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ΔE–uniformity–tuned color sets

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Abstract

Color charts are used in many color imaging contexts to sample a color space or the effects of a color transformation. A rich variety of approaches exist here for determining the set of colors contained in a color chart and the specific choices of colors and their cardinality both then impact the goodness of the color imaging processes that use them. A key example here are the color charts used for building ICC color profiles. Such charts are often variations on uniform samplings in either an RGB or a CMYK device color space. At the same time, the accuracy of an ICC profile is judged in terms of color difference (ΔEs), and a chart that is uniformly distributed in those terms would be advantageous. This paper presents an approach to explicitly tuning color charts for uniformity in arbitrary color spaces and for arbitrary color difference metrics, using computational optimization over a hexagonally close packed mesh. Results show a substantial improvement in the uniformity of ΔE differences between each chart color and its neighbors.

Introduction

Color imaging is performed on the basis of discrete sets of color samples, often composed into charts that can be output or captured with an imaging device and also measured using a colorimeter or spectrophotometer. The resulting corresponding pairs of device–dependent and device–independent values can be used to build color models and ICC profiles or to characterize and evaluate performance.

In printing, a typical example is the use of color charts for calibration, linearization and profiling, where the resulting data is the basis of exercising color control, and for then evaluating performance in terms of consistency and accuracy.

A key consideration here is the relationship between the composition of such color charts and the performance of a color imaging solution on the one hand, and the reliability of performance evaluation on the other.

Since the importance of color charts and their relationship to both color models and the goodness of color evaluation has long been understood, the next section will provide some background. This will be followed by setting out a new approach to optimizing color chart uniformity and reviewing its performance, before drawing conclusions and setting out next steps.

Background

For input device characterization, the most typical approach is to use some form of uniform sampling. E.g., for devices controlled in RGB, sets of s^n RGB triples (where *s* is the number of samples per each of the *n* dimensions), evenly spaced in RGB space are common (e.g., charts composed of 9^3=729 RGBs). Such overall uniform samplings can be extended by denser samplings for some specific parts of the device color space (e.g., additional samples on the R=G=B neutral axis or in the skin tone region).

Where the device color space is CMYK, more elaborate variants of the uniform sampling approach can be used, like the reduction of sampling steps as values in the black channel (K) increase. E.g., in the ISO 12642-2:2006 chart [1], *s*=9 when K=0% and *s*=4 when K=80%. A further departure from strict uniformity in device color space is to have the per-dimension samples spaced non-uniformly. The ISO 12642 chart, e.g., uses [0%, 40%, 70%, 100%] levels for an *s*=4 sampling instead of [0%, 33%, 66%, 100%], based on typical device to colorimetry relationships in printing systems and the desire to have a spacing that is more uniform colorimetrically.

A particularly elegant evolution of uniform sampling is Mahy's Incomplete Localized Neugebauer Process [2], where the starting point is a sampling that has fixed levels per dimension, followed by the removal of some of the resulting nodes that yield a subdivision of the whole space not into hyper-cubes, but into a tessellation of hyper-oblongs of varying sizes. An alternative, applicable to both RGB and CMYK, is to apply greedy search RANSAC or genetic algorithm optimization [3] to identify a reduced number of color chart samples that perform comparably with a larger set or uniform ones.

Instead of defining charts in device color terms, they can also be specified colorimetrically. A simple approach here is to start with a uniform, cubic sampling of a color gamut's enclosing oblong in CIELAB and identifying samples enclosed inside that gamut from among the spanning uniform sampling lattice. This can be further supplemented by additional samples that are outside the color gamut by one layer in the regular lattice and gamut mapping them onto the gamut boundary [4].

To further improve the uniformity of distribution of a set of colorimetries, Urban *et al.* have proposed a multigrid optimization that transform CIELAB to a space where a given ΔE metric is Euclidean distance [5]. Particularly relevant here is the use of CIE ΔE2000 (ΔE00) [6] for such Euclidization, since several standards (e.g., for contract proofing [7]) use it to set targets and thresholds. More recently, Ahrens et al. have trained multilayer perceptron neural networks with unsupervised learning to address the same challenge [8]. While this type of approach, where a space is made more uniform in terms of a given ΔE metric has strong benefits for deriving uniformly–distributed colors to use in a characterization or evaluation chart, they have two shortcomings that the present paper seeks to address.

First, that the use of cubic or oblong based tessellations does not yield optimal neighbor distance distributions and second, that wholesale color space uniformization in terms of a ΔE metric will have residual errors that may adversely impact chart color selection. The following two sections set out solutions to both of these challenges.

