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Modeling the Relationship between Haze and Attenuation Coefficient from Image-Based Measurements of Translucent Liquids

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Abstract

This study focuses on exploring the relationship between haze and the intrinsic optical properties of translucent materials through image-based measurements conducted in a real-world setting. The research adopts water-based samples mixed with milk and black tea, enabling the investigation of materials with varying absorption and scattering properties. We quantify haze using an image-based measurement system and estimate lateral attenuation coefficient with a translucency meter device. A linear regression model was established, relating haze to the logarithm of the product of sample thickness and the effective lateral attenuation coefficient. This finding contributes to advancing the understanding the appearance of translucent materials and has potential industrial applications.

Introduction

Transparent and translucent materials play a pivotal role in numerous manufacturing industries, where their appearance properties significantly influence product design and quality [1]. An important appearance attribute of translucent materials is haze, which describes how a material affects the contrast of the background when viewing through the material. According to the ASTM Standard Terminology of Appearance [2], haze is *"the scattering of light by a specimen responsible for the apparent reduction in contrast of objects viewed through it"*. Further, haze is characterized by *"the percent of transmitted light that is scattered so that its direction deviates more than a specified angle from the direction of the incident beam."* Rooted in optical scattering and attenuation, haze is a visual attribute that serves as an essential factor for characterizing the appearance of translucent materials [3].

Besides surface effects, the ability of a material to absorb or scatter light is its intrinsic property that governs its optical behavior and influences its haze. A deeper understanding of the interplay between haze and the optical properties is essential for predicting and controlling material appearance in manufacturing processes. Typically, materials with higher absorption and scattering ability exhibit greater haze. However, there is a lack of studies that provide insight on the relationship between haze and optical parameters, since optical parameters are usually measured in a very controlled environment, while haze measurement instruments are related to industrial process monitoring.

One major challenge in such studies is the quantification of haze and absorption and scattering properties. ASTM Standard D1003 [4] outlines two procedures for measuring haze: one is utilizing a hazemeter, an instrument specifically designed to measure light scattering and transmission, and another one is employing a spectrophotometer equipped with an integrating sphere to assess the scattering properties of the material. However, such specialized instruments are not commonly accessible to industry professionals, making standardized haze measurement challenging and costly.

The absorption coefficient quantifies the fraction of light energy absorbed per unit distance as light travels through a material, with inverse length units, typically cm^{-1} or mm^{-1}. The scattering coefficient describes how strongly a material disperses light in different directions, per unit distance. Measurement of these coefficients often requires solving inverse problems to estimate each component based on the total reduction in radiation, which requires sophisticated setups and computational techniques. As a practical alternative, an effective attenuation coefficient is often used. This coefficient quantifies the combined effect of absorption and scattering on the reduction of light intensity as it travels through a medium, providing an aggregate measure of how a material diminishes light energy due to both phenomena.

Given the limited access to specialized instruments and diverse samples, this project employed alternative methods from the state-of-the-art. For haze quantification, Busato *et al.* [5] proposed an image-based estimation method. Based on the idea that haze represents the reduction of contrast, they adopted the Edge Spread Function (ESF) to derive an image-based haze index. The authors validated their image-based haze index by demonstrating its correspondence with measurements under the ASTM D1003 standard. In this work, we re-implemented their method for estimating haze. To measure effective lateral attenuation coefficient, we used DiaStron TSL850 translucency meter. This index focuses on attenuation in directions orthogonal to the light source and reflects the combined effects of absorption and scattering. Our intention is not developing a scattering model for rendering, but an empirical, image-based method to link measurable attenuation and thickness with perceptual haze, which is practical for characterizing materials in the wild as well as in industrial quality control.

Inspired by Chadwick *et al.* [6, 7], we selected mixtures of water with milk or milk and tea as our samples. These mixtures provide a diverse range of absorption and scattering properties. By varying the concentration of milk and tea, we controlled the scattering and absorption properties of the samples, respectively. The use of liquids also enabled us to easily adjust the sample thickness, allowing for a systematic investigation of haze and attenuation properties.

The primary contribution of this work is a quantitative model, which links haze with material thickness and lateral attenuation coefficient, where any of the three can be estimated if the other two are known. The parameters of this model would also be used to characterize a material. This will have practical implications for computer vision and material appearance applications.

