Gamut Calculation of Color Reproduction Devices

Marc Mahy
Agfa-Gevaert N.V., Mortsel, Belgium

Abstract

More advanced color management systems rely on the availability of the gamut description of a color reproduction device. The problem of calculating such a gamut description, however, has no definitive solution yet.

In this paper we explain that several kinds of surfaces exist in colorant space, that map to the gamut boundaries in color space. The different kinds of gamut boundaries that should be taken into account do not only depend on the mathematical model that describes the color mixing behavior of the reproduction device, but also depend on the colorant domain (e.g. colorant limitation) and the number of colorants of the printer.

The color gamut boundaries can be determined mathematically for any printer model and any number of inks. For some models however, such as the Neugebauer equations, most kinds of gamut boundaries can be described analytically.

Introduction

There exist two classes of methods to obtain a color gamut description.[1] According to a first approach, a large set of samples spanning the printable range is measured and empirical methods are used to estimate the gamut boundaries. A second approach is based on an analytical model of the printer that relates color values in function of colorant values.

In this paper the second approach has been selected because we believe, provided that a good model of the printing system is available, it leads to the most accurate gamut description.

The analytical model that describes the color reproduction device is called the printer model. It relates color values in function of a number of independent variables with which the printer can be addressed. Those independent variables are called colorants and are denoted as c_1, c_2, ..., c_n. A typical example is the classic CMYK offset system where we have four colorants, i.e. the cyan (C), magenta (M), yellow (Y) and black (K) ink. For purposes of generality we assume that the colorants range from 0 to 100%. A color reproduction device with n colorants is called an n-ink process.

The colorant domain is a region in an n-dimensional space that contains all the possible colorant combinations. In general the colorant domain is an n-dimensional cube. The space in which the colorant domain is represented is called the colorant space. In some cases however the number of colorant combinations have to be limited for practical reasons. A typical example is the limitation of the maximum amount of ink in offset to 340%. In this paper we assume that colorant limitations are given by a number of linear inequalities of the colorant values, referred to as ink limitations. In the previous example for the offset system, the ink limitation would be

$$c_1 + c_2 + c_3 + c_4 \leq 340\% \quad (1)$$

Colors on the other hand are represented in a 3-dimensional space. Typically a CIE based color system such as XYZ or CIELAB can be used.[2] In this paper however we allow that colors can also be specified with other non-CIE based color spaces. A typical example would be red, green and blue densities that are commonly used in photography.

We assume that the printer model is a continuous function from colorant space to color space. The range of colors obtained by transforming the colorant combinations of the colorant domain to color space is called the color gamut of the n-ink process.

In the following paragraphs the gamut of an n-ink process will be described by its boundaries. We will indicate that there are several kinds of boundaries that should be taken into account. The presence of different kinds of boundaries depends on several factors such as the printer model, the number of colorants and the colorant domain.

Color Gamut Boundaries

Introduction

Because the relations between color and colorant values are continuous functions and because the colorant domain is connected in colorant space, the resulting gamut in color space is also connected. The transformation of the colorants of an ink process with more than two inks results in general in a volume in color space. Such a gamut is completely determined if its boundaries are known in color space. In a 3-dimensional space, boundaries are 2-dimensional surfaces. Due to the continuous relations between color and colorants, colorant boundaries in colorant space have to be searched for that map on these color boundaries in color space.

3-Ink Process

In ref. 3 we indicated that two kinds of boundaries have to be taken into account, i.e. physical and natural boundaries. If no ink limitations are specified, the physical colorant boundaries correspond to the faces of the colorant cube in colorant space that are transformed to the physical color boundaries in color space by the printer model. These physical boundaries are often considered as gamut boundaries because they are the border between physical realizable and unrealizable colorant combinations. In reality this is not always the case because if natural boundaries are involved some physical color boundaries lie inside the color gamut. A physical colorant boundary can be seen as a 2-ink process because for each face of the colorant cube one colorant is constant.

Natural colorant boundaries on the other hand are surfaces in colorant space that in general do not coincide with one of the boundaries of the colorant domain, but they may be transformed to gamut boundaries in color space by the printer model. An important characteristic of natural boundaries is that if the colorant domain is intersected by a natural colorant boundary, there will be multiple colorant combinations inside the colorant domain to reproduce some colors inside the color gamut. In ref. 3 an example of a Neugebauer process is presented showing this effect. The inversion of the Neugebauer equations to find all the possible colorant combinations to reach a given color is presented in ref. 4.

