Spectrophotometric Image Analysis of Fine Art Paintings

Henri Maître, Francis Schmitt, Jean-Pierre Crettez,
Yifeng Wu and Jon Yngve Hardeberg
École Nationale Supérieure des Télécommunications, Département Images
Paris, Cedex, France

Abstract

We propose a method for the reconstruction of the spectral reflectance function of every pixel of a fine art painting, from a series of acquisitions made through commercially available chromatic filters. We rigorously determine the minimal number of necessary filters, as well as their choice in an available set. We give the exact reconstruction formulae of the spectral reflectance functions, as well as the most adequate way to display the painting on a monitor of known colorimetric parameters.

Introduction

The scientific analysis of masterpieces in fine arts, in particular of paintings, is based on a very sophisticated technical tool-box, which has been developed over the last century, and which takes advantage of the most advanced technologies: X-rays, magnetic resonance, and beam accelerators for instance. Developed parallelly, digital imaging allows to keep faithful images of these paintings, providing the highest resolution as demanded by the expert, the perfect geometry of the original document and the fidelity of colors.

The color of the painting is the manifestation of the different pigment and varnish layers to the human perception, under given lighting conditions. It a very important aspect of the painting, but nevertheless limited. For many restoration or art-history oriented applications, it is of major importance to proceed into a more complete analysis of the painting, and to obtain some insight into the spectrophotometrical properties of the pigments.

This study is usually carried out using spectrophotometers which accurately analyze the light absorption of a small area of the painting. When a high spatial resolution is required, micro-spectrophotometers, which allow to focus the analysis onto a very small part of the painting, are used. But in both cases, the slowness of the measure prohibits the processing of the entire painting with a sufficient resolution. Quite recently, imaging systems named hyper-spectral scanners have been developed, mostly for remote sensing applications. Based on the use of a diffracting element placed in front of a 2D-sensing array, these systems allow the acquisition, in one shot, of several hundred measurements at different wavelengths for each pixel of a line. By scanning the painting with this line, it would be possible to obtain a complete spectrophotometric analysis. Unfortunately these hyperspectral systems are still very expensive and difficult to operate, limiting their current use to military and spacial applications.

We present a method that is less expensive. It is based on digital imaging techniques in which chromatic filters are introduced between the camera and the painting. It is well known that with 3 well-chosen filters, it is possible to obtain a good reconstruction of the color tristimulus values of the reference human observer as defined in colorimetry.[1] Our aim is to reconstruct as precisely as possible the spectral reflectance curve using more than three filters. We present here an original solution, taking advantage of the properties of the pigments used in classical painting. For this reason it can not be easily extended to post-classical painting where non-conventional materials are used: fluorescent or acrylic paintings, incorporated materials, etc.

Painting Digitization

Camera Calibration

A direct analysis of the painting is made with a digital camera, avoiding any chemical step. The painting is observed under a normal incidence angle, it is lighted with a broad band source, under an angle of about 45 degrees, following the CIE recommendations. We denote the spectral density of the illuminant by s_λ, the spectral transmittance of the optical systems in front of the detector array by o_λ, and the spectral sensitivity of the array by c_λ. The camera response to a painting area covered by a pigment with reflectance r_λ is equal to:

$$\chi = \Gamma\left(\int_{\lambda_1}^{\lambda_2} r_\lambda s_\lambda o_\lambda c_\lambda d\lambda\right) = \Gamma\left(\int_{\lambda_1}^{\lambda_2} r_\lambda \omega_\lambda d\lambda\right) \tag{1}$$

where Γ denotes eventual non-linearities of the camera, and ω_λ the system unknowns. After calibration, by inverting Γ, the spectral response $\hat{\chi}$ of an ideally linear camera is determined.

If the spectral domain of interest is sampled in N wavelength intervals, the response $\hat{\chi}$ to a given pixel may be described by $\hat{\chi}$

$$\hat{\chi} = \sum_{n=1}^{N} r_n \omega_n$$

or, using matrix notation $\hat{\chi} = \bar{\mathbf{r}}^t \omega$. Let us consider the vector ω describing the system unknowns. For that, we measure the responses $\hat{\chi}_P$, $P = 1...P$, of P pigments, $P \geq N$, with known reflectances $\mathbf{r}_p$. Let $\mathbf{R} = [\mathbf{r}_1 \mathbf{r}_2 ... \mathbf{r}_P]$, is an N-line, P-column matrix and we have $\hat{\chi}_{1...P} = \mathbf{R}^t \mathbf{u}$, where $\hat{\chi}_{1...P}$

$[\hat{\chi}_1\ \hat{\chi}_2 \cdots \hat{\chi}_P]^t$. When **R** is of rank N, we may derive ω from the following equation:

$$\omega = \left[\mathbf{R}\mathbf{R}^t\right]^{-1}\mathbf{R}\hat{\chi}_{1\ldots P} = R^{-}\hat{\chi}_{1\ldots P} \qquad (2)$$

where $\mathbf{R}^-$ denotes the N-line, P-column pseudo-inverse of $\mathbf{R}^t$.

