Q}^T\, [\mathcal{Q}\mathcal{Q}^t]^{-1}$ is called the *pseudo*-inverse of $\mathcal{Q}$. Substituting (5a) and (5b) into (7):

$$\mathcal{T} = \mathcal{X}^t\mathcal{C}\mathcal{C}^t\mathcal{D}[\mathcal{D}^t\mathcal{C}\mathcal{C}^t\mathcal{D}]^{-1} \tag{8}$$

We can see from (8) that $\mathcal{T}$ depends only on the 31×31 color signal auto-correlation matrix $\mathcal{C}\mathcal{C}^t$ and the $31 \times m$ device sensitivities $\mathcal{D}$.

Maximum Ignorance Color Correction

Under the conventional maximum ignorance assumption we assume that every color signal occurs with equal likelihood: every component of the matrix $\mathcal{C}$ is drawn uniformly and randomly from the interval [- 1,1]. Relative to this assumption it can be shown[12] that the correlation matrix $\mathcal{C}\mathcal{C}^t$ is proportional to of the 31×31 identity matrix :$\mathcal{I}$ (assuming that the number of color signals in C is large). Substituting $\mathcal{C}\mathcal{C}^t = \alpha\mathcal{I}$ (where α is a scalar of proportionality) into (8), it follows that,

$$\mathcal{T} = \mathcal{X}^t\mathcal{D}[\mathcal{D}^t\mathcal{D}]^{-1} \tag{9}$$

Note that $\mathcal{T}$ defined above depends only on the human observer and color device sensitivities and as such defines a calibration free color correction transform. Importantly it can be shown that $\mathcal{T}$ minimizes:

$$\|\mathcal{T}\mathcal{D}^t - \mathcal{X}^t\|_F \tag{10}$$

That is, the transform which fits device sensor spectral sensitivities onto the human observer matching curves is exactly the same as the transform which maps device RGBs to tristimuli under the maximum ignorance assumption.

Unfortunately, to arrive at the simple formula defined in (9) we had to use sleight of hand. In particular we assumed that the entries in $\mathcal{C}$ were drawn from the interval [-1,1]. This assumption does not bear scrutiny since $\mathcal{C}$ denotes the power at the ith wavelength of light for the jth color signal and power is a strictly positive quantity! Figure 1 illustrates the problem.

Maximum Ignorance with Positivity

Under the maximum ignorance assumption with positivity we assume that all color signal, **with strictly positive power** (i.e. they are physically plausible spectra), occur with equal likelihood. To implement this assumption we assume that every component of the matrix $\mathcal{C}$ is drawn uniformly and randomly from the interval [0,1]. Relative to this all positive maximum ignorance assumption it can be shown[2] that the correlation matrix $\mathcal{C}\mathcal{C}^t$ must be equal to:

$$\left[\mathcal{C}\mathcal{C}^t\right]_{ij} = \begin{matrix} \frac{\alpha}{3}(i = j) \\ \frac{\alpha}{4}(i \neq j) \end{matrix} \tag{11}$$

where as before α or is a scalar of proportionality. An illustration of the structure of the 31×31 (for $\alpha = 1$) matrix $\mathcal{C}\mathcal{C}^t$ is drawn in Figure 1.

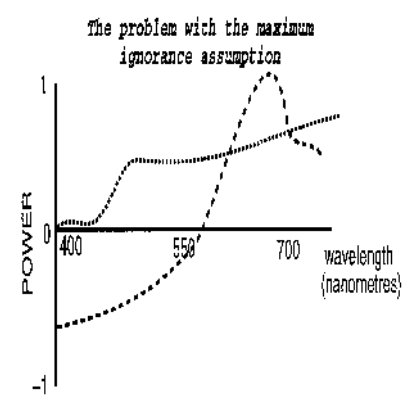

Figure 1. Two equally likely color signal spectra (according to the maximum ignorance assumption). However, the dashed spectrum has negative power and this is physically impossible.