Hexagonal close packing

To obtain a set of color samples that uniformly span a given color gamut, the first step is to specify a target lattice whose nodes are uniformly spaced. Looking more closely at a cubic lattice reveals that neighbor distances are not uniform, but instead take on three values, as a function of cube side length *l*:

l, $\sqrt{2}l$ and $\sqrt{3}l$. In other words, even in a perfectly uniform color space (in terms of a given ΔE), a cubic lattice does not contain uniformly distributed samples as far as neighbor distances are concerned.

Instead of a cubic–based lattice, using a hexagonal close packing (HCP) lattice [9] does indeed result in all neighbor distances being of length l, where l is the side of the tessellating triangular orthobicupola [10] (Fig. 1).

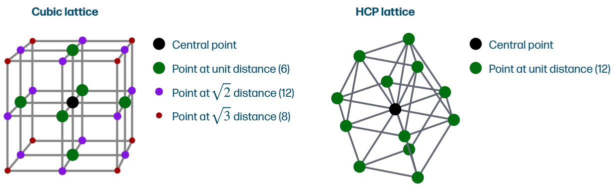


Figure 1. *Unit cubic versus triangular orthobicupola based hexagonal close packing (HCP) lattices with neighbor distances from central point.*

Spanning a test color gamut with a volume of 260K cubic CIELAB units (Fig. 2), the two strategies will result in 79 vertices for the cubic and 110 vertices for the HCP lattice respectively, for l=15 (Fig. 3). The different cardinalities are a consequence of HCP having a higher packing density [11].

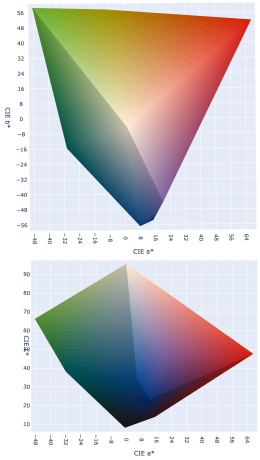


Figure 2. *A test color gamut in CIELAB.*

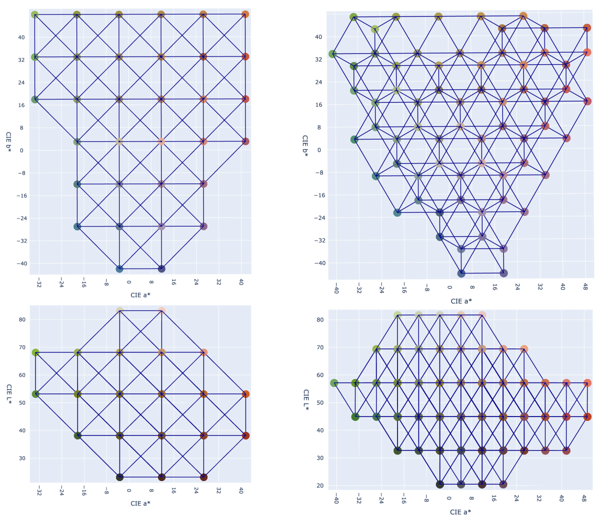
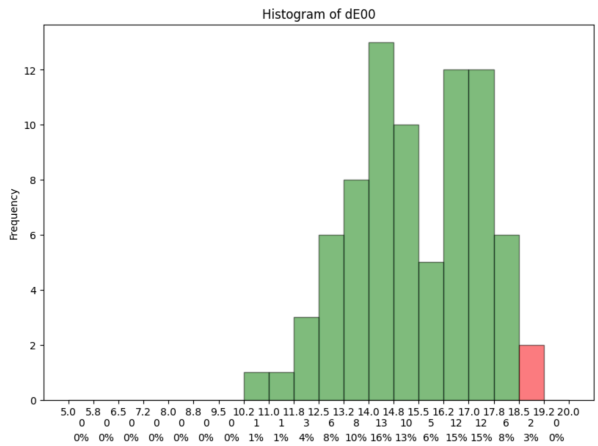


Figure 3. *Cubic (left) and HCP (right) lattices spanning a color gamut with a volume of 260K cubic CIELAB units. Orthographic projections shown onto a*b* (top) and a*L* planes (bottom). Note that all blue edges here are of the same length of 15 units.*

While the distribution of neighbor color differences of the cubic lattice is composed of Euclidean distances at 15, $\sqrt{2} * 15$ and $\sqrt{3} * 15$, as set out in Fig. 1, for the HCP case it is a perfect Dirac distribution where all Euclidean distances are at 15. Evaluating the same lattices in terms of ΔE00 paints a different picture, with the following distributions (Fig. 4). As can be seen, already the move from cubic to HCP lattice results in a narrowing of the neighbor ΔE00 distributions, where the 90% range is |12.33-18.31| = 5.98 ΔE00 and the cubic case and |8.89-13.84|=4.85 for HCP.