A point in color space belongs to a natural color boundary if in this point the color can be changed in less than three independent directions for all possible infinitesimal changes of the colorants. From a mathematical point of view this means that the rank of the Jacobian is less than three.Iftheprintermodelofthe3-inkprocessisgivenby

$$\begin{aligned} X &= \Phi_x(c_1, c_2, c_3) \\ Y &= \Phi_y(c_1, c_2, c_3) \\ Z &= \Phi_z(c_1, c_2, c_3) \end{aligned} \tag{2}$$

a natural boundary is reached if the determinant of the Jacobian is zero, i.e.

$$\det\begin{pmatrix} \frac{\partial X}{\partial c_1} & \frac{\partial Y}{\partial c_1} & \frac{\partial Z}{\partial c_1} \\ \frac{\partial X}{\partial c_2} & \frac{\partial Y}{\partial c_2} & \frac{\partial Z}{\partial c_2} \\ \frac{\partial X}{\partial c_3} & \frac{\partial Y}{\partial c_3} & \frac{\partial Z}{\partial c_3} \end{pmatrix} = 0 \tag{3}$$

with X, Y, Z the color values, c_1, c_2, c_3 the colorant values.

The occurrence and shape of natural boundaries depend on the printer model, as shown in the following four examples.

In the case of a linear relation between color values and colorant values, no physical boundaries are present.

For a color display the printer model can be written as[5]

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} c_1^\gamma \\ c_2^\gamma \\ c_3^\gamma \end{pmatrix} \tag{4}$$

with γ the gamma of the display (typically 2.2), (c_1,c_2,c_3) the values for the red, green and blue CRT.

A natural colorant boundary is met if c_1=0 or c_2=0 or c_3 = 0. In this case the natural colorant boundaries coincide with three of the physical colorant boundaries.

For the photographic process, the Lambert-Beer law can be used as printer model.[5] If the color values correspond to three multi-spectral values, measured by making use of three delta filters, this law can be written as

$$\begin{aligned} R &= k_r e^{-c_1 a_{11} - c_2 a_{12} - c_3 a_{13}} \\ G &= k_g e^{-c_1 a_{21} - c_2 a_{22} - c_3 a_{23}} \\ B &= k_b e^{-c_1 a_{31} - c_2 a_{32} - c_3 a_{33}} \end{aligned} \tag{5}$$

with c_1, c_2, c_3 the amounts of the cyan, magenta and yellow dye

R, G, B the color values

a_{ij} and k_i constants

For this process the natural colorant boundaries are planes at infinity.

If the Neugebauer equations[6] are used and c_1 and c_2 are considered as parameters, the natural boundaries can be written as a quadratic function in c_3, i.e.

$$\begin{aligned} & c_3^2\left(k_0 + k_1 c_1 + k_2 c_2 + k_{12} c_1 c_2\right) + \\ & c_3\left(l_0 + l_1 c_1 + l_{11} c_1^2 + l_{112} c_1^2 c_2 + l_2 c_2 + l_{22} c_2^2 + l_{122} c_1 c_2^2 + l_{12} c_1 c_2\right) + \\ & \left(m_0 + m_1 c_1 + m_{11} c_1^2 + m_{112} c_1^2 c_2 + m_2 c_2 + m_{22} c_2^2 + m_{122} c_1 c_2^2 + m_{12} c_1 c_2\right) = 0 \end{aligned} \tag{6}$$

with k_i, l_i and m_i constants.

This indicates that for every couple (c_1, c_2) a set of two values c_3 can be found that meet the equation. If these two values of c_3 are real and at least one of the two colorant combinations (c_1, c_2, c_3) belongs to the colorant domain, the corresponding color lies on a natural boundary. The sets of two solutions c_3 that correspond to couples (c_1, c_2) form two natural boundaries.

It may look contradictory at first that the actual color gamut of a printing process can be larger than what is predicted by the physical color boundaries, and hence, that a color on a physical color boundary sometimes lies inside the color gamut. This paradox can only be explained by the presence of at least one other colorant combination that renders that same color and does not lie on a physical color boundary nor on a natural color boundary. In that case colorant changes can be found that result in color changes in any direction, including in a direction that goes beyond the limits set by the first colorant combination. The above reasoning demonstrates that a close relationship exists between the presence of multiple colorant solutions for certain colors and the occurrence of natural boundaries.