Methodology

Samples

Materials. The samples used in this study are liquids, specifically water mixed with milk and water mixed with milk and black tea. The addition of black tea introduces absorption to the samples, as milk alone is predominantly whitish and exhibits minimal light absorption. By varying the concentrations of these mixtures, we create samples with differing abilities of absorption and scattering.

To ensure the samples are suitable for the study, they must meet two criteria. First, the materials used for measuring light attenuation with the TSL850 needed to be sufficiently opaque and thick to guarantee a semi-infinite material hypothesis in decent lab conditions. Second, for haze measurement, the thinner samples derived from these materials had to be transparent enough to allow visual observation of the background. These constraints limited the range of suitable concentration variations for the mixtures. We finally prepared:

- 10 variations of water mixed with milk, referred to as "milk" for convenience.
- 8 variations of water mixed with milk and black tea. Similarly, we refer to them as "milktea" in the following sections.

Each variation is considered an independent material due to its unique optical properties. We named these materials following the format of {liquid type} + {numerical suffix}, where the greater suffix number means more milk or milk-tea was added to this mixture.

Thinner samples. For each liquid variation, we create multiple samples with different thicknesses. The number of thickness levels ranges from four to eight, depending on the concentration of the liquid. In total, we generated 111 samples, with:

- 62 samples made from water mixed with milk.
- 49 samples made from water mixed with milk and black tea.

Each sample is contained in a cylindrical glass container, and the thickness of the liquid in the cylindrical bottle was measured using a standard ruler. Measurements were taken at four evenly spaced positions around the container's circumference to account for potential variations in table level. The ruler was aligned vertically to minimize parallax error, and the average of the four measurements was calculated as the final thickness.

Image-based Haze Metric

We used the above-mentioned method proposed by Busato *et al.* [5]. The procedure begins with capturing a reference image of a checkerboard pattern without any sample placed on it. Regions of Interest (ROIs) are selected from the reference image, and their ESFs are computed. Contrast values are then derived from these ESFs to serve as the reference contrast. Next, the samples are placed on the checkerboard, and images are captured with the same procedure. The ESFs for the same ROIs are calculated, and their contrast values are measured. The reduction in contrast caused by the sample is determined as the ratio of the sample contrast to the reference contrast. This ratio is used as the haze metric. This relative measurement approach enhances flexibility by eliminating dependence on absolute luminance values, making it adaptable to different conditions.

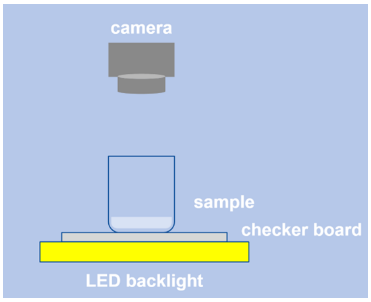

Figure 1. Schematic diagram of the acquisition setup.

Experiment setup. In our experiment, a Nikon D5600 camera was positioned vertically above the sample to capture images with a resolution of 6016×4016 pixels. The illumination source was an LED backlight, provided by the Macbeth PLT 1620 Multi-Purpose Light Box. A printed checkerboard pattern, comprising 2×2 alternating black and white patches, was placed on top of the LED backlight as the background. The sample was positioned at the center of the checkerboard, ensuring it covered two vertical and two horizontal edges of the pattern. The experimental setup is illustrated in Figure 1. During image capture, the camera was operated in manual mode with fixed exposure settings: ISO 400, an aperture of f/9, and a shutter speed of 1/100 seconds. All captured images were saved in their raw format to ensure maximum image quality for subsequent processing.

Image processing. After capturing the raw images, we converted them into the TIFF file format with dcraw (default settings) and used Photoshop to manually select four Regions of Interest (ROIs) with a resolution of 480 × 480 pixels. Each ROI contains and only contains one edge and is carefully selected to avoid noises such as shadow or caustics. Figure 2 shows an example of selected ROIs in an image.

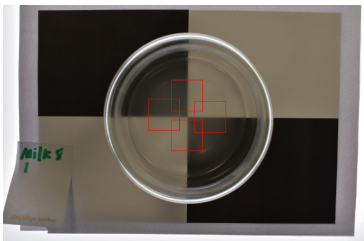

Figure 2. Example of captured image and ROIs selection. Red boxes are the selected ROIs. In addition to the backlight, the room light was also on when taking this illustrative photograph.