Determination of an Unknown Pigment Reflectance

Returning to Equation 1 with r_λ now unknown, but with known ω_λ, and using a set of K chromatic filters with known transmittance $\phi_{\lambda,k}$, $k = 1 \ldots K$, we determine the camera response to the pigment r_λ through the kth filter by:

$$\chi_k = \Phi\left(\int_{\lambda_2}^{\lambda_1} r_\lambda \phi_{\lambda,k}\omega_\lambda d\lambda\right) \qquad (3)$$

Denoting as Θ the known N-line, K-column matrix with general term $\phi_{nk}\omega_n$, correcting the system non-linearities and returning to matrix notation, we obtain:

$$\hat{\chi}_{1\ldots K} = \Theta^t\mathbf{r}, \qquad (4)$$

where

$$\hat{\chi}_{1\ldots K} = \left[\hat{x}_1\hat{x}_2\ldots\hat{x}_k\right]^t.$$

From this equation, the unknown pigment reflectance may be perfectly determined (up to numerical errors) if matrix Θ is of full rank N as follows:

$$r = \left[\Theta\Theta^t\right]^{-1}\Theta\hat{\chi}_{1\ldots K} = \Theta^{-}\hat{\chi}_{1\ldots K} \qquad (5)$$

Monitor Display

As a particular application example of this method, we may use the previous equations to reproduce the painting on a monitor. From the known characteristics of the primaries of the CRT used we first determine the associated colorimetric functions $\bar{c}_{\lambda,i}$. We also determine radiometrically the non-linearity functions γ_i of the 3 monitor channels. For an assumed illuminant S_λ (not necessarily the same as for the acquisition stage) we obtain the 3 γ-corrected tristimulus values C_i, to be fed into the 3 inputs of the CRT:

$$C_i = \left(\int_{\lambda_1}^{\lambda_2}\bar{c}_{\lambda,i}S_\lambda r_\lambda d\lambda\right)^{//\gamma i}, \qquad (6)$$

which becomes, in a matrix form:

$$C = \left(\Psi^t\mathbf{r}\right)^{1/\gamma} = \left(\Psi^t\Theta^{-}\hat{\chi}_{1\ldots K}\right)^{1/\gamma}, \qquad (7)$$

where Ψ is the N-line, 3-column matrix with elements $\bar{c}_{ni}S_n$.

Choice of the Analysis Filters

Pigment Space, Eigen-Pigments

The number of filters K needed to cover the entire visible domain is a free parameter. In the most general case, a perfect reconstruction requires an infinite number of filters to be able to reproduce the narrowest signal discontinuities. We have not found in the literature an exact answer to this problem, although some concurrent techniques exist.[2-6]

Pigments used in traditional painting all exhibit rather regular and smooth absorption curves, without narrow peaks nor discontinuities (Figure 1). We based our study on a significant set of color patches provided by the National Gallery in London (courtesy of David Saunders), providing the essential pigments used in the restoration of old paintings. This set contains $P = 64$ pure pigments covering all the shades of colors of the spectrum. Each pigment has been analyzed spectrophotometrically with a resolution of 2 nm in the visible domain from 380 to 780 nm ($N = 201$).

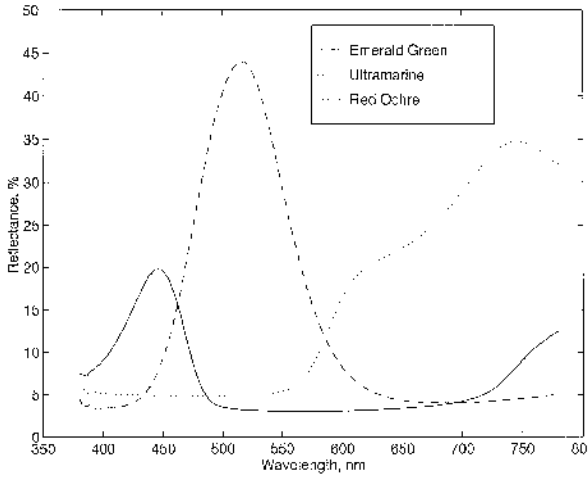

Figure 1. Spectral reflectance of three pure pigments

We want to develop an algebraic modelisation and adopt a linear law for the mixing of pigments, in disagreement with Beer's law. Beer's law,[7] a multiplicative law on the spectral transmittance of mixed pigments, leads to experimental difficulties, which are unfortunately not easy to solve. We then want to determine the minimal number of filters $\tilde{K}$ needed to restitute in N points the spectral reflectance of the P pigments. Let **R** denote the N-line, P-column matrix of the pigment reflectances, as defined earlier. The dimension of the space spanned by **R** is given by the rank of matrix **A**, obtained by a Singular Value Decomposition (SVD) of **R**.