The gamut of the 3-ink process is obtained by taking the envelope of both the physical and natural color boundaries.

4-Ink Process

In case of a 4-ink process the concept of natural and physical boundaries as defined for the 3-ink process can be extended to the 4-ink process.[3] Also in this case physical boundaries are 2-ink processes with the other two inks set at their minimum or maximum value. On the other hand, a natural boundary is met if colors can only be changed in at most two independent directions for all possible infinitesimal changes of the four colorants. Mathematically this means that the rank of the Jacobian, a 3×4 matrix, is less than three. Apart from these two kinds of boundaries a new kind has to be taken into account.[3] We called these boundaries hybrid boundaries because they can be seen as a mixture between natural and physical boundaries, i.e. these hybrid boundaries correspond to the natural boundaries of the 3-ink processes obtained by setting one of the four colorants in the printer model to its minimum or maximum value. These 3-ink processes are called the extracted 3-ink processes of the 4-ink process. In practice however, natural boundaries are very rare, so the gamut of a 4-ink process is obtained by taking the union of the gamuts of all the extracted 3-ink processes of the 4-ink process.

n-Ink process

In case of an n-ink process, such as the Hexachrome process, in theory more than three kinds of boundaries have to be taken into account. However, in practice also in this case the gamut can be obtained by taking the union of the gamuts of all the extracted 3-ink processes of the n-ink process.

Color Gamut Boundaries with Ink Limitations

The gamut of an n-ink process of which the colorant domain is limited by an ink limitation corresponds to the union of the extracted 3-ink processes. The ink limitation is inherited by the extracted 3-ink processes, i.e. in the ink limitation the colorants that are constant to determine the corresponding 3-ink process are replaced by their constant value.

The gamut of a 3-ink process with an ink limitation is obtained by taking into account not only the physical and natural boundaries but also the extra boundary imposed by the ink limitation, i.e. the condition for which the colorant limitation, specified as an inequality, holds. This boundary is called the ink limitation boundary. Also in the case of physical colorant boundaries, seen as 2-ink processes, the ink limitation is inherited. The envelope of all these color boundaries constitutes the color gamut.

Analytical Calculation of Color Gamut Cross Sections

A useful gamut descriptor is obtained by intersecting the color gamut with parallel planes. Typically these cross sections correspond to equidistant “slices” perpendicular to the L* axis in CIELAB space. If the printer model is specified in the XYZ space, this is equivalent with making cross sections with constant Y-planes which are then transformed to CIELAB.

In ref. 3 we indicated that it is possible to determine these cross sections analytically in the case of the Neugebauer printer model for the physical gamut boundaries. It is possible to show that also in the case of ink limitation boundaries, the gamut boundary can be calculated analytically.

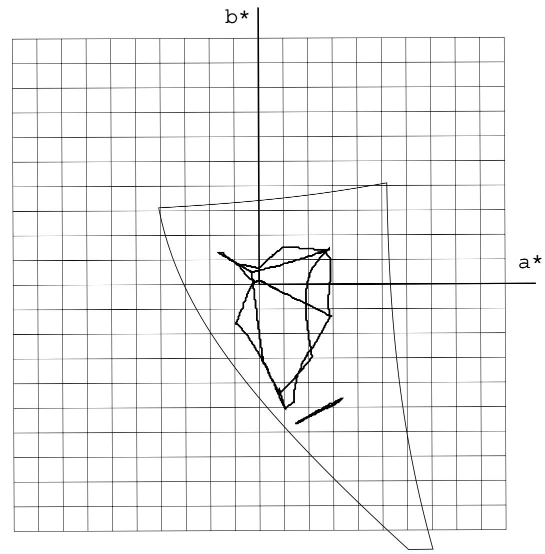

Figure 1. Cross section with a constant lightness plane (L = 30) of the gamut of an ink jet printer, represented in CIELAB. As printer model, the localized Neugebauer equations are used. In this case only the physical boundaries are taken into account.*