To further reduce the range of neighbor differences, an option would be to construct a HCP lattice in a transformation of CIELAB Euclidized with respect to ΔE00 [5], [8]. However, since such spaces are constructed to provide an approximation of ΔE00 by their Euclidean distances overall, their errors relative to ΔE00 would reduce the neighbor–distance uniformity of the resulting set.

```
Min dEs at vertex ( de00 )
Min     5th     25th    Median  75th    95th    Max
10.74   12.33   14.12   15.16   16.96   18.31   18.62
```




```
Min dEs at vertex ( de00 )
Min     5th     25th    Median  75th    95th    Max
7.44    8.89    10.08   10.98   12.29   13.84   14.26
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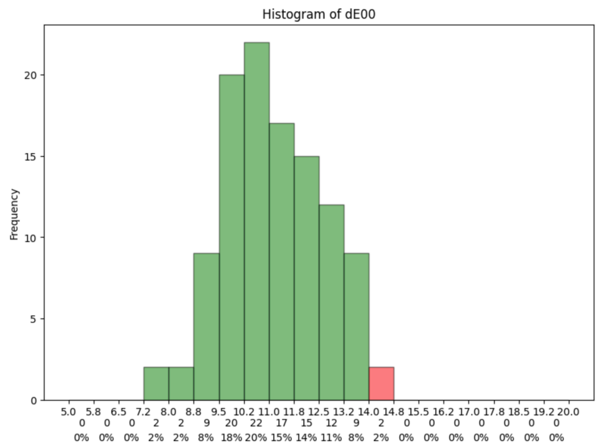


Figure 4. *Distribution of neighbor differences in a cubic (top) and HCP (bottom) lattice spanning the test color gamut.*

ΔE–tuning a color set

An alternative to overall Euclidization is to Euclidize only the pair distances in a lattice, where error can be minimized for a discrete set of distances rather than having to be minimized over an entire space.

Before reviewing the optimization process, it is useful to introduce the concept of total edge error T, which can be computed as follows:

$$T = \left|\sum_{i=1}^{n} \|e_i\| - n\|e_d\|\right| \qquad (1)$$

where $\|e_i\|$ is the length of the i-th of n edges and $\|e_d\|$ is the desired edge length after optimization. T then represents the degree of mismatch between a set of edges and their desired total length.

The starting point of optimization then is an HCP lattice that encloses and exceeds a test gamut for which a neighbor–distance uniform color set is to be generated. Each node of the lattice can then be traversed sequentially and the following node coordinate optimization performed:

1. Identify the n edges between the current node and nodes that precede it in order of traversal. Restricting edges in this way means that only edges between the current node and previous nodes, that will no longer be modified, are considered.
2. Iterate the following until the total edge error T (Eq. 1) is below a chosen threshold (e.g., 0.01) or until an iteration limit is reached:
   a. Compute the n edge lengths using ΔE00.
   b. Compute total edge length error T.
   c. Use a local search strategy (e.g., gradient descent) to identify a new candidate for the current node's CIELAB coordinates that minimizes T.
3. Intersect the resulting lattice with the test gamut and keep only in–gamut nodes and edges connecting them.

The end result of the above optimization is a distorted HCP lattice in CIELAB whose edge lengths are optimized for uniformity in ΔE00. Other color spaces and ΔE metrics can equally be used in the above optimization process.

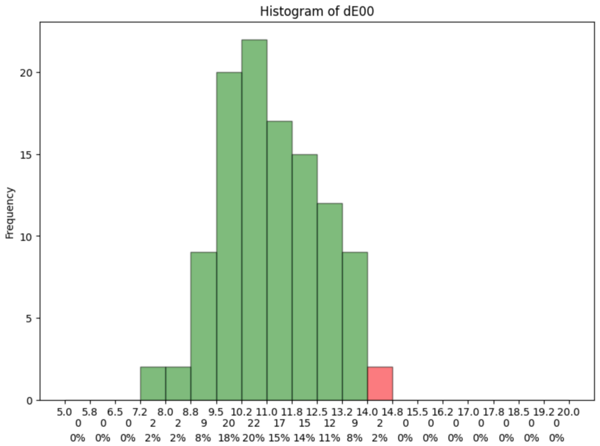
Results and discussion

To test the above approach, the gamut shown in Fig. 2 was used as a starting point. An HCP lattice with edge length l=15 units in CIELAB was then generated to enclose it, and the tuning process set out in the previous section was executed. Computing the distribution of neighbor edge lengths with ΔE00 then yielded a distribution with a 90% range of |8.41-10.68| = 2.27 ΔE00, compared with the |8.89-13.84|=4.85 ΔE00 range of the initial HCP lattice, which is a 53% reduction (Fig. 5). Compared against a conventional cubic lattice, where the 90% range was |12.33-18.31| = 5.98, this represents a 62% reduction of variation.

The resulting lattice can be seen in Fig. 6, where it shows a drawing together of nodes around the neutral axis and a spreading out as chroma increases, which is consistent with the difference between Euclidean distance in CIELAB (i.e., ΔE76) and ΔE00, which predicts higher differences for grays than for more highly chromatic color pairs. Interesting to see is also that the effect of the tuning is nearly L* independent, which again aligns well with the structure of ΔE00.

To illustrate the effect of the ΔE tuning further, Fig. 7 shows charts corresponding to the cubic lattice and the ΔE00–tuned HCP one. Also, by looking at the charts, it can be seen that the ΔE00 tuning results in differences that are more visually uniform.