$$\mathbf{R} = \mathbf{U}\mathbf{A}\mathbf{V}^t \qquad (8)$$

When examining carefully the eigen-values, it may be seen that the 12 largest represent 98.2% of the global energy (Figure 2). Therefore we propose to replace Equation 8 by:

$$\mathbf{R} = \tilde{\mathbf{R}} = \mathbf{U}\mathbf{O}_{\tilde{K}}\mathbf{A}\mathbf{V}^t \qquad (9)$$

where $O_{\tilde{K}}$ represents the projection onto the $\tilde{K} = 12$ first Eigen-vectors. These vectors, $\tilde{U} = \mathbf{U}\mathbf{O}_{\tilde{K}}$ are denoted **Eigen-pigments**. They do not correspond to realistic pigments since their reflectance is not guaranteed to be positive (Figure 3). They span a space $\mathcal{P}$, a sub-space of the space spanned by all the pigments, i.e. of the approximations $\tilde{\mathbf{r}}$ of the real pigments **r**. From now on we will only try to preserve the reconstruction of $\tilde{\mathbf{r}}$.

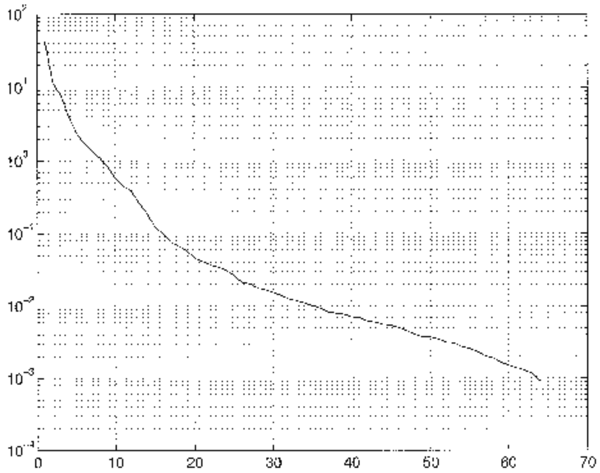

Figure 2. Singular values of the A matrix (in log scale.

Filter Selection

We know the matrix Θ obtained from the spectral transmittances of several commercially available filters (Wratten, Hoffman, Schott). We want to select the $\tilde{K}$ best filters for our reconstruction. For each filter we build the unitary vector

$$y_k = \phi_k / \|\phi_k\|$$

aligned on the *k*th filter, its projection on the *j*th Eigen-vector is $\tilde{\mathbf{u}}^t_j \mathbf{y}_k$ and its projection in $\mathcal{P}$ is $\mathbf{g}_k = \tilde{U}^t \mathbf{y}_k$We present two methods for the filter selection:

1. As a first basis vector $\mathbf{b}_1 = \mathbf{y}_{k_1}$ we choose the one with a projection in $\mathcal{P}$ of maximal norm:

$$\|g_{k_1}\| \geq \|g_k\|$$

The second filter is chosen among the other filters such that the projection on $\mathcal{P}$ of its component orthogonal to $\mathbf{b}_1 = \mathbf{y}_{k_1}$ is maximal:

$$\left\| y_{k_2} - \left(b_l^t y_{k_2}\right) b_l \right\| \geq \left\| yk - \left(b_l^t y_k\right) b_l \right\|, \forall k \neq k_{2,} \qquad (10)$$

continuing for the *k*th vector which is chosen if it maximizes its projection in $\mathcal{P}$ of its component normal to the space generated by $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{k-1}$.