Metric Derivation. Busato *et al.* [5] use the Michelson contrast to define haze, which is widely used in appearance measurement [8] and is expressed as:

$$C = \frac{L_{max} - L_{min}}{L_{max} + L_{min}} \quad (1)$$

where L_{max} and L_{min} are the maximum or minimum luminance values of the regions, respectively. To approximate the maximum and minimum luminance on the ROIs we selected, we utilize the ESF, which represents the spatial luminance distribution as light transitions across a sharp edge and is depicted as a profile of grayscale intensity G as a function of position. The maximum luminance is identified in regions where the grayscale intensity values stabilize at their highest levels. Similarly, the minimum luminance is determined in regions where the grayscale intensity values stabilize at their lowest levels. Figure 3 shows an example of ESF, with the red boxes indicating the regions of maximum and minimum luminance. To mitigate noise, we use the mean grayscale values from these stable regions rather than single-pixel values to represent L_{max} and L_{min}. To compute the ESF of the ROIs, we first convert the ROIs to grayscale and use MATLAB's MTFdh toolbox [9] for the calculation of ESFs. After obtaining the ESFs, the contrast is calculated using the formula:

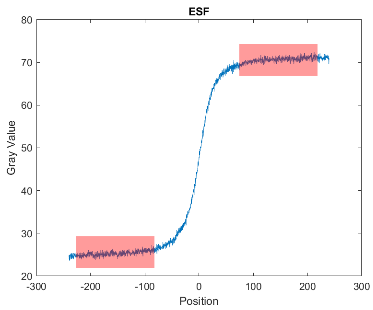

***Figure 3.** An example of ESF. The regions in the red boxes are the regions of minimum and maximum luminance.*

$$C = \frac{\overline{G}_{bright} - \overline{G}_{dark}}{\overline{G}_{bright} + \overline{G}_{dark}} \tag{2}$$

where $\overline{G}_{dark}$ and $\overline{G}_{bright}$ are the average gray values at the dark and bright stable regions. Then we average the four ROIs.

Once the contrast is determined, the haze is calculated using the formula:

$$Haze = (1 - \frac{C_{sample}}{C_{ref}}) \times 100\% \tag{3}$$

where C_{sample} is the contrast of the background look through the sample, and C_{ref} is the contrast of the reference background, which is the background looked through the empty glass container in our experiment setup, to account for any potential offset a glass container may introduce.

Attenuation Coefficient Measurement

We used Dia-Stron TSL850 translucency meter to quantify light attenuation inside the material. The device is equipped with three LEDs that emit light into the material, with peak wavelengths at 630 nm, 525 nm, and 472 nm, respectively, from the normal direction. A 20 mm photodiode array detects the light scattered back from the sample material to estimate the light attenuation. We obtained the effective lateral attenuation coefficient, denoted as μ_{eff}, using the built-in functions of the device. It quantifies the combined absorption and scattering abilities of the material ($\mu_{eff} = \sqrt{3\mu_a(\mu_a + \mu_s)}$, where μ_a and μ_s are absorption and reduced scattering coefficients, respectively [10]). According to the TSL850 manual, the calculated μ_{eff} is proportional to material's properties of absorption and of scattering. Our measurements are consistent with this.

Measurement Framework

In summary, the measurement framework is illustrated in Figure 4. The first step is to measure the attenuation coefficient for a specific liquid with TSL850. This begins by placing the liquid in a 500 mL cylindrical container, ensuring a thickness sufficient to eliminate background influence during measurement. All liquids were measured at an approximate thickness of 100 mm. The TLS850 is then used to measure attenuation. To minimize uncertainty, each liquid sample is measured eight times from different positions and angles, and the mean and standard deviation are computed.

Before capturing samples' images, an image of an empty glass cylindrical container on a checkerboard pattern is captured to serve as the reference for estimating the samples' haze. Then, a smaller portion of the liquid that has been measured with TLS850 is added to this cylindrical container for haze measurement. sample's thickness is measured using a ruler. The container with the liquid sample is then placed on the checkerboard, and images are captured to evaluate the haze. This process is repeated for various thickness levels by gradually adding more liquid to the container.

Once all thickness measurements for a specific liquid are completed, the liquid concentration is adjusted, and the entire procedure is repeated for the new concentration.