```
Min dEs at vertex ( de00 )
Min     5th     25th     Median  75th    95th    Max
7.44    8.89    10.08    10.98   12.29   13.84   14.26
```




```
Min dEs at vertex ( de00 )
Min     5th     25th     Median  75th    95th    Max
6.53    8.41    9.85     10.16   10.36   10.68   11.42
```

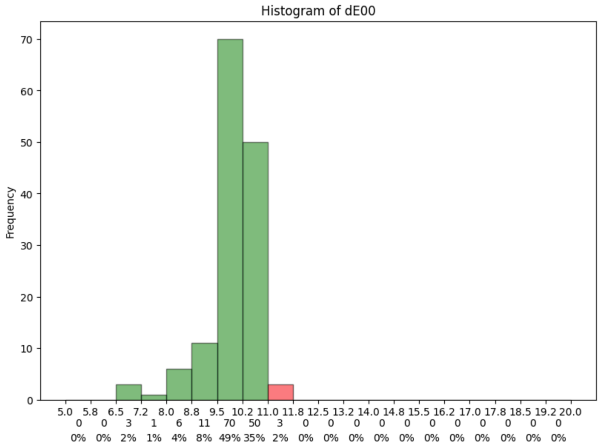


Figure 5. *Distribution of neighbor differences in an HCP (top) and ΔE00–tuned HCP (bottom) lattice spanning the test color gamut.*

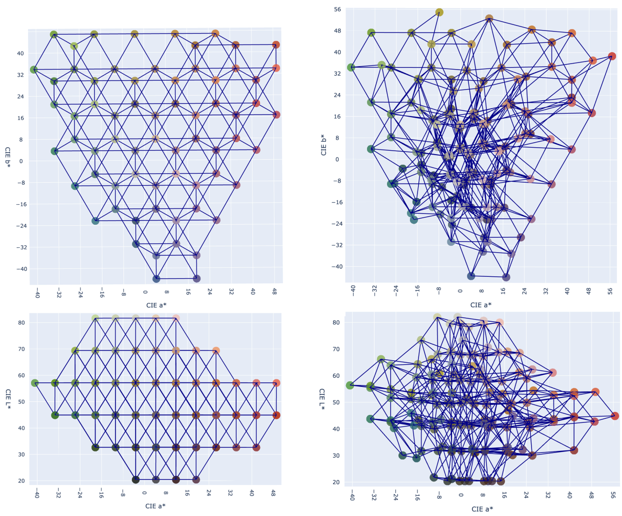


Figure 6. *HCP (left) and ΔE00–tuned HCP (right) lattice spanning the test color gamut.*

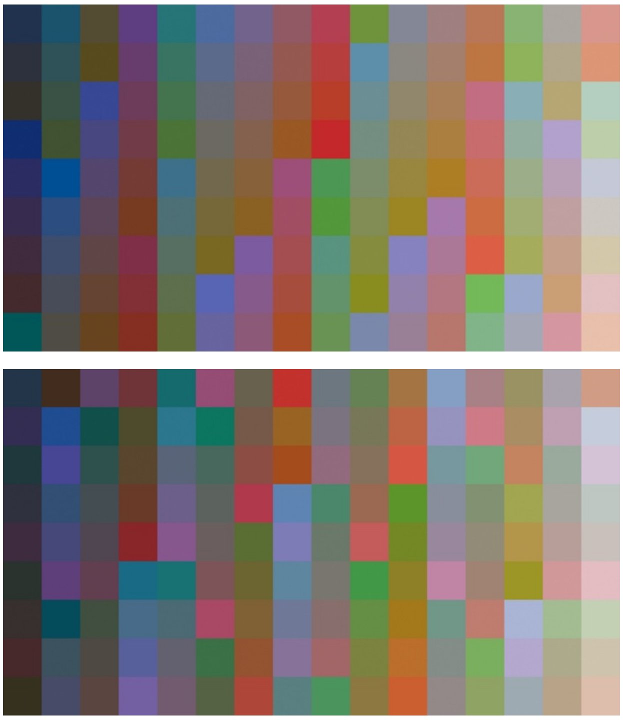


Figure 7. *Cubic (top) and ΔE00–tuned HCP (bottom) color charts.*

A further comparison that can be performed is with respect to CAM16–UCS [12], [13], where Euclidean distance has been shown to preform similarly in uniformity to ΔE00 [14]. Generating a HCP grid, with edge length l=11(chosen to yield a comparable sampling density over the test gamut to l=15 in ΔE00) and visualizing it in CIELAB (Fig. 8) shows broad similarities in terms of the relative density for neutrals versus higher–chroma pars of color space. Interesting to note is the more regular structure of the UCS grid in CIELAB as compared to the grid tuned to ΔE00.

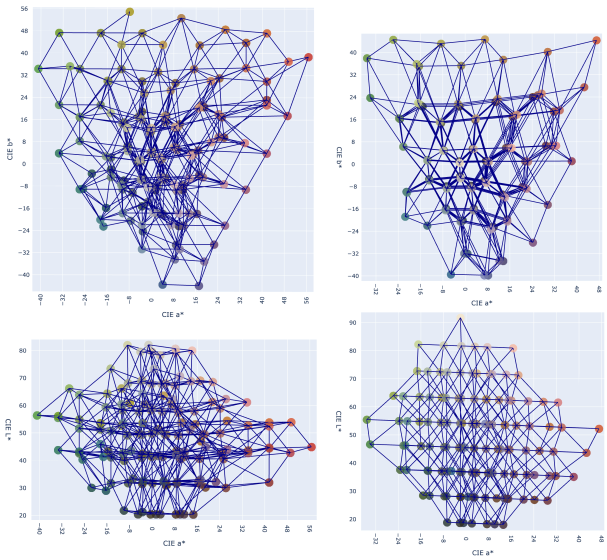


Figure 8. *CIELAB representation of ΔE00–tuned HCP (left) and CAM16–UCS HCP (right) lattices spanning the test color gamut.*

While ΔE00 and Euclidean distance in CAM16–UCS have been shown to similarly agree with the ground truth data, being able to tune a set of colors in terms of ΔE00 has particular benefits when it comes to using the resulting chart for characterizing systems that are then evaluated using ΔE00. In other words, when it is not only visual uniformity but uniformity in terms of a specific color difference equation that matters, the ΔE–tuning presented here is particularly advantageous.