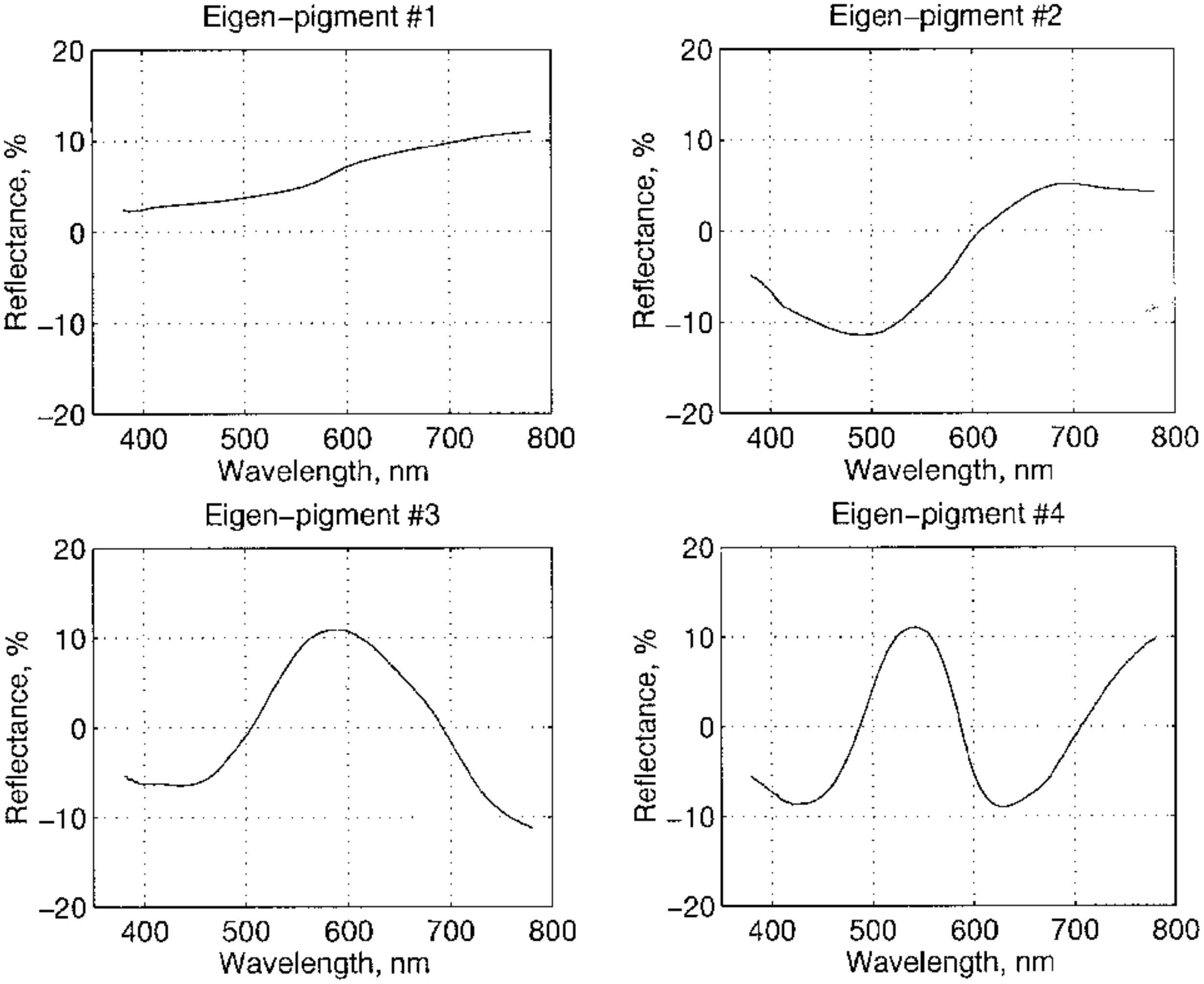

Figure 3. The four largest eigen-vectors of the basis: three contain negative reflectances without any physical meaning.

2. Starting from the set of projections $\mathbf{g}_k$, we choose the one of maximal norm as the first basis vector $\mathbf{b}_1 = \mathbf{y}_{k_1}$. The second filter $\mathbf{b}_2$ is then the filter $\mathbf{y}_{k_2}$ which maximizes the component orthogonal to g_{k_1} of its projection g_{k_1}:

$$\left\| g_{k_2} - \left(g_{k_1}^t g_{k_2} \right) g_{k_1} \right\| \geq \left\| g_k - \left(g_{k_1}^t g_k \right) g_{k_1} \right\|, \forall k \neq k_{2,} \tag{11}$$

Let $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{q-1}$ denote the $q - 1$ first selected filters, and $\mathcal{F}$ the space spanned by their $q - 1$ projections in $\mathcal{P}$. The filter $\mathbf{b}_q$ is then chosen as the filter $\mathbf{b}_q = \mathbf{y}_{k_q}$, whose projection $\mathbf{g}_{k_q}$ has the largest component orthogonal to $\mathcal{F}$.