Results

Relationship between Haze and Thickness

We first examine the relationship between haze and thickness. Since each material has only four to eight different thickness levels, there is insufficient data to robustly fit a model. Instead, we plot the measured haze index against thickness to gain a conceptual understanding of their relationship. The resulting plot is shown in Figure 5. Different colors and styles are used to distinguish samples derived from different materials. Additionally, samples from the same material are connected by lines to clearly illustrate the trend of haze change as thickness increases. We observe that the haze index values increase with thickness. However, the relationship is logarithmic: as the thickness increases, the rate of increase in the haze index gradually slows down.

Relationship between Haze and $t \cdot \mu_{eff}$

We compute the product of μ_{eff} and the sample thickness (t), which reflects the cumulative light attenuation effect across the sample. We evaluate the mean μ_{eff} across the RGB channels to provide a more generalized perspective. Refer to Fig. 6. To distinguish between the two types of liquid, we use dots to represent milk and use triangles to represent milktea. Different colors indicate different concentrations.

From these plots, We can observe that haze index increases as $t \cdot \mu$ increases. This indicates that for a fixed thickness, the haze index rises when μ_{eff} increases. Since this trend is consistently observed in both milk and milktea samples, we can infer that introducing absorption does not alter the general relationship between haze and μ_{eff}.

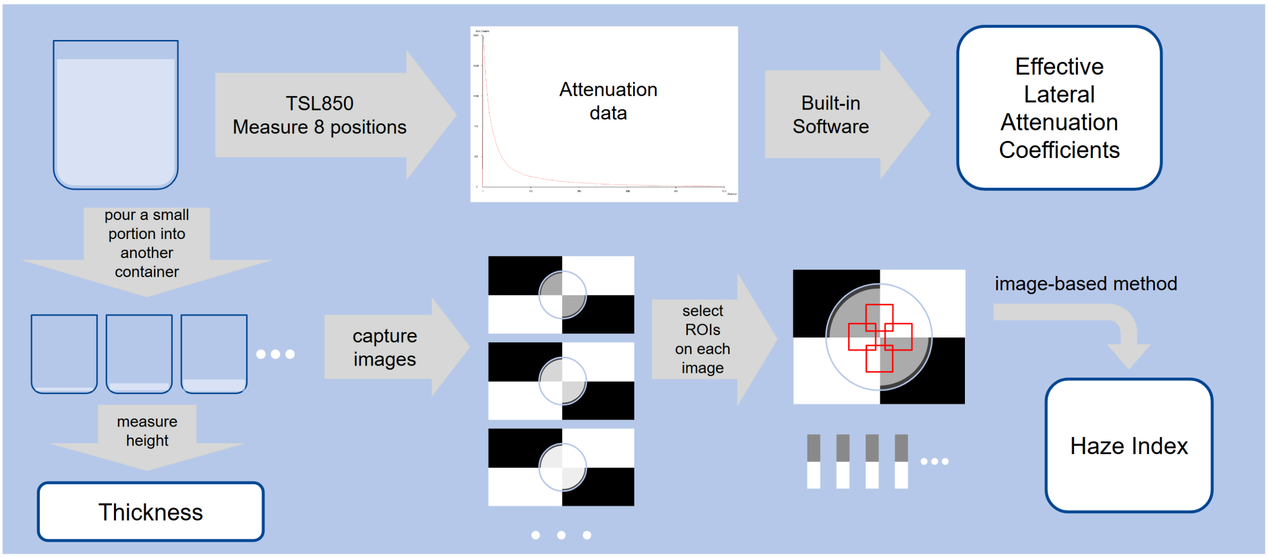

Figure 4. *Proposed measurement framework. The procedure starts by measuring effective lateral attenuation coefficient using a TLS850 device. The amount of liquid is large enough to occlude the background. Afterward, a smaller portion of the liquid (small enough to see through) is moved to a different container, and its thickness is measured with a ruler. Thickness can be varied by adding more liquid. Afterward, the container is placed on a checkerboard and its image is captured, out of which, we extract ROIs and calculate haze index based on ESF.*

