https://doi.org/10.2352/CIC.2023.31.1.39

Analyzing perceptual uniformity using Jacobian determinants

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Abstract

The CIE *LAB* color space was established by color scientists to be approximately perceptually uniform. The $Y'C_BC_R$ color space is widely assumed by video engineers to be approximately perceptually uniform. However, these two color spaces have quite different transforms from tristimuli (radiometric) coordinates; they clearly cannot have the same perceptual performance.

It is instructive to ask: Where in color space is $Y'C_BC_R$ quantization the worst, when evaluated in terms of *LAB*? And, conversely, where in color space does *LAB* quantize most coarsely, when evaluated in Euclidean $Y'C_BC_R$ difference?

The *Jacobian* corresponding to a color coordinate in 3-space is a 3×3 matrix of partial derivatives. The determinant of that matrix is analogous to volume. We compute numerical Jacobian determinants to explore how unit *LAB* volumes at sample points spanning a target color space map to volumes in $Y'C_BC_R$. Where that volume is quite large, we expect poor perceptual performance of $Y'C_BC_R$. Where that volume is quite small, $Y'C_BC_R$ is over-quantizing, and may have poor codeword utilization – but in such regions it's reasonable to suspect the performance of *LAB*.

We present "heat maps" that visualize where $Y'C_BC_R$ performs poorly compared to *LAB* (and vice versa).

Background

Helmholtz, Schrödinger, Stiles, MacAdam and others explored the mapping from radiometric spaces to perceptually uniform spaces using "line elements." MacAdam used the line element concept to establish his colour difference ellipsoids – or, when projected to two dimensions, ellipses. In the classic book by Wyszecki & Stiles [5, pp. 654–689], the last section (8.4), comprising fully 5% of the body text of the book, is devoted to line elements.

Line elements are usually formulated to establish an analytic mapping from a radiometric space to a perceptual space. Here, we have a more modest goal: We seek to compare spaces that are intended to be roughly perceptual uniform to begin with, and we analyze color spaces numerically instead of analytically.

Since its standardization by CIE in 1976, *LAB* has been presumed to be perceptually uniform – that is, unit Euclidean distance in *LAB* coordinates approximates a just noticeable difference (JND), or equivalently, a just unnoticeable difference.

Since the standardization of color television in 1953, analog luma and chroma component signals [*Y′, U, V*] have been presumed to be perceptually uniform, based upon the observation that analog noise is approximately equally distributed throughout color space. In modern times, digital luma/chroma components [Y', C_B, C_R] have been presumed to be perceptually uniform, where quantization error takes the place of analog noise. In eight-bit digital video components, unit difference is commonly taken to approximate one JND. Ten-bit components were standardized, and are now widely used. Ten-bit components are assumed to have performance somewhat better than one JND for a unit increment in any component. Ten-bit components are used to minimize accumulation of error from cascaded processing and compression/decompression steps. Details are found in Poynton's text [2].

A long-standing question of the present authors is this: If color scientists consider *LAB* representation to be roughly perceptually uniform, and video engineers consider eight-bit $Y'C_BC_R$ to be roughly perceptually uniform, how do those two schemes compare? To elaborate: At what colors in $Y'C_BC_R$ space does unit increment in any of the three coordinates $Y'C_BC_R$ produce *LAB* ΔE that is significantly larger than unity? That condition would indicate a region in $Y'C_BC_R$ that is likely to exhibit quantization problems. Conversely, at what colors in $Y'C_BC_R$ space does unit increment in any of the three coordinates produce and ΔE that is significantly smaller than unity? That condition would indicate a region in $Y'C_BC_R$ that is over-quantized; $Y'C_BC_R$ space would be wasteful of coding values in any such region.

We analyze $Y'C_BC_R$; however, with suitable scaling, *R′G′B′* (eg, sRGB) would yield identical results, subject only to our discrete approximations. The mapping from *R′G′B′ to* $Y'C_BC_R$ is an affine transform, and for $Y'C_BC_R$ scaled [0…1, ±1, ±1], the unit volume of *R′G′B′* is preserved in $Y'C_BC_R$. Our choice of $Y'C_BC_R$ allows more intuitive visualizations in chroma planes.

In this work we use $Y'C_BC_R$ for HD as standardized in BT.709/BT.1886. We limit ourselves to color values in the range from black to diffuse white. We exclude color values above diffuse white, such as colors that might arise from direct light sources and specular highlights. Representation of color values above the portrayal of diffuse white – perhaps 3 or 5 times higher – is the distinguishing feature of high dynamic range (HDR). However, we exclude these values because applications of HDR do not typically demand high accuracy for colors outside the representation of diffusely reflecting surfaces. Furthermore, in imaging, specular highlights and direct light sources in the image typically occupy small areas (or equivalently, have small angular subtense), and so are not expected to be critical in terms of perceptual uniformity.

Implementation

In this section, we describe the algorithm to compute the Jacobian determinants. A summary of the algorithm is presented in Table 1.

We start by choosing a representative color space: *R′G′B′* with BT.709/BT.1886 primaries, BT.1886 EOTF, D_{65} white, and peak luminance of 320 $cd \cdot m^{-2}$ (typical of consumer television viewing or office computer monitor use). We analyze SDR, where peak white is assumed equivalent to the portrayal of diffuse white. The gamut volume of this *R′G′B′* space is unity.