Color correction under an assumption of maximum ignorance with positivity simply involves substituting the correlation matrix defined in (11) (and illustrated in Figure 2) into Equation (8).

$$CC^t = \begin{bmatrix} 1/3 & 1/4 & 1/4 & \cdots & 1/4 & 1/4 \\ 1/4 & 1/3 & 1/4 & & & 1/4 \\ 1/4 & 1/4 & \ddots & & & \vdots \\ \vdots & & & \ddots & & \vdots \\ & & & \ddots & 1/4 & 1/4 \\ 1/4 & & & 1/4 & 1/3 & 1/4 \\ 1/4 & 1/4 & \cdots & 1/4 & 1/4 & 1/3 \end{bmatrix}$$

Figure 2. The color signal correlation matrix derived under the assumption of maximum ignorance with positivity. All diagonal and non-diagonal entries are equal to 1/3 and 1/4 respectively.

Simulation Experiments

We carried out the following simulation experiment. First we calculated the color correction transform which takes the responses of a SONY DXC-930 camera to XYZ tristimulus values (where the viewing illuminant for camera and standard observer is D65) using both the maximum ignorance and the maximum ignorance with positivity assumptions. live then calculated the camera responses for two real sets of reflectance spectra: the 462 Munsell spectra measured by Nickerson[7] and the 170 Object spectra measured by Vrhel et a1.[11] The camera responses were then mapped to XYZ tristimuli using the maximum ignorance (with and without positivity) transforms. The colorimetric perfor mance (CIE Lab error) of these mappings is summarized in Table 1. For the data considered, the correction based on maximum ignorance with positivity delivers much better performance.

Table 1: Statistics for CIELABΔE^*_{ab} values comparing XYZ tristimuli values with corrected camera RGBs (both camera and XYZ tristimuli are calculated with respect to D65). Two correction transforms are compared: first, one derived using maximum ignorance assumption (Max Ig.) and second, one derived under maximum-ignorance with positivity (Max Ig. +ve).

Data Set	Median ΔE^*_{ab}	Mean ΔE^*_{ab}
Munsell (Max Ig.)	3.30	3.94
Munsell (Max Ig. +ve).	2.38	3.23
Object (Max Ig.)	3.27	4.13
Object (Max Ig. +ve)	2.13	3.14

We repeated this simulation experiment for the Sharp JX450 scanner[1] sensitivities. Results are shown in Table 2. Again performance is improved: under the maximum ignorance assumption with positivity correction error is roughly halved. It is worth noting that for many applications a CIE Lab error of between 2 and 5 is acceptable;[5,6,8] only the maximum ignorance assumption with positivity delivers performance in this range for both camera and scanner sen-

Table 2: Statistics for CIELAB ΔE^*_{ab} values comparing XYZ tristimuli values with corrected scanner RGBs (XYZ tristimuli are calculated with respect to D65). Two correction transforms are compared: first, one derived using maximum-ignorance assumption (Max Ig.) and second, one derived under the maximum-ignorance with positivity (Max Ig. +ve).

Data Set	Median ΔE^*_{ab}	Mean ΔE^*_{ab}
Munsell (Max Ig.)	7.93	8.09
Munsell (Max Ig. +ve)	4.27	5.45
Object (Max Ig.)	7.70	7.92
Object (Max Ig. +ve)	4.34	5.98

Conclusion

In this paper we present a new maximum ignorance, zero calibrations method for color correction. Key to the method is the assumption that all, strictly positive, spectra are equally like to occur. This information, coupled with the device sensitivities is sufficient to define a linear transform for mapping device RGBs to XYZ tristimuli.

The method improves on the conventional maximum ignorance zero calibration method, which operates under the assumption that all spectra with both positive and negative power, in two respects. First, negative power spectra have no physical meaning and, as such, should be ignored in considering color correction. Second, the all positive maximum ignorance assumption delivers substantially improved correction performance.

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