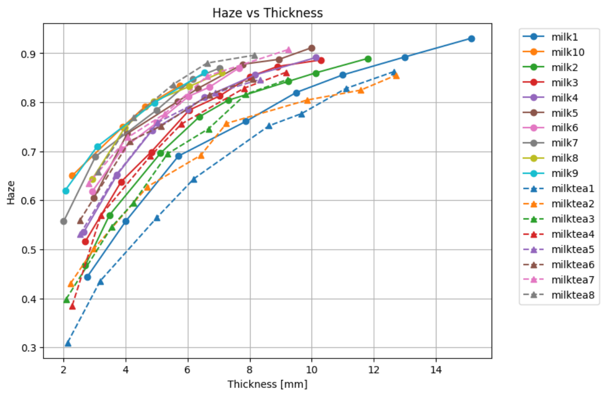

Figure 5. *Haze as a function of thickness.*

A logarithmic relationship can be observed in the figure, suggesting a likely linear relationship between haze and the logarithm of $t \cdot \mu_{eff}$. To test this assumption, we also plot haze with respect to $log_{10}(t \cdot \mu_{eff})$ in the same figure. Another reason for selecting a logarithmic function was the fact that according to Beer-Lambert law, light attenuation in a medium happens exponentially, which a logarithm is expected to linearize. The calculated correlation coefficients for all data and different subsets of data, including milk and milktea, across the R, G, and B channels and their mean, are summarized in Table 1.

It can be observed that most data points lie around a linear curve when logarithm is used. The correlation coefficients presented in Table 1 are notably high, further supporting the assumption of linearity. Based on these observations, we propose using a linear model to represent the relationship between haze and $log_{10}(t \cdot \mu)$. It is also worth noting that, since t is measured in millimeters (mm) and μ_{eff} is in inverse millimeters (mm^{-1}), the term $log_{10}(t \cdot \mu)$ is unitless, consistent with the unitless nature of the haze index.

Table 1. Correlation Between Haze and $\log_{10}(t \cdot \mu_{eff})$

Sample Set	Channel	Correlation Coefficient
milk	Mean	0.9736
milk	R	0.9609
milk	G	0.9710
milk	B	0.9769
milktea	Mean	0.9700
milktea	R	0.9499
milktea	G	0.9706
milktea	B	0.9802
all	Mean	0.9533
all	R	0.9451
all	G	0.9534
all	B	0.9502

Vu *et al.* [11] previously found that the relationship between transmittance and perceptual translucency aligns with Stevens' power law. Inspired by this finding, we also fit our data to a power function model. However, the results did not demonstrate a better fit compared to the logarithmic model.

Linear Regression

In our case, we consider the haze index the dependent variable. We only have one independent variable, $log_{10}(t \cdot \mu_{eff})$. Thus, the linear model can be formulated as:

$$Haze = w_0 + w_1 \cdot log_{10}(t \cdot \mu_{eff}) \qquad (4)$$

We employ the least squares method to fit this linear model to our data. To implement the fitting process, we use the LinearRegression class from the Python machine learning library, scikit-learn. The evaluation metrics of the fit are R^2 and Root Mean Square Error (RMSE). The results of the linear regression for each dataset are summarized in Table 2. Additionally, we visualize the regression lines alongside the corresponding data points in Fig. 7. The closer the data points lie to the regression line, the better the

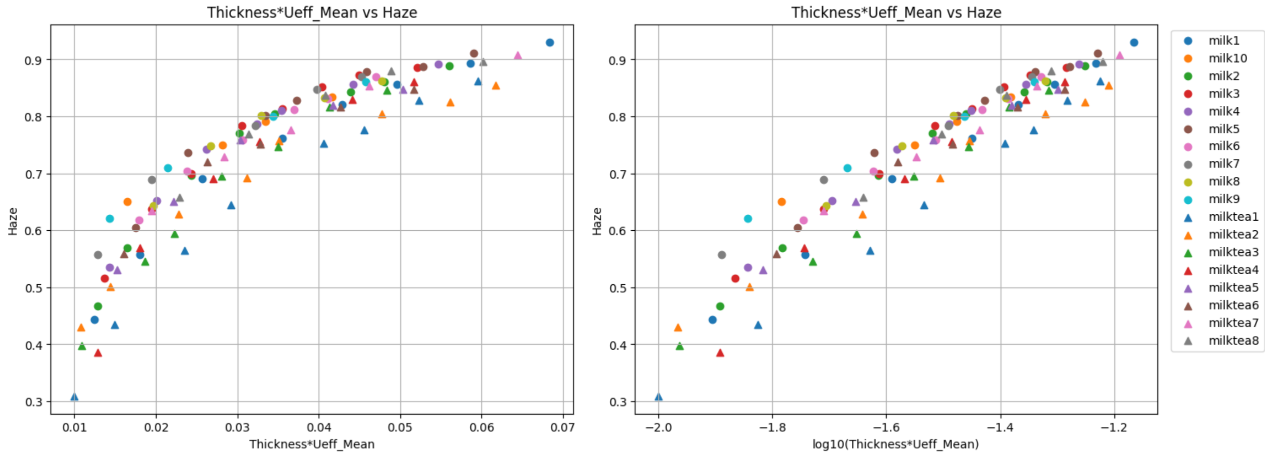

Figure 6. *Haze as a function of $t \cdot \mu_{eff}$ (left) and $log_{10}(t \cdot \mu_{eff})$ (right). The mean of RGB is reported. Similar trend was observed for individual channels.*

Table 2. Regression Results for Different Sample Sets

Sample	Chnl	w_1	w_0	R^2	RMSE
milk	Mean	0.2578	1.6571	0.9480	0.0268
milk	R	0.2533	1.6627	0.9233	0.0325
milk	G	0.2571	1.6495	0.9428	0.0281
milk	B	0.2568	1.6389	0.9543	0.0251
milktea	Mean	0.2879	1.7110	0.9409	0.0363
milktea	R	0.2838	1.7349	0.9023	0.0467
milktea	G	0.2878	1.7071	0.9420	0.0359
milktea	B	0.2883	1.6833	0.9608	0.0296
all	Mean	0.2734	1.6891	0.9089	0.0407
all	R	0.2708	1.7088	0.8932	0.0441
all	G	0.2732	1.6838	0.9090	0.0407
all	B	0.2700	1.6565	0.9028	0.0421

Table 3. Summary of 5-fold validation for Linear Regression Results for Different Sample Sets and Channels

Sample Set	μ_{eff} Channel	Mean R^2	Mean RMSE
milk	Mean	0.947	0.026
milk	R	0.919	0.032
milk	G	0.940	0.028
milk	B	0.955	0.024
milktea	Mean	0.928	0.037
milktea	R	0.881	0.048
milktea	G	0.930	0.036
milktea	B	0.953	0.030
all	Mean	0.893	0.041
all	R	0.875	0.045
all	G	0.893	0.041
all	B	0.884	0.043

model fits the observed data. The numerical results along with the visualizations demonstrate that linear models can effectively capture the relationship between haze and $log(t \cdot \mu_{eff})$.

Model Evaluation

Evaluation solely by R^2 and RMSE values for training data points may not be sufficient due to the risk of overfitting. Therefore, we employ a 5-fold cross-validation approach. This configuration balances the need for an adequate number of training data points while preserving enough validation data to reliably assess the model's generalization performance. The mean R^2 and RMSE values calculated across the folds for each type of liquid and each channel are presented in Table 3. It can be observed that while the evaluation metrics from the cross-validation show slightly worse performance compared to the metrics calculated on the training data (Table 2), the model's performance remains good, suggesting the linear relationship between haze and $log(t \cdot \mu)$.

Discussion and Concluding Remarks

Versatility of the model. This study demonstrates that haze increases with absorption and scattering, providing experimental evidence to support theoretical predictions. By establishing a linear relationship between haze and the logarithm of the product of thickness (t) and the effective lateral attenuation coefficient (μ_{eff}), we present a practical framework for characterizing haze in translucent materials. With measured thickness and μ_{eff}, haze can be predicted using the formula: $Haze = w_0 + w_1 \cdot log_{10}(t \cdot \mu_{eff})$, where (w_0, w_1) slightly vary across material types and chromatic channels but are approximately $(1.1619 \pm 0.0283, 0.2717 \pm 0.1267)$.

Considering that $log_{10}(t \cdot \mu_{eff}) = log_{10}(t) + log_{10}(\mu_{eff})$, any of the parameters can be easily estimated if the other two are known. For instance, with image-based measurement of haze and physical measurement of material's thickness, its attenuation coefficient can be estimated without TLS850. In computer vision applications, haze measurement of a material with known μ_{eff} may be useful for estimating thickness.

Better fitness for milk than milktea. From Table 3, we observe that the model built using all data shows slightly worse fit performance compared to the models built separately for milk or milktea samples. This suggests that the optimal model representing the relationship between haze and $t \cdot \mu_{eff}$ may differ for these two material types. Moreover, the model fits milk samples better than milktea samples, as the latter involves more absorption in addition to scattering. Absorption and scattering may contribute to haze in slightly different ways. For milk samples, which primarily exhibit scattering, the linear relationship between haze and $log_{10}(t \cdot \mu_{eff})$ is stronger. In contrast, milktea samples, which involve the absorption component, introduce additional complexity that the linear model struggles to capture. Future studies to inves-

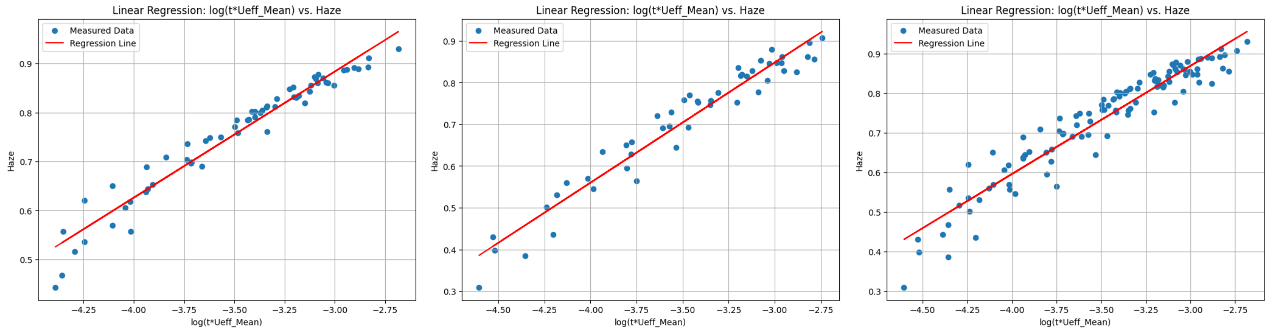

Figure 7. *Visualization of linear regression on milk (left), milktea (middle), and all (right) samples. Average of all channels.*

tigate absorption and scattering individually are needed.

Worse fitness in R channel. When comparing the fitness across channels, it shows that the R channel has the worst performance, as reflected by lower R^2 and higher RMSE. This indicates that the relationship between haze and the optical properties of absorption and scattering is wavelength-dependent. A deeper investigation should be conducted in the future.

Worse fitness in low $t \cdot \mu$ value regions. Observing the visualization of the linear regression model in Fig. 7, higher deviations in the low $t \cdot \mu$ value regions, which are located on the left side of the plots, are evident. These deviations indicate that the linear model struggles to fit the data accurately in this range. One possible explanation is measurement inaccuracies for very small values, where errors are often more pronounced. Alternatively, this observation may suggest the presence of a nonlinear relationship between haze and $log_{10}(t \cdot \mu)$ in low $t \cdot \mu$ value regions, which was previously observed in [12]. Therefore, more precise measurement methods are necessary for samples with low $t \cdot \mu$ values. In the future, noise correction or non-linear modeling can be used to better capture the behavior in this range.

Limitation of the Logarithmic Model. Logarithmic functions inherently approach negative infinity as the independent variable approaches zero, which can theoretically result in negative haze index values. Although this scenario is unlikely to occur in practice, it limits the model's theoretical comprehensiveness. For example, consider the model for haze and the mean μ_{eff} of milk materials:

$$Haze = 1.6571 + 0.2578 \cdot log_{10}(t \cdot \mu_{eff}) \tag{5}$$

According to this equation, a negative haze index would occur if $t \cdot \mu_{eff} < 3.73 \times 10^{-7}$. This value is far below the sensitivity range of ordinary measurement techniques and is therefore not practically relevant. However, it does indicate a fundamental limitation of the logarithmic model in representing haze for extremely thin or very transparent (low μ_{eff}) materials.

Future Work. Several questions remain that merit future work: first, more, beyond lab-scale materials and wavelength-dependent nature of haze should be covered. Second, in addition to attenuation coefficient, the relationship with haze can be more accurately modeled by measuring absorption and scattering coefficients separately. The method by Nguyen *et al.* [10] can be used for this. Furthermore, w_0 and w_1 parameters should be investigated for specific materials and determining factors should be found for their values. A more advanced model should also include the exact concentrations of milk and black tea. Our approach can be also tested against specialized commercial instruments (e.g. Novo-Haze TX Transmission Hazemeter). Apart from this, it is useful also to model the second translucency attribute – *clarity*, for which, Busato *et al.* [5] proposed a measurement method based on MTF – Modulation Transfer Function. And finally, the findings should be verified with subjective psychophysical studies.

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