A prerequisite to computing our volume elements is to compute the gamut volume of each of the two spaces to be compared. These gamut volumes will be used later to normalize our volume elements. The volume ratio normalization accounts for correspondence of the same set of color samples in both perceptual color representations. We compute gamut volume in the manner of the Dolby white paper [1]; details of the scheme are discussed by Poynton and his colleagues [4]. We choose 11^3 (1331) sample points distributed throughout *R′G′B′* space using uniform intervals of each or *R′*, *G′*, and *B′* across the range of signal values 0 to 1 according to the inverse-EOTF of BT.1886.

We seek to develop the Jacobian determinant ratio of two color spaces to be compared, as a function of several hundred or several thousand sample colors perceptually distributed within the reference *R'G'B'* color space. In principle, we could densely sample *R'G'B'* space; an advantage of that approach is that the *R'G'B'* sample points are – by construction – within gamut. However, that approach would require calculation covering 3D space. We chose a different, 2.5D approach: We compute Jacobian determinant values only at *L** levels to be visualized. We use seven *L** levels; at each level, we compute across an 801×801 lattice of [*a**, *b**] coordinates (that is, at *a** and *b** increments of 0.25 across the range from -100 to +100). A disadvantage of this scheme is that gamut testing is necessary. At each point, we transform to both of our comparison spaces, *LAB* and $Y'C_BC_R$.

For each of the two spaces – symbolize them *S*1 and *S*2 – we compute discrete versions of the Jacobian matrix associated with each sample point. We form the discrete approximations by offsetting suitable combinations of [$\pm0.5\varepsilon$, 0, 0], [0, $\pm0.5\varepsilon$, 0], and [0, 0, $\pm0.5\varepsilon$], surrounding each sample point with six nearby points offset by a small amount on each of the three axes. We chose ε to be $2.5 \cdot 10^{-5}$ (0.000250) as a compromise: small enough to avoid out-of-gamut excursions, and large enough that floating-point numerical errors are insignificant. (We use the expedient of gamut-testing just the sample points, not all the cuboid vertices.)

We then take the determinant of the discretized Jacobian matrix, then normalize each of these determinants by the appropriate gamut volume, computed earlier. These scaled determinants approximate the volumes associated with a putative JND – think of each volume element as a cuboid, or as an ellipsoid – at each sample point of the two spaces.

We are not interested in the scaled Jacobian determinants *per se* at each sample point, because we wish to make no assumptions of what constitutes a JND in either space; we seek only to compare the two spaces. What we need is the *ratio* of is the ratio of color coordinate differences that comprise the elements of the Jacobian determinant matrix, scaled by the gamut volume of the corresponding color space. Symbolizing the two color spaces *S*1 and *S*2, if the ratio of volumes $v_2 : v_1$ is 10, then space 1 is underquantized compared to space 2; its perceptual performance is liable to be poor. If the ratio $v_2 : v_1$ is 0.1, then space 1 is overquantized compared to space 2; its codeword utilization is liable to be poor.

Visualization

In Figure 1, we visualize volume ratios mapped in *LAB* coordinates using a set of colorized heat map slices, with the associated color indices from 0.25 to 4 on a logarithmic scale. As a convenience in interpreting the plots, we indicated (by “stars”) the coordinates of primaries and secondaries at *L** white value 75 (relative luminance about 0.48).

The test case selected has a peak white and diffuse white luminance of 320 $\mathrm{cd \cdot m^{-2}}$. Since *LAB* is calculated relative to diffuse white and BT.1886 is calculated relative to peak white, Figure 1 is independent of the (radiometric) luminance level of 320 $\mathrm{cd \cdot m^{-2}}$. The results shown in Figure 1 are dependent on the ratio of peak white to diffuse white. When applying this scaled Jacobian determinant value visualization to compare other color representations and color perception approximations, the specific luminance level of the diffuse white or peak white can change the visualization for some alternate test cases.

Discussion

The “core” of the *LAB* plot for *L** 50, 75, and 90 indicates that perceptual performance of *LAB* and $Y'C_BC_R$ are roughly comparable. This is not a trivial conclusion: Normalizing color volumes guarantees that both spaces are comparable in terms of their counts of JNDs, but that says nothing about the distribution.

It is evident at all *L** levels, but particularly above *L** 10, that $Y'C_BC_R$ overquantizes compared to *LAB* at colors where at least one primary is near unity. We attribute this behavior to $Y'C_BC_R$ forming its achromatic (luma) component from a weighted sum of nonlinear primary contributions, whereas *LAB* forms its achromatic (*L**) component from a nonlinear mapping of weighted tristimuli. *LAB* exhibits *constant luminance* (CL) behaviour, whereby once [*L*, a*, b**] components are formed, altering *L** does not alter the luminance of the decoded *XYZ* tristimuli. Although $Y'C_BC_R$ approximates CL behaviour, mathematically it has *non-constant luminance* (NCL) behaviour (also called *non-constant luminance coding,* NCLC), whereby changing C_B or C_R “in the channel”– for example, by colour subsampling – alters decoded luminance. In NCL/NCLC systems, the monochrome component is symbolized *Y'* and called *luma* to distinguish $Y'C_BC_R$ encoding from true CL coding. See Poynton [2].

At *L** of 25 and below, the plots indicate a predominance of normalized volume ratios between 0.25 and 0.5: $Y'C_BC_R$ underquantizes compared to *LAB*.

References

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