New Quality Measures for a Set of Color Sensors ~ Weighted Quality Factor, Spectral Characteristic Restorability Index and Color Reproducibility Index ~

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Abstract

New quality measures for a set of color sensors—weighted quality factor q_e, spectral characteristic restorability index q_r and color reproducibility index Q—are proposed to practically evaluate color reproduction quality.

Because these quantities take account of object color spectral characteristics, they are more reasonable and useful than previously-proposed quality measures. Simulation results clearly show a good relation between the proposed indices and color reproduction errors after a linear color correction.

Introduction

It is well known that spectral sensitivities for color image input devices (scanners, cameras etc.) should satisfy the Luther condition. Neugebauer's quality factor q[1] is known as a measure to evaluate how well a sensor satisfies the Luther condition. q is calculated by Equation (1).

$$q = \sum_{i=1}^{3} a_i^2 / |\bar{\mathbf{s}}|^2 \tag{1}$$

where $\bar{\mathbf{s}}$ is a spectral sensibility vector of a sensor, $\{a_i\}$ are expansion coefficients, when $\bar{\mathbf{s}}$ is expanded by orthonormal bases ($\bar{\mathbf{e}}_1$, $\bar{\mathbf{e}}_2$, $\bar{\mathbf{e}}_3$) in a space spanned by human cone sensitivities (human visual subspace). q is the squared directional cosine of $\bar{\mathbf{s}}$, projected onto the human visual subspace.

However, q has the following problems.

1. As q is evaluated for each individual sensor, it is impossible to evaluate color reproducibility for a set of three sensors.
2. Object colors are postulated to distribute uniformly over an entire color measurement space, i.e. q is not based on realistic color distribution.

When three output signals (S_1, S_2, S_3) are obtained from three sensors whose spectral sensitivities are ($\bar{\mathbf{s}}_1$, $\bar{\mathbf{s}}_2$, $\bar{\mathbf{s}}_3$), a linear conversion, Equation (2) is conventionally applied to display them on an RGB monitor, using optimal coefficients $\{a_{ij}\}$.

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} \tag{2}$$

In this case, for the RGB representation, spectral sensitivities ($\bar{\mathbf{r}}$, $\bar{\mathbf{g}}$, $\bar{\mathbf{b}}$) are considered to be a linear combination of ($\bar{\mathbf{s}}_1$, $\bar{\mathbf{s}}_2$, $\bar{\mathbf{s}}_3$), and the quality factor for ($\bar{\mathbf{r}}$, $\bar{\mathbf{g}}$, $\bar{\mathbf{b}}$) should have the same value as that for ($\bar{\mathbf{s}}_1$, $\bar{\mathbf{s}}_2$, $\bar{\mathbf{s}}_3$). In 1993, Vora and Trussell proposed a new measure ν for the goodness of sensor set sensitivities.[2] ν is defined as Equation (3).

$$\nu \equiv \sum_{i=1}^{3} q_i / 3 \tag{3}$$

where q_i (i = 1, 2, 3) are Neugebauer's quality factors for $\bar{\mathrm{f}}_i$, which are orthonormal bases for a subspace spanned by sensor sensitivities (the sensor subspace). In the linear spectral measurement space, ν can be interpreted as a quantity related to the angle formed by the human visual subspace and the sensor subspace. If the angle is small, ν is near 1.

Though this ν solved the above mentioned problem 1, problem 2 remains unsolved. For example, object color spectral reflectivities are similar to those at neighboring wavelengths. Maloney and Wandell constructed a color constancy theory based on the fact that almost all object spectral reflectivities can be described by three or four principal components.[3] This fact apparently explains why many color scanners with poor q's can reproduce scanned color images well, as long as a linear 3×3 matrix color correction is applied. Therefore the condition $\nu \cong 1$ is a sufficient condition but not a necessary condition for color reproduction. The purpose of this paper is to propose new color reproducibility indices that can explain this phenomenon.