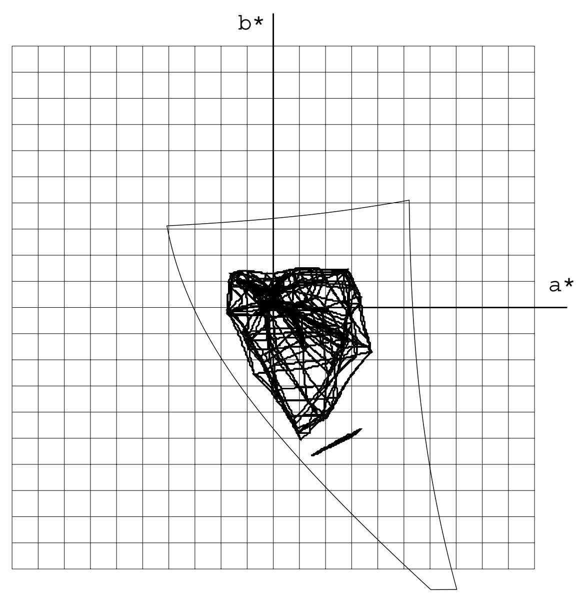

Figure 2. Cross section of all the cells of the localized Neugebauer process of the same printer as in Figure 1.

Examples

The Neugebauer equations are not accurate enough to model real color printers. Therefore we make use of the localized Neugebauer model.[7] The localized Neugebauer equations characterize the printer by dividing the colorant domain in a number of non-overlapping cells. Per cell a set of Neugebauer equations is used. Hence, the gamut of such a system can be obtained by taking the union of the gamuts of all the cells in the colorant domain.

In Figures 1 and 2 gamut cross sections are represented of an ink jet printer, modeled with the Localized Neugebauer equations. They are intersections in CIELAB with a constant lightness plane with L* = 30. In Figure 1 only the physical boundaries are taken into account, in Figure 2 the cross sections of all the cells of the Localized Neugebauer model are represented. The cross section in Figure 2 is considerably larger than the cross section in Figure 1 and hence this process is an example of a 4-ink process for which also hybrid boundaries should be taken into account.

Another example of the use of gamuts is the study of the effect of printing parameters. In Figure 3-4 for example, gamut representations are given of the offset CMYK process printed with ABS (Agfa Balanced Screening, the conventional screening technology of Agfa) 150 lpi and CR (Cristal Raster, the stochastic screening technology of Agfa) 21 mu. In Figure 3 the number of colors per lightness cross section is given, in Figure 4 the number of colors per hue plane. The colors are counted in CIELAB on a grid with a grid size of 1 CIELAB unit. Both figures show that printing with cristal raster has a larger gamut, apart from colors in the orange region, than printing with the conventional screening method.

In Figure 5-6, similar plots are shown as in Figure 3-4 but for different ink limitations, i.e. 400 % (no limitation), 250 %, 200 %, 150 % 100 % and 50 % for the CMYK offset process printed with ABS 150 lpi. It is remarkable that almost no gamut reduction takes place for ink limitations down to 250 %.