Conclusions and next steps

Composing color charts in a way that favors their distribution in terms of a color difference equation is a challenge that has previously been addressed by using color spaces constructed to have such ΔEs as their Euclidean distance. While the use of such general approaches is good, a new approach was presented here that allows for the use of arbitrary ΔEs for distributing colors in an arbitrary color space such that neighbor differences within such color sets are close to being uniform.

This was achieved by combining a hexagonal close packing lattice as the starting point, followed by a sequential, iterative optimization process considering all neighbors of a color within a set. Results of the optimization showed a significant narrowing of the spread of neighbor ΔEs that also translated to a visually more uniform color chart.

In terms of next steps, the color accuracy benefits of using such ΔE–tuned charts, in terms of the chose ΔE, will be evaluated and the potential to apply the ΔE tuning to other types of color charts, beyond the output imaging systems one considered here, will be explored.

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Ján Morovič received his Ph.D. in color science from the University of Derby (UK) in 1998, where he then worked as a lecturer. Since 2003 he has been at Hewlett-Packard in Barcelona as a senior color scientist and later master technologist. He has also served as the director of CIE Division 8 on Image Technology and Wiley and Sons have published his ‘Color Gamut Mapping’ book. He is the author of over 120 papers and has filed 180+ US patents (145 granted).

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