Color Correction by a Linear Matrix

If the object spectral characteristics distribution in the spectral measurement space is described by three principal components, accurate color reproduction can be attained through the following steps.[4]

1. Each object spectral characteristic is restored from three sensor measurements.
2. Accurate tristimulus values (e.g. X, Y, Z) are estimated

from the restored object spectral characteristics and known color matching functions.

In this case, the spectral characteristic vector of an arbitrary object **p** can be represented by a mean vector $\mathbf{p_0}$ and three principal components $\mathbf{p_1} \sim \mathbf{p_3}$ as follows.

$$\mathbf{p} = \mathbf{p_0} + \sum_{\mathbf{m}=1}^{3} b_m \mathbf{p_m} \tag{4}$$

Each spectral characteristic is represented by three coefficients $b_1 \sim b_3$. $b_1 \sim b_3$ can be solved and accurate tristimulus values calculated from sensor outputs $S_1 \sim S_3$ with Equation (5).

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = D \cdot \mathbf{C}^{-1} \begin{pmatrix} S_1 - S_{10} \\ S_2 - S_{20} \\ S_3 - S_{30} \end{pmatrix}$$

$$\text{where } \mathbf{C} = \begin{pmatrix} \bar{\mathbf{s}}_1 \cdot \mathbf{p}_1 & \bar{\mathbf{s}}_1 \cdot \mathbf{p}_2 & \bar{\mathbf{s}}_1 \cdot \mathbf{p}_3 \\ \bar{\mathbf{s}}_2 \cdot \mathbf{p}_1 & \bar{\mathbf{s}}_2 \cdot \mathbf{p}_2 & \bar{\mathbf{s}}_2 \cdot \mathbf{p}_3 \\ \bar{\mathbf{s}}_3 \cdot \mathbf{p}_1 & \bar{\mathbf{s}}_3 \cdot \mathbf{p}_2 & \bar{\mathbf{s}}_3 \cdot \mathbf{p}_3 \end{pmatrix}, S_i = \bar{\mathbf{s}}_\mathbf{i} \cdot \mathbf{p_0}$$

$$\mathbf{D} = \begin{pmatrix} \bar{\mathbf{x}} \cdot \mathbf{p_1} & \bar{\mathbf{x}} \cdot \mathbf{p_2} & \bar{\mathbf{x}} \cdot \mathbf{p_3} \\ \bar{\mathbf{y}} \cdot \mathbf{p_1} & \bar{\mathbf{y}} \cdot \mathbf{p_2} & \bar{\mathbf{y}} \cdot \mathbf{p_3} \\ \bar{\mathbf{z}} \cdot \mathbf{p_1} & \bar{\mathbf{z}} \cdot \mathbf{p_2} & \bar{\mathbf{z}} \cdot \mathbf{p_3} \end{pmatrix}$$

$$\text{and } X_0 = \bar{\mathbf{x}} \cdot \mathbf{p}_0, Y_0 = \bar{\mathbf{y}} \cdot \mathbf{p}_0, Z_0 = \bar{\mathbf{z}} \cdot \mathbf{p}_0 \tag{5}$$

Because $\mathbf{D} \cdot \mathbf{C}^{-1}$ is reduced to a 3×3 matrix, Equation (5) is very similar to Equation (2), and the color correction obtained with Equation (5) may be equivalent to that obtained with Equation (2). Experiments were carried out to verify this hypothesis. One thousand color patches were output by a thermal sublimation color printer and their spectral reflectances were measured from 380nm to 730nm at 10nm intervals. Eight sensor sets were simulated as follows.

- Color matching functions $\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}$ of CIE-1931, sampled at the above wavelength range and interval ('X36').
- Six different sensor sets were generated from 'X36' by altering sampling intervals. The interval for 'X18' was 20nm. 'X18' had the same sensitivity at 380nm, 400nm etc. but zero sensitivity at 390nm, 410nm etc. The intervals for 'X12', 'X9', 'X6', 'X4' and 'X3' were 30nm, 40nm, 60nm, 90nm and 120nm, respectively.
- A conventional sensor set manufacturing method was simulated. Red, green and blue sensor sensitivities were generated by multiplying silicon photo diode sensitivity and the transmittances of Kodak wratten filters 29, 61 and 47, respectively (SiD).