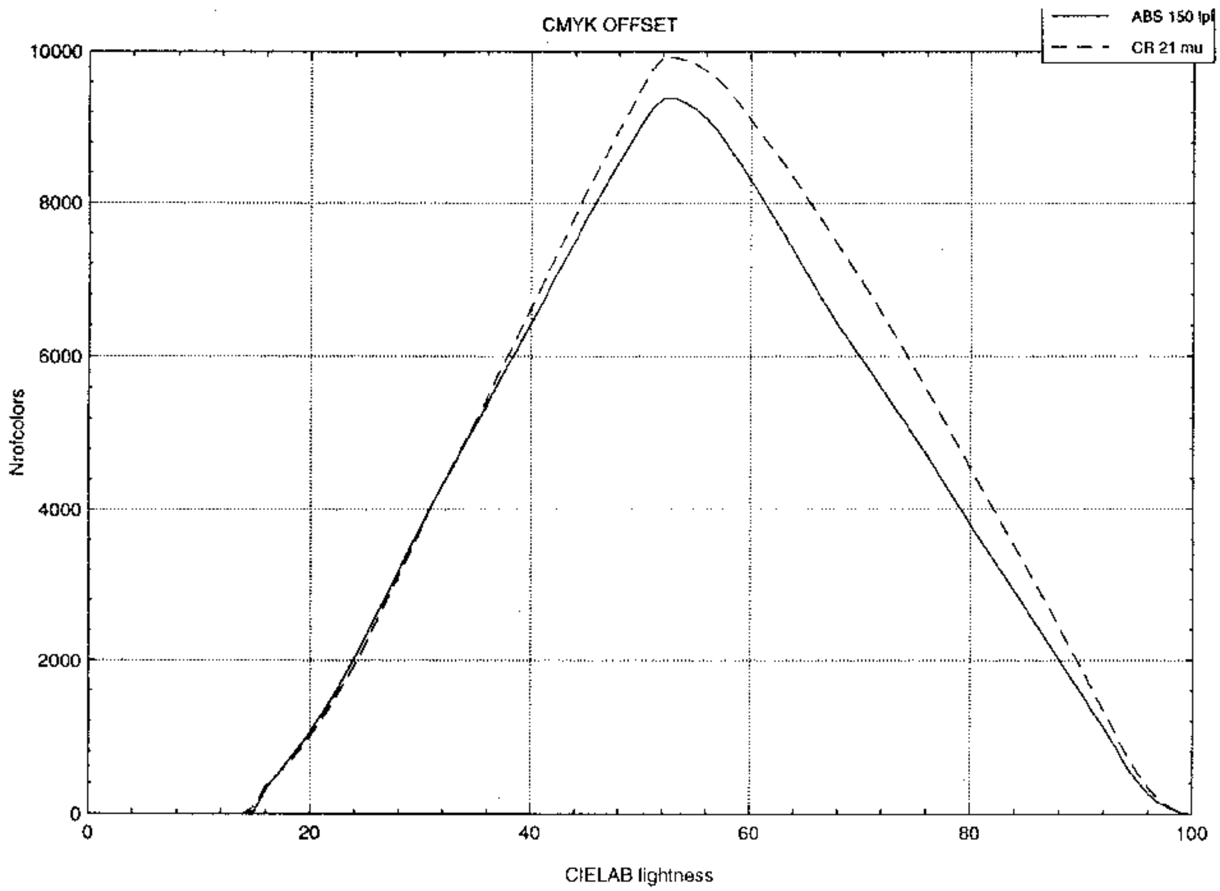

Figure 3. Number of colors per lightness cross section of the gamut of a CMYK process, printed with ABS (Agfa Balanced Screening) 150 lpi and CR (Cristal Raster) 21 mu. The colors are counted in CIELAB on a grid with grid size of 1 CIELAB unit. The localized Neugebauer equations are used to model the process.

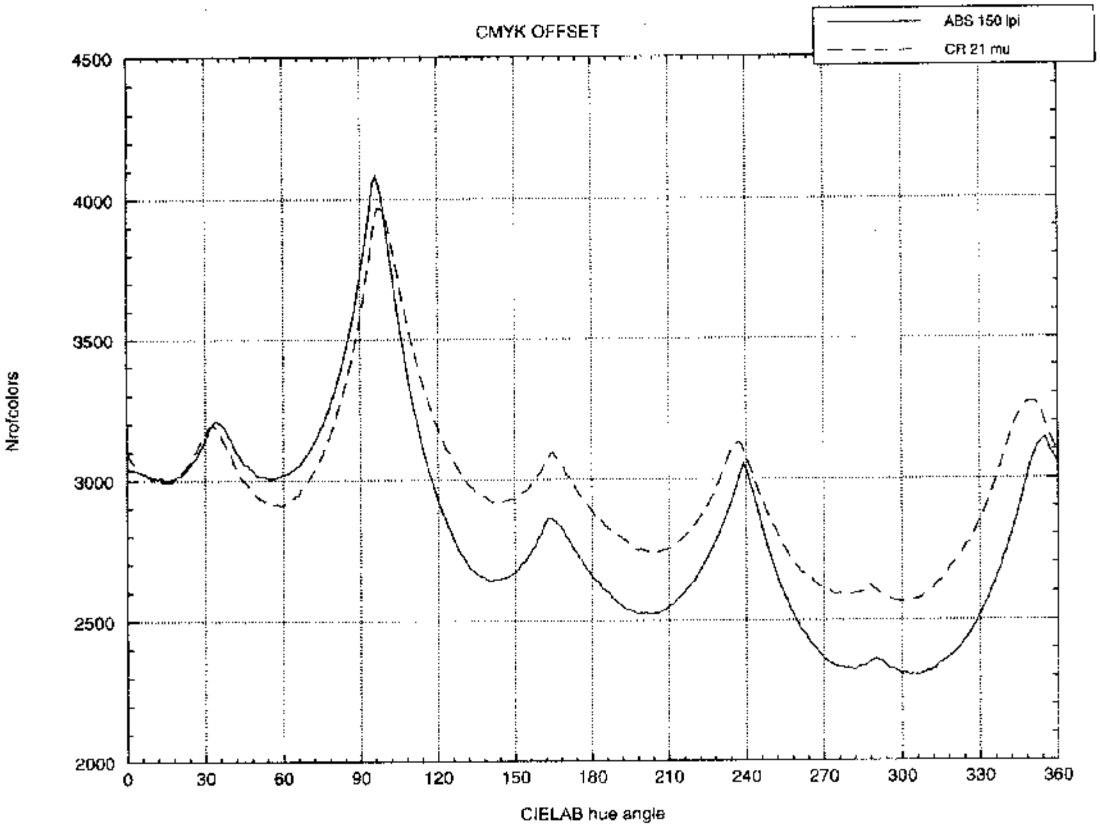

Figure 4. Number of colors per hue cross section of the gamut of the same processes as in Figure 3.

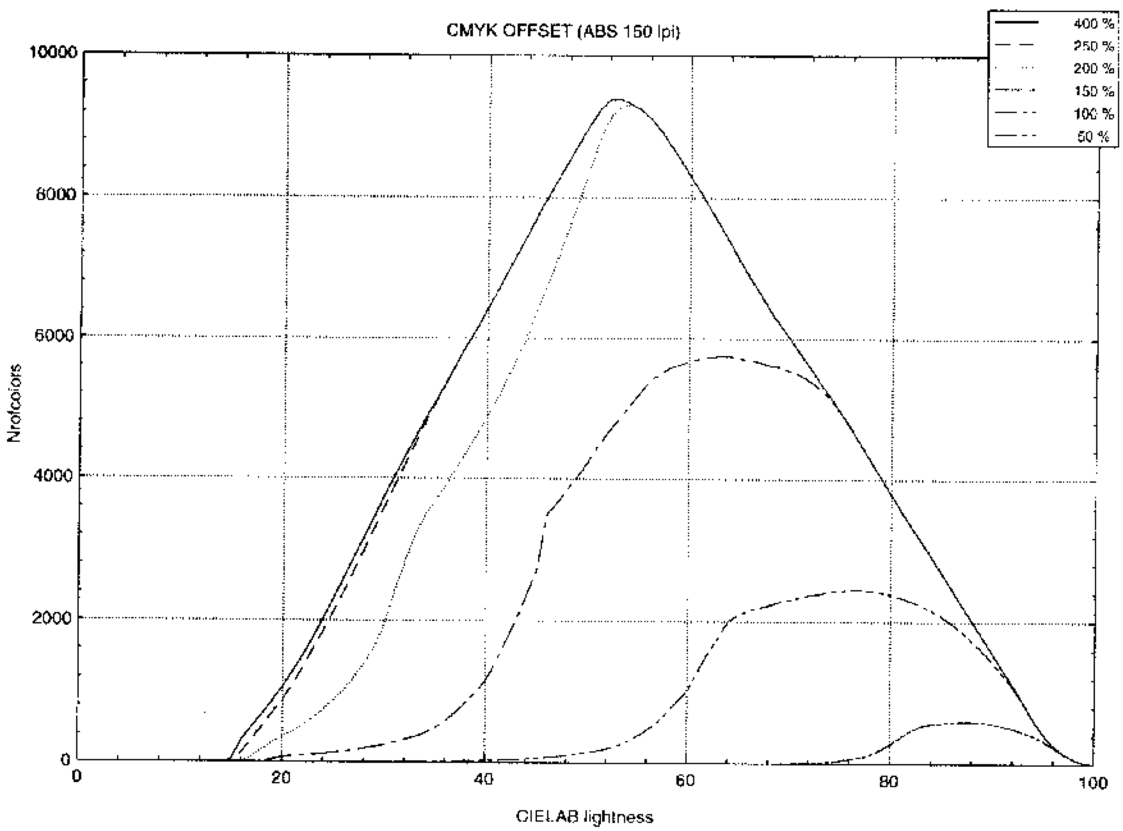

Figure 5. Number of colors per lightness cross section of the same offset CMYK (ABS 150 lpi) process as in Figure 3 for different ink limitations.

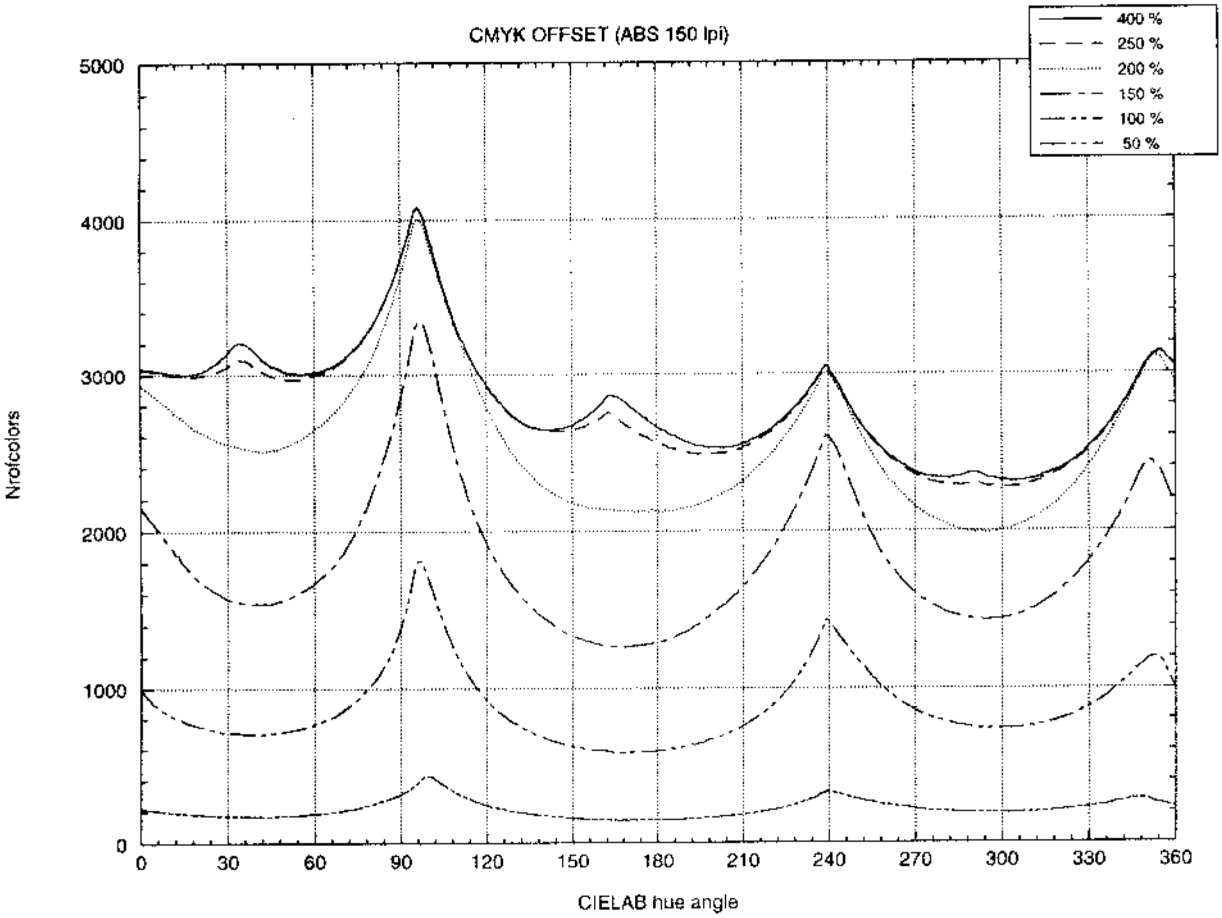

Figure 6. Number of colors per hue cross section of the same offset CMYK (ABS 150 lpi) process as in Figure 3 for different ink limitations.

References

1. William Kress and Michael Stevens , Derivation of 3-dimensional gamut descriptors for graphic arts output devices, *TAGA Proceedings* 1994, pp. 199-214.
2. Gunter Wyszecki, W. S. Stiles, Color Science: Concepts and Methods, Quantitative Data and Formulae, second edition, John Wiley & Sons, 1982.
3. M. Mahy. P. Delabastita, Calculation of color gamuts based on the Neugebauer model, accepted for publication in *Color. Res. Appl.*
4. M. Mahy. P. Delabastita, Inversion of the Neugebauer equations, accepted for publication in Color. Res. Appl.
5. R. W. G. Hunt, The reproduction of colour, 2nd Ed., Fountain Press, London, 1967.
6. Yule J. A. C., "Principles of Color Reproduction", John Wiley & Sons, Inc., New York, 1967.
7. Rollestone Robert and Balasubramanian Raja, Accuracy of Various Types of Neugebauer Model, 1993 *IS&T and SID's Color Imaging Conference: Transforms & Transportability of Color*, pp. 32-37.