The second method appears to be the best in terms of quality of the results when $\tilde{K}$ is greater than approximately 10. It allows to determine the set of $\tilde{K}$ optimal real filters. For practical applications, we have found that with 10 to 12 filters chosen according to this method, it is possible to reconstruct any of the r_λ curves with a good accuracy. Figures 4 and 5 show the reconstruction for the yellow Mercuric Iodide pigment, comparing the two methods. Among the 64 pigments of our study, the Mercuric Iodide spectrum was the most difficult to reconstruct, since it showed a very steep transition near 590 nm.

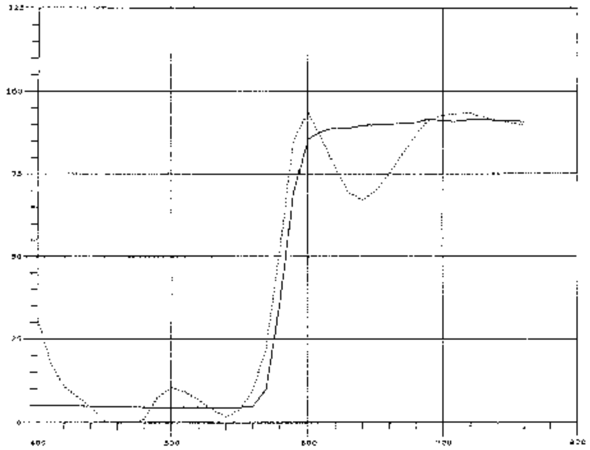

Figure 4. Reconstruction for the Mercuric Iodide pigment from 11 filters by the first method. Severe differences may be detected.

Conclusion

Using the smooth spectral properties of pigments used in classical painting, it is possible to analyze with a small number of filters the spectral response of a painting, at every pixel. When applying such methods, difficulties may nevertheless appear if the signal to be analyzed is very weak (very dark paintings). The solution of the linear equation system may become hazardous if the camera does not provide a signal of very high quality (12 bit per pixel seem necessary).

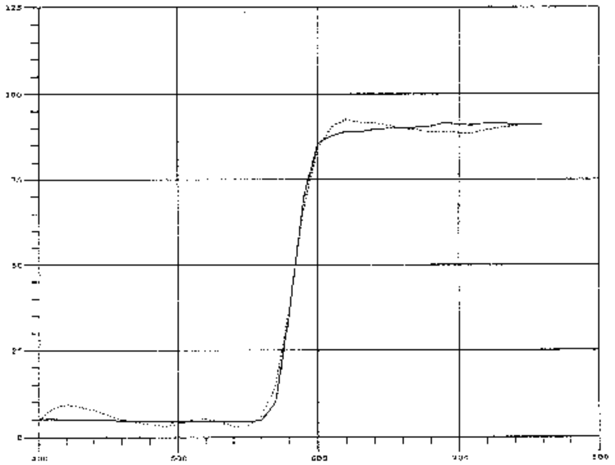

Figure 5. Reconstruction for the Mercuric Iodide pigment from 11 filters by the second method. The curves are in excellent agreement.

Acknowledgments

This study has been carried out in the framework of the European Project Esprit 2649 VASARI. It has taken benefit from the help of the Laboratoires de Recherche des Musées de France and the National Gallery of London. The contributions of Axelle Camus-Abonneau, Christophe Camus, Farouk Goulam-Ally and Manfred Müller are acknowledged.

References

1. G. Wyszecki and W. S. Stiles. *Color Science—Concepts and Methods, Quantitative Data and Formulae,* 2nd ed., John Wiley, New York, 1982.
2. P. L. Vora, H. J. Trussel, L. Iwan, “A Mathematical Method for the Design of Color Scanning Filters”, *SPIE Symp. on Electr. Imaging, Science & Techn. San Jose*, California, 1993.
3. M. J. Vrhel, H. J. Trussell, “Color Correction using Principal Components”, *Color Research and Applications*, vol **17**, 5, pp 328-338, 1992.
4. A. F. H. Goetz, Imaging Spectrometry: An Image Analysis and Interpretation Challenge, *9th SCIA*, Uppsala, Sweden, pp. 619-631, 1995.
5. D. H. Marimont, B. A. Wandell*,* Linear Models of Surface Illuminant Spectra, *JOSA*, **11**, pp. 1905-1913, 1992.
6. D. Sherman, J. E. Farrell*,* “When to use Linear Models for Color Calibration”, *IS&T/SID’s 2nd Color Imaging Conf., Color Science, Systems and Appl.*, pp. 33-36, 1994.
7. W. G. Driscoll, ed. *Handbook of Optics,* p. 8-12, McGraw-Hill, 1978.