

Transforming Cone Fundamentals to Color Matching Functions for use by iccMAX-based color management

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Abstract

The use of the 1939 standard colorimetric observer for modern display calibration and color management has proven to be problematic in that it doesn't adequately predict the color matching of actual human observers. Recent research has resulted in the ability to identify physiologically based Cone Fundamental (CF) curves that more accurately predict individual color matching. However, the challenge still remains of adapting CF curves for the use of imagery and color specification based on the 2-degree 1931 standard observer. The use of iccMAX-based color management requires well defined relationships between custom observers and the 2-degree 1931 standard observer to be provided.

In this paper, mathematical relationships and principles between cone fundamentals and color matching functions relative to viewing primary lights are outlined. Methods of conversion between cone fundamentals and color matching functions are explored and compared along with the proposed use of Wpt based material adjustment transforms to create color matching functions that provide backwards compatibility with legacy standard observer colorimetry.

Introduction

Advances in vision science have provided measurements and understanding of the variability of the various components of the human visual system.^{12,13,14} This allows for the direct modeling of the cone excitation responses (cone fundamentals) used by the visual system to determine color perception. An example of this is the CIE 2006 observer cone fundamentals that are determined for variations of age and field size.^{3,11}

Using cone fundamentals effectively is a challenge because a large part of color science, as it has evolved over the last century, has been based on tristimulus values that result from the use of color matching functions. Examples include: color appearance models⁶, color spaces⁹, color difference equations¹, vision models⁸, and other means of identifying or representing relationships between colors.

More importantly, the vast majority of color imagery is based on the standard 1931 observer¹⁹ and the basis of ICC profile connection¹⁸ is the 1931 standard observer. Additionally, color management using iccMAX²⁰ profiles allows for alternate observers to the 1931 standard observer to be used for color profile connection, but there is still a requirement that the custom color matching functions be defined having well defined relationships to the 1931 standard observer.

Establishing meaningful relationships and transformations between cone fundamentals and color matching functions that will enable an effective use of cone fundamentals with existing color science approaches and iccMAX color management is the focus of this work.

Relationships between cone fundamentals and color matching functions

The metrology of color (colorimetry) is based on the principle of color matching. Normal human color vision is trichromatic - having only three types of photoreceptors (cones) for determining color perception.¹⁷ Additionally, the human visual system (HVS) has experimentally been determined (to a first approximation) to be linear and additive.¹⁷

A color matching system can be described as an apparatus with a view of a bipartite field that allows for a colored light in one field to be compared and matched with a combination of primary lights in the second field. A color match is determined with such a system by an observer adjusting the intensities of the primaries until a match to the test stimulus is found (with negative primary intensities represented as intensities of the primaries added to the test stimulus). A color matching system can be mathematically described as the relation involving the excitation response of the cones (cone fundamentals) to both the spectral power distribution (SPD) of a test light as well as the SPD resulting from a linear combination of three primary lights used for color matching.¹⁷ Thus:

$$\mathbf{F}\mathbf{s} \sim \mathbf{F}\mathbf{P}\mathbf{c} \quad (1)$$

Where:

The \sim is a perceptual equivalence operator indicating that the two stimuli match perceptually to an observer with cone fundamentals defined by \mathbf{F} ,

\mathbf{F} is a matrix representing the cone fundamentals of the observer's HVS,

\mathbf{s} is a column vector containing the SPD of a test light

\mathbf{P} is a matrix containing column vectors with the SPD for each of the primary matching lights

\mathbf{c} is a column vector that indicates the relative intensities of the primaries needed to obtain a visual match by the observer with cone fundamentals \mathbf{F} .

Directly determining cone fundamentals is challenging because the cone excitation responses are a result of multiple aspects of the visual system (including optics, pigment encoding, and neural processing).¹² As colorimetry was developed, the direct determination of cone fundamentals was not performed. Instead, combinations of primaries were determined for color matches for single wavelengths of light.¹⁶ Thus:

$$\mathbf{F}\mathbf{s}_\lambda \sim \mathbf{F}\mathbf{P}\mathbf{c}_\lambda \quad (2)$$

Where:

\mathbf{F} , and \mathbf{P} are the same as in Eq. 1,

\mathbf{s}_λ is a column vector containing the SPD of a test light that only has a "unit" amount of light with a wavelength of λ ,

\mathbf{c}_λ is a column vector that indicates the relative intensities of the primaries needed to obtain a visual match of a light defined by \mathbf{s}_λ .

A set of color matching functions for an observer can then be defined by combining the results of the set of observer's experimental matches related to Eq. 2 as follows:

$$\begin{aligned} \mathbf{F}[s_{\lambda_1} \dots s_{\lambda_n}] &\sim \mathbf{FP}[c_{\lambda_1} \dots c_{\lambda_n}] \\ \mathbf{FI} &\sim \mathbf{FPC} \\ \mathbf{F} &\sim \mathbf{FPC} \end{aligned} \quad (3)$$

Therefore, a color matching system provides a relationship between cone fundamentals, color matching primaries, and color matching functions as defined in the following relationship:

$$\mathbf{Fs} \sim \mathbf{FPCs} \quad (4)$$

The essence of Eq. 4 is that the color matching functions with associated color matching primaries (**PC**) convert the SPD of the test light into a metamer of the test light for the observer with cone fundamentals defined by **F**.

Interestingly, based on Eq. 4 the following observations can be made:

1. Any invertible transformation of the cone fundamentals doesn't change the color matching characteristics of the system (Luther-Ives condition).^{7,10} Thus:

$$(\mathbf{MF})\mathbf{s} \sim (\mathbf{MF})\mathbf{PCs} \quad (5)$$

2. Color matching functions can be expressed as a linear transform of the cone fundamentals. This combined with the previous observation means that color matching functions can be used in place of cone fundamentals for defining relationships in color matching systems.² Thus:

$$(\mathbf{FP})^{-1}\mathbf{Fs} \sim \mathbf{Cs} \quad (6)$$

$$\mathbf{MFs} \sim \mathbf{Cs} \quad (7)$$

3. The choice of primaries and color matching functions is not singular. Any simultaneous manipulation of the primaries and color matching functions that result in transforming the test stimulus to a metamer of the test stimulus for the observer can be used. Thus:

$$\begin{aligned} \mathbf{Fs} &\sim \mathbf{FPM}^{-1}\mathbf{MCs} \\ \mathbf{Fs} &\sim \mathbf{F}(\mathbf{PM}^{-1})(\mathbf{MC})\mathbf{s} \\ \mathbf{Fs} &\sim \mathbf{FP}_M\mathbf{C}_M\mathbf{s} \end{aligned} \quad (8)$$

4. The use of metamers of the primaries for color matching implies no change in the cone fundamentals or the color matching functions. This means that the choice of primaries for color matching is not unique.⁴ Thus:

$$\begin{aligned} \mathbf{Fs} &\sim \mathbf{FP}_B\mathbf{Cs} \\ \mathbf{Fs} &\sim \mathbf{F}(\mathbf{P} + \mathbf{B})\mathbf{Cs} \\ \mathbf{Fs} &\sim \mathbf{FPCs} + \mathbf{FBCs} \\ \mathbf{Fs} &\sim \mathbf{FPCs} + \emptyset \\ \mathbf{Fs} &\sim \mathbf{FPCs} \end{aligned} \quad (9)$$

Where:

B defines a metameric black vector.

5. The pseudo-inverse of the color matching functions can be used as a satisfactory replacement of the primaries. This is due to the fact

that the result of multiplying matrices **PC** is functionally equivalent to an identity matrix for the purposes of color matching. Thus:

$$\begin{aligned} \mathbf{Fs} &\sim \mathbf{FPCs} \\ &\vdots \\ \mathbf{I} &= \mathbf{PC} \\ \mathbf{P} &= \mathbf{C}^T(\mathbf{CC}^T)^{-1} = \mathit{pinv}(\mathbf{C}) \end{aligned} \quad (10)$$

6. Conversely, the color matching functions predict the primaries. Therefore, the pseudo-inverse of the primaries can be used as a satisfactory replacement of the color matching functions. Thus:

$$\begin{aligned} \mathbf{I} &= \mathbf{CP} \\ \mathbf{C} &= \mathbf{P}^T(\mathbf{PP}^T)^{-1} = \mathit{pinv}(\mathbf{P}) \end{aligned} \quad (11)$$

7. Color matches for the observer can simply be defined by the intensities of the primaries (or tristimulus values) without direct mention or use of the cone fundamentals or primaries themselves. This is because for the purposes of color metrology, the interim primary intensities provided by the color matching functions in Eq. 4 give a satisfactory metric for defining a color match for the observer.¹⁶ Thus:

$$\mathbf{c} = \mathbf{Cs} \quad (12)$$

Color matching basis for transforming fundamentals

Having established relationships between cone fundamentals and color matching functions for a single observer in the previous section, we now consider establishing relationships between color matching functions of a reference observer and cone fundamentals for alternate observers.

It is proposed that there are two general approaches for defining transforms of cone fundamentals to color matching functions relative to a reference observer. Both can be defined relative to the use of one or more color matching systems. The first approach involves preserving the color matching primaries, the second involves preserving the tristimulus values.

Transformations that preserve color matching primaries

The first approach for converting cone fundamentals to color matching functions involves a preservation of color matching primaries. From an observational perspective, an apparatus is set up for performing trichromatic color matching. All observers use the same three primary lights (real or physically non-realizable) to perform color matching. The primaries are linearly independent of one another and based upon the color matching functions of a reference observer. The color matching functions for alternate observers is found by application of Eq. 6 and Eq. 10 as follows:

$$\begin{aligned} \mathbf{C}_{\text{alt}} &= (\mathbf{F}_{\text{alt}}\mathbf{P}_{\text{ref}})^{-1}\mathbf{F}_{\text{alt}} \\ \mathbf{C}_{\text{alt}} &= \left(\mathbf{F}_{\text{alt}}\mathbf{C}_{\text{ref}}^T(\mathbf{C}_{\text{ref}}\mathbf{C}_{\text{ref}}^T)^{-1} \right)^{-1} \mathbf{F}_{\text{alt}} \end{aligned} \quad (13)$$

The color matching functions found using Eq. 13 are the result of performing pseudo-inverse operations. This essentially corresponds to applying a least-squares optimization to predict the primaries. This means that the prediction of the primaries may only be approximate. An additional step can be performed to ensure that the scale of the prediction of the tristimulus **Y** primary is the same by scaling the color matching functions uniformly. This results in no

change in chromaticity and corresponds to a slight uniform change in intensity of all the primaries. This is done as follows:

$$\begin{aligned} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} &= \mathbf{C}_{\text{ref}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} &= \mathbf{C}_{\text{alt}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \mathbf{C}'_{\text{alt}} &= k\mathbf{C}_{\text{alt}} \\ \text{Where: } k &= \left(\frac{y_1}{y_2}\right) \end{aligned} \quad (14)$$

It is important to note that the color matching functions defined in \mathbf{C}'_{alt} may only approximate the primaries defined by \mathbf{C}_{ref} . They represent the closest possible expression of color matching with a metamer/paramer of the primaries for the reference observer.

Prediction of Primaries by Observers

From Eq. 11, if the color matching functions for a set of observers are all based on the same primaries (defined by the matrix \mathbf{P}) then the color matching functions applied to this matrix will always result in an identity matrix. Thus:

$$\mathbf{I} = \mathbf{C}_i \mathbf{P} \text{ for each observer } \mathbf{C}_i \quad (15)$$

When color matching functions for the i^{th} observer produce the same set of primaries (and the primaries are unknown), then the common primaries can be found using Eq. 15 with a regression that includes all the observers. Thus:

$$\begin{aligned} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \vdots \\ \mathbf{I}_n \end{bmatrix} &= \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \vdots \\ \mathbf{C}_n \end{bmatrix} \mathbf{P} \\ \mathbf{P} &= \text{pinv} \left(\begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \\ \vdots \\ \mathbf{C}_n \end{bmatrix} \right) \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \vdots \\ \mathbf{I}_n \end{bmatrix} \end{aligned} \quad (16)$$

Where:

\mathbf{P} is a matrix containing the primaries,
 \mathbf{C}_i is a matrix containing color matching functions for the i^{th} observer, and
 \mathbf{I}_i is an identity matrix for the i^{th} observer.

When estimating a set of primaries for a set of color matching functions it is important to apply the estimated primaries to the color matching functions, thus ensuring that they always result in an identity matrix. Otherwise, it is likely that the set of color matching functions are not based on a common set of primaries, and this may result in the tristimulus values from the observers having no comparable basis relative to a single set of color matching primaries.

Transformations that preserve tristimulus values

The second approach for converting cone fundamentals to color matching functions involves a preservation of tristimulus values. From an observational perspective, an apparatus is used for performing trichromatic color matching with a set of three primary lights (real or physically non-realizable) that can easily be replaced. A reference observer picks a set of primaries to match colors with the apparatus. Alternate observers then individually use a potentially different set of personal primary lights (real or imaginary) that result in color matches that have identical (or as close as possible) primary

intensity combinations as the primary intensity combinations for the reference observer for the same matched colors.

In other words, differences in tristimulus values are minimized and primaries are allowed to vary. The relationship between reference color matching functions and alternate cone fundamentals is established using Eq. 7 and Eq. 12 in the following manner:

$$\begin{aligned} \mathbf{C}_{\text{alt}} &\approx \mathbf{C}_{\text{ref}} \\ \mathbf{C}_{\text{alt}} \mathbf{s} &\approx \mathbf{C}_{\text{ref}} \mathbf{s} \\ \mathbf{M} \mathbf{F}_{\text{alt}} &\approx \mathbf{C}_{\text{ref}} \\ \text{where:} \\ \mathbf{C}_{\text{alt}} &= \mathbf{M} \mathbf{F}_{\text{alt}} \end{aligned} \quad (17)$$

The minimization technique used to determine the conversion matrix (\mathbf{M}) in Eq. 17 may or may not encompass some sense of shared perception by the different observers. Three different methods will be discussed.

Direct Regression

With the first technique of determining conversion matrix \mathbf{M} the pseudo-inverse of alternate cone fundamentals is applied to the reference color matching functions.² Thus:

$$\begin{aligned} \mathbf{M} \mathbf{F}_{\text{alt}} &\approx \mathbf{C}_{\text{ref}} \\ \mathbf{M} &\approx \mathbf{C}_{\text{ref}} \text{pinv}(\mathbf{F}_{\text{alt}}) \\ &\vdots \\ \mathbf{C}_{\text{alt}} &= \mathbf{C}_{\text{ref}} \mathbf{F}_{\text{alt}}^T (\mathbf{F}_{\text{alt}} \mathbf{F}_{\text{alt}}^T)^{-1} \mathbf{F}_{\text{alt}} \end{aligned} \quad (18)$$

Color Correction

The method used for camera color correction can be applied to define matrix \mathbf{M} which has similarities to the Direct Regression approach. In this case tristimulus values are determined for a set of known material objects (aka patches on a color checker chart) for both the reference and alternate observer. Matrix \mathbf{M} then represents a linear transform that approximates tristimulus values for the alternate observer from tristimulus values for the reference observer. Matrix \mathbf{M} can be found using the pseudo-inverse of the tristimulus values as follows:

$$\begin{aligned} \mathbf{M} \mathbf{c}_{\text{alt}} &\approx \mathbf{c}_{\text{ref}} \\ \mathbf{M} [\mathbf{c}_{1,\text{alt}} \dots \mathbf{c}_{n,\text{alt}}] &\approx [\mathbf{c}_{1,\text{ref}} \dots \mathbf{c}_{n,\text{ref}}] \\ &\vdots \\ \mathbf{M} &= [\mathbf{c}_{1,\text{ref}} \dots \mathbf{c}_{n,\text{ref}}] \text{pinv}([\mathbf{c}_{1,\text{alt}} \dots \mathbf{c}_{n,\text{alt}}]) \end{aligned} \quad (19)$$

Where:

$\mathbf{c}_{i,\text{obs}}$ represents the tristimulus values for the i^{th} object for the obs observer.

Unlike the direct regression approach, in this case matrix \mathbf{M} is dependent on the set of material objects chosen for the regression. The distribution and spectral selectivity of the objects will impact the sameness of tristimulus values for arbitrary color matches.

Normalized white point adjustment

This technique involves using a reference white to “normalize” or “adapt” the tristimulus values so that they match for a reference white or illuminant. This can be done in conjunction with the direct regression or color correction methods. This involves determining tristimulus values of the white/illuminant for both the reference observer and the adjusted alternate observer and then adjusting the \mathbf{M} matrix² as follows:

$$\begin{bmatrix} X_{\text{ref}} \\ Y_{\text{ref}} \\ Z_{\text{ref}} \end{bmatrix} = \mathbf{C}_{\text{ref}} \mathbf{s}_{\text{ill}}$$

$$\begin{bmatrix} X_{\text{alt}} \\ Y_{\text{alt}} \\ Z_{\text{alt}} \end{bmatrix} = \mathbf{M}_{\text{alt}} \mathbf{F}_{\text{alt}} \mathbf{s}_{\text{ill}} \quad (20)$$

$$\mathbf{M} = \begin{bmatrix} X_{\text{ref}}/X_{\text{alt}} & 0 & 0 \\ 0 & Y_{\text{ref}}/Y_{\text{alt}} & 0 \\ 0 & 0 & Y_{\text{ref}}/Y_{\text{alt}} \end{bmatrix} \mathbf{M}_{\text{alt}}$$

Where:

X, Y, Z are tristimulus values

\mathbf{C}_{ref} is the CMF of the reference observer

$\mathbf{M}_{\text{alt}} \mathbf{F}_{\text{alt}}$ is the CMF of the transformed

\mathbf{s}_{ill} is the SPD of the reference white / illuminant

Normalizing the color matching functions to predict the same tristimulus values for an illuminant results in color matching functions having a different set of primaries for each observer.

Using a Waypoint Material Adjustment Transform

A fourth technique is proposed of converting cone fundamentals to color matching functions preserving tristimulus values that defines matrix \mathbf{M} in Eq. 7 using a Material Adjustment Transform (MAT). In previous research, a method was proposed that establishes a waypoint (Wpt) normalization matrix \mathbf{A} for a given observer and illuminant combination that transforms cone excitations / tristimulus values to an opponency based unified representation of color designated by the coordinates (W, p, t). Wpt normalization minimizes differences due to changes in observer and/or illuminant by minimizing differences in lightness, chroma, and hue based on a set of reference spectral reflectance colors.⁵

Conversion from cone fundamentals to color matching functions can be accomplished by applying a Material Adjustment Transform (MAT) that is defined by concatenating a Wpt normalization matrix optimized for the cone fundamentals with an inverse Wpt normalization matrix optimized for the reference color matching functions with both matrices optimized using the same illuminant that defines the white point. Thus:

$$\mathbf{M} = (\mathbf{A}_{\text{alt}})^{-1} \mathbf{A}_{\text{basis}} \quad (21)$$

$$\mathbf{C}_{\text{alt}} = (\mathbf{A}_{\text{alt}})^{-1} \mathbf{A}_{\text{basis}} \mathbf{C}_{\text{basis}}$$

Where:

$\mathbf{A}_{\text{basis}} = T(\mathbf{C}_{\text{basis}}, \mathbf{L}_{\text{basis}})$ uses the Wpt normalization procedure $T()$ based on an observer defined by $\mathbf{C}_{\text{basis}}$, and basis illuminant defined by $\mathbf{L}_{\text{basis}}$, and

$\mathbf{A}_{\text{alt}} = T(\mathbf{F}_{\text{alt}}, \mathbf{L}_{\text{basis}})$ uses the Wpt normalization procedure $T()$ based on an observer defined by \mathbf{F}_{alt} , and basis illuminant defined by $\mathbf{L}_{\text{basis}}$.

To ensure that the alternate color matching functions have the same relative scale, Eq. 21 can be used substituting \mathbf{C}_{ref} for $\mathbf{C}_{\text{basis}}$. This is akin to having the reference observer establish tristimulus values for white as well as colors that all have the same perceived lightness and chroma, and then adjusting the tristimulus values of these same

colors for the alternate observer so that the tristimulus values of the white match with minimized differences in tristimulus values for the other colors.

Using Custom Observers with iccMAX

Legacy ICC color management¹⁸ uses Profile Connection Spaces based on colorimetry for the 1931 Standard 2-degree observer under a D50 illuminant. A PCS provides a device-independent bridge between transforms that use device-specific color encodings. All transforms are encoding using D50/Standard 2-degree colorimetry with chromatic adaptation required for different viewing conditions. The iccMAX color management²⁰ framework allows for both custom observers and lighting conditions to be used for either determining colorimetry from spectral color representations or for connecting profiles with a custom colorimetric Profile Connection Space (PCS).

Profile Connection Conditions

Three Profile Connection Condition (PCC) tags are required in a profile when spectral transforms or custom colorimetry is used. The Spectral Viewing Conditions (SVCN) tag defines the observer and lighting used for the colorimetry encoded in the colorimetric profile transforms. Thus the SVCN tag provides the encoding of custom color matching functions as well as the spectral power distribution of the desired illuminant/light source for the colorimetric PCS associated with a profile.

The Custom To Standard PCC (C2SP) and Standard To Custom PCC (S2CP) tags provide encoding of transforms (usually as 3x3 matrices) that convert between standard D50/Standard 2-degree colorimetry and custom colorimetry when profiles are connected that use colorimetry for different observers or lighting conditions.

Additionally, PCC tags from alternate iccMAX files can be substituted when making profile connections thus configuring both the late binding of the observer and lighting when spectral data is involved as well as the conversions between custom and standard colorimetry.

Considerations for PCC creation

There is some flexibility in how PCC tags are populated provided that the C2SP/S2CP tags are consistent with the SVCN tag. Different linear transforms of color matching functions in the SVCN tag result in corresponding linear transforms of the C2SP/S2CP tags, and the choice of the illuminant/lighting SPD also results in corresponding adjustments to the C2SP/S2CP tags. Let's consider a soft proofing scenario for the purposes of populating PCC tags for the use in iccMAX display profiles involving two different displays and a scene of one or more physical objects under a light source with the scene depicted on both displays.



Figure 1. Soft proofing scenario

Ideally there is a visual match between the two displays and the scene. Thus, light from corresponding points on the displays and

the scene is metameric which can be expressed mathematically as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{d1} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{d2} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{scene} \quad (22)$$

Where:

$[X \ Y \ Z]_s^T$ represents the colorimetry of situation s .

Typically, displays use a D65 white point so it is also desirable that the colorimetry of the display white points matches the colorimetry of a D65 illuminant. Thus:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{white,d1} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{white,d2} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{D65} \quad (23)$$

Therefore, for display color management it is proposed that the SVCN tag be populated with the SPD of the D65 illuminant.

Equations 22 and 23 use common observer CMFs and work equally well for any observer if the CMFs correctly correspond to the observer. However, relationships between a custom observer and the standard observer are needed for populating PCC tags in iccMAX profiles. Therefore, for the purpose of populating a SVCN tag it is proposed that the following relationships are also achieved:

$$\begin{bmatrix} X_{obs} \\ Y_{obs} \\ Z_{obs} \end{bmatrix}_{D65} = \begin{bmatrix} X_{std} \\ Y_{std} \\ Z_{std} \end{bmatrix}_{D65} \quad (24)$$

$$\begin{bmatrix} X_{obs} \\ Y_{obs} \\ Z_{obs} \end{bmatrix}_{scene,D65} \cong \begin{bmatrix} X_{std} \\ Y_{std} \\ Z_{std} \end{bmatrix}_{scene,D65} \quad (25)$$

Scene reflectances generally use smooth and broad spectral reflectances that have less color inconstancy than display primaries. Additionally, having a well-defined relationship with the standard observer helps with populating the C2SP/S2CP tags.

Comparisons of CF to CMF approaches

Preservation of primaries, direct regression with white point normalization, color correction with white point normalization, and Wpt MAT approaches were used to generate CMFs for CIE 2006 2-degree CFs, Asano's ten 2-degree categorical observers²², and the CIE 2015 2-degree observer.²³ The resulting CMFs are shown in Figure 2.

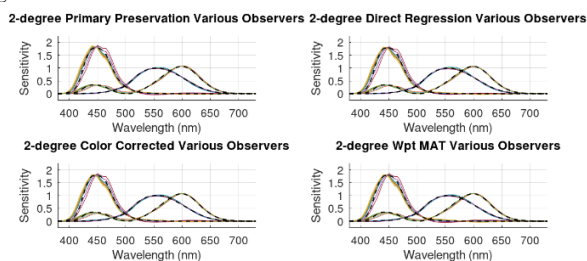


Figure 2. CMFs from different transform approaches. Top-left: primary preservation, top-right: direct regression, bottom-left: color corrected, bottom-right: Wpt MAT

The resulting CMFs for each of the approaches are very similar with only slight differences being noted.

Primaries for the standard 2-degree observer were found using Eq. 10 and are shown in Figure 3.

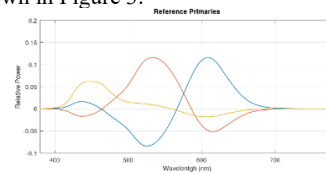


Figure 3. Primary SPDs associated with the standard 2-degree observer

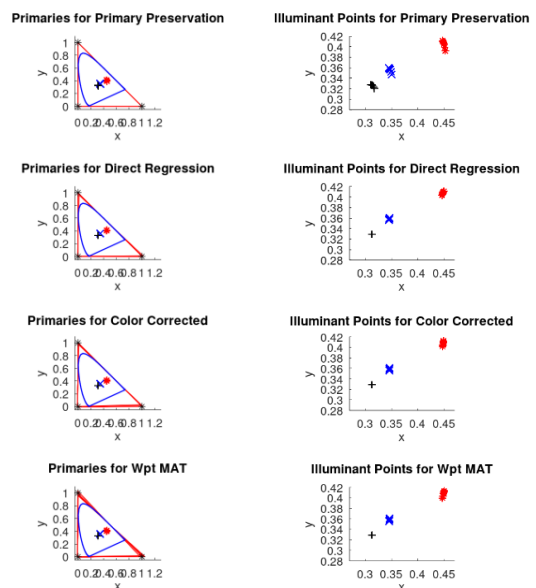


Figure 4. left: Prediction of observer primaries; right: observer D65 (+) D50 (X) and illuminant A (*) chromaticity points; top: primary preservation; second row: direct regression; third row: color corrected; bottom: Wpt MAT

Prediction of primary chromaticity point ranges, and D65 and D50 illuminant chromaticity points were determined for the same set of observers for each of the CMF conversion approaches with results shown in Figure 4.

The difference between primary preservation and preserving tristimulus values is evident in Figure 4 with the primary preservation approach exactly predicting the primary chromaticity points with variability for the other tristimulus preserving approaches. The Wpt MAT approach appears to have the greatest variability in primary prediction. Conversely, the tristimulus preserving approaches exactly predict the chromaticity points of the D65 illuminant due to normalization being applied in each case. These results indicate that tristimulus preserving approaches satisfy the conditions of Eq. 24.

CIELAB values from tristimulus values were determined for the reflectances from the Standard Object Color Spectra database (SOCS)²¹ under D65 for the standard 1931 2-degree observer and adjusted observers used in Figure 2. CIEDE2000 color differences were then made between the reference observer and adjusted

observes with mean, 95%, and maximum color differences. The results are shown in Table 1.

Table 1 - SOCS Reflectance CIEDE2000 Color Differences

	Mean	95 th percentile	Max
<i>Predict Primary</i>	0.56	1.56	8.62
<i>Direct Regression</i>	0.53	1.45	7.17
<i>Color Correction</i>	0.55	1.57	6.91
<i>Wpt MAT</i>	0.61	1.77	9.99

The mean results are close and generally indicate on average the differences are below the threshold of being noticeable for all approaches. The Direct Regression approach appears to have the best mean and 95th percentile results, while the Color Correction approach had the lowest maximum difference. In each of these cases it appears that all the approaches satisfy the conditions of Eq. 25.

Conclusions

Various relationships between cone fundamentals and color matching functions were detailed, leading to discussing methods of converting cone fundamentals to color matching functions by preserving color matching primaries or preserving tristimulus values. It was observed that for iccMAX color management it is desirable for tristimulus values from adjusted CMFs to match the tristimulus values for the standard D65 illuminant as well as closely matching the results for surface objects. This results in custom CMFs that are interchangeable with the standard 2-degree observer. Use in iccMAX profiles will also require PCC transforms between D50 colorimetry and D65 colorimetry.

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