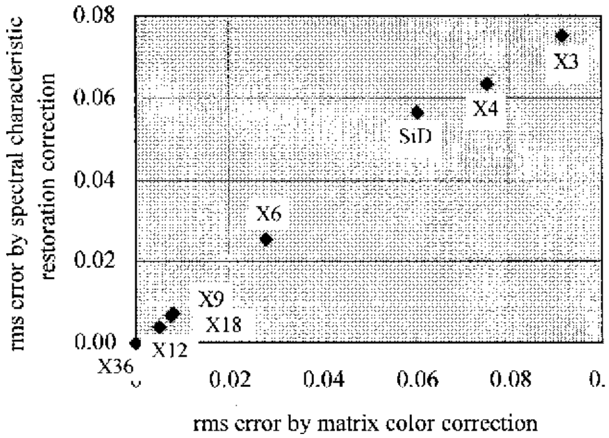

Figure 1. Relation between two color corrections.

Using these data, matrix correction, Equation (2), and spectral characteristic restoration correction, Equation (5), were compared. Figure 1 shows that these two corrections are highly correlated and equivalent to each other.

New Indices for Color Reproduction Quality Evaluation

The color correction by spectral characteristic restoration is perfectly accurate in two cases.

- Sensor subspace is in coincidence with human visual subspace.
- Object spectral characteristic is completely described by three principal components and can be restored from three sensor outputs.

These two conditions are independent of each other. However, they are dependent on object spectral characteristic distribution. The two indices defined in this section correspond to these two conditions.

Weighted Quality Factor

Table 1 shows v's for the above mentioned simulated sensor sets. Comparing Table 1 and Figure 1, color reproducibility has little to do with v values. For example, X18 has very similar sensitivity to X36, except that it oscillates with very high frequency. Its high color reproducibility means that high frequency variation in spectral sensitivity has little relation to color reproduction quality. Major color distribution variances are described by a small number of principal components whose reflectivity changes smoothly with wavelength.

A weighted quality factor q_e is defined as Equation (6), which evaluates similarity of sensor subspace to human visual subspace, considering variances of principal components.

Table 1. *v*'s for Sensor Sets

X36	X18	X12	X9	X6	X4	X3	SiD
1	0.532	0.323	0.247	0.171	0.095	0.097	0.637

$$q_e = \frac{\sum_{j=1}^{N} \sigma_j^2 \frac{\min\left(E_j, F_j\right)}{\max\left(E_j, F_j\right)}}{\sum_{j=1}^{N} \sigma_j^2}$$

$$\text{where} \quad E_j = \frac{\sigma_j^2 \sum_{i=1}^{M} \left(\mathbf{p_j} \cdot \bar{\mathbf{e}}_\mathbf{i}\right)^2}{\sum_{\mathbf{k}=1}^{N} \left\{\sigma_k^2 \sum_{i=1}^{M} \left(\mathbf{p_k} \cdot \bar{\mathbf{e}}_\mathbf{i}\right)^2\right\}}, \tag{6}$$

$$F_j = \frac{\sigma_j^2 \sum_{i=1}^{M} \left(\mathbf{p_j} \cdot \bar{\mathbf{f}}_\mathbf{i}\right)^2}{\sum_{\mathbf{k}=1}^{N} \left\{\sigma_j^2 \sum_{i=1}^{M} \left(\mathbf{p_k} \cdot \bar{\mathbf{f}}_\mathbf{i}\right)^2\right\}}$$

Figure 2 shows the relation between q_e and rms error by spectral characteristic restoration correction for the above mentioned seven sensitivities. It is observed that the nearer to 1 q_e is, the smaller the error is.

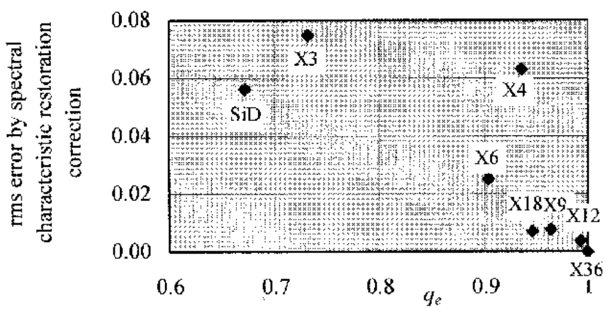

Figure 2. Relation between q_e and color correction error.

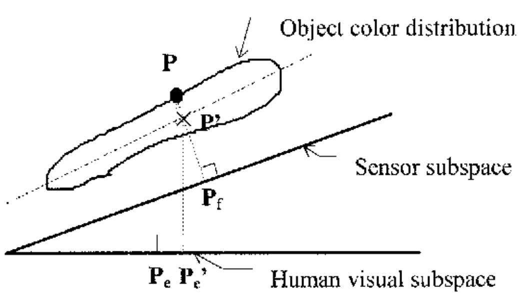

Figure 3. Color correction error by spectral characteristic restoration.

Spectral Characteristic Restorability Index

To perfectly restore an object's spectral characteristic by Equation (5), object color distribution should be confined in three dimensions. This is illustrated in Figure 3. Three-dimensional subspaces (human visual subspace and sensor subspace) are represented by a straight (one dimensional) line for simplicity. In Figure 3, object color distribution has variances in dimensions other than those of the three principal components. A color **P** is sensed as $\mathbf{P}_\mathrm{e}$ by human vision and as $\mathbf{P}_\mathrm{f}$ by a sensor set. Equation (5) restores the characteristic to **P'** and projects it onto the human visual space as $\mathbf{P'}_\mathrm{e}$, which is not equal to $\mathbf{P}_\mathrm{e}$. The following two conditions are necessary to make the restroration error small.

- Spectral characteristic distribution is small in directions other than those of the three principal components.
- The subspace spanned by the three principal components is nearly parallel to the sensor subspace.

The condition a) is easily understood from Figure 3. Figure 4 shows a case where the condition b) is not satisfied. Spectral characteristic restorability index q_r is defined as Equation (7), considering the two conditions.

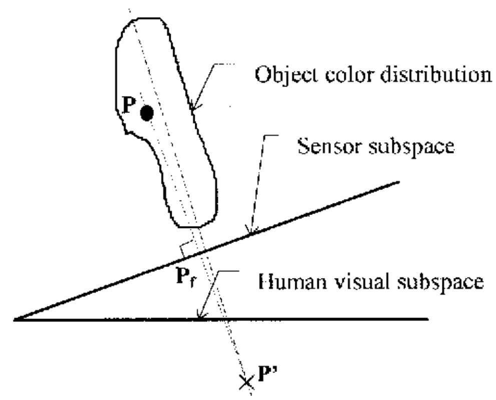

Figure 4. Object color restoration error is large, when color distribution is nearly perpendicular to the sensor subspace.

$$q_r = \left(\frac{\sum_{m=1}^{M} \sigma_m^2}{\sum_{m=1}^{N} \sigma_m^2}\right) \cdot \left(\frac{\sum_{m=1}^{M} \sigma_m^2 \left\{\sum_{k=1}^{M} (\mathbf{p}_m \cdot \mathbf{f_k})^2\right\}}{\sum_{m=1}^{N} \sigma_m^2 \left\{\sum_{m=1}^{M} (\mathbf{p}_m \cdot \mathbf{f_k})^2\right\}}\right) \tag{7}$$

The first term explains the condition a) and the second term explains the condition b). The relation between q_r and color correction error is evaluated, hypothesizing that M (= 3, 4, 5, 6) principal components are perfectly restored by M ideal sensors. In this case, the second term is always 1. Figure 5 shows the simulation result. P3, P4, P5 and P6 show the number of restored principal components. As can be seen from the figure, the relation between q_r and color correction error is nearly linear. Calculation for SiD is also depicted for reference.

Color Reproducibility Index

It has been made clear that matrix color correction error can be broadly expressed by the weighted quality factor and spectral characteristic restorability index. The last problem is how to construct a color reproducibility index that totally evaluates sensor set color reproduction. Conditions to be satisfied are as follows.

- If $q_e = 1$, then $Q = 1$.
- If $q_r = 1$, then $Q = 1$.

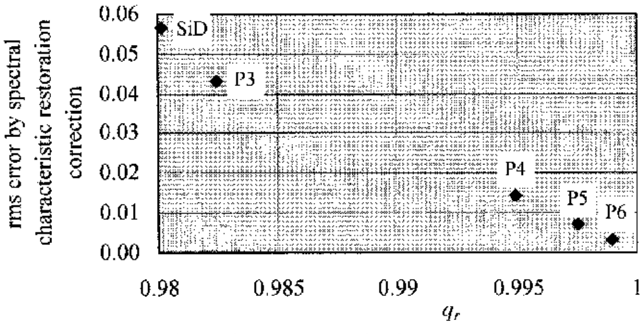

Figure 5. Relation between qr and color correction error.

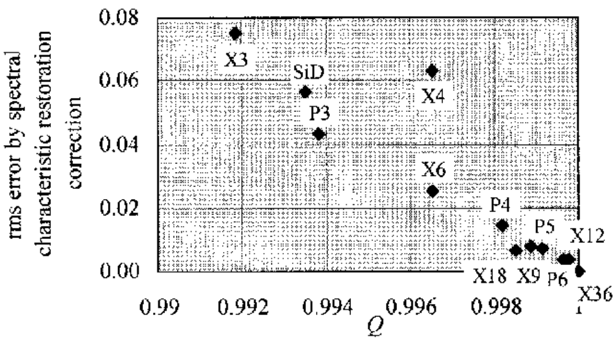

Figure 6. Relation between Q and color correction.

The simplest quantity that satisfies the two conditions is proposed in Equation (8).

$$Q = 1 - (1 - q_e)(1 - q_r) \quad (8)$$

The above-mentioned twelve sensor sets (X36 ~ X3, P3 ~ P6 and SiD) were evaluated and are plotted in Figure 6. Again, the relation between Q and correction error is almost linear, thus clearly demonstrating the validity of Q as a means of evaluating color reproducibility.

Conclusion

This paper has clarified that matrix color correction is equivalent to another type of color correction where an object's spectral characteristic is restored from sensor outputs and re-projected onto the human visual subspace, assuming the object color distribution is low dimensional.

Based on this model, weighted quality factor q_e and spectral characteristic restorability index q_r as well as color reproducibility index Q were proposed for evaluating the color reproduction quality of a sensor set. The practical usefulness of these indices was demonstrated through an experiment applying them to color patches printed by a thermal sublimation printer.

References

1. H. E. J. Neugebauer, "Quality Factor for Filters whose Spectral Transmittances are Different from Color Mixture Curves, and its Application to Color Photography", *J. Opt. Soc. Am.* Vol. **46**, No.10, 821-824 (1956)
2. P. L. Vora and H. J. Trussell, "Measure of Goodness of a Set of Color-Scanning Filters", *J. Opt. Soc. Am. A*, Vol. **10**, No. 7, 1499-1503 (1993)
3. L. T. Maloney and B. A. Wandell, "Color Constancy: A Method for Recovering Surface Spectral Reflectance", *J. Opt. Soc. Am. A*, Vol. **3**, No. 1, 29-33(1986)
4. T. Saito, "Color Reproduction of Image Scanner by Estimation of the Principal Spectral Distributions", *IEICE Technical Report*, IE90-72(1990) (in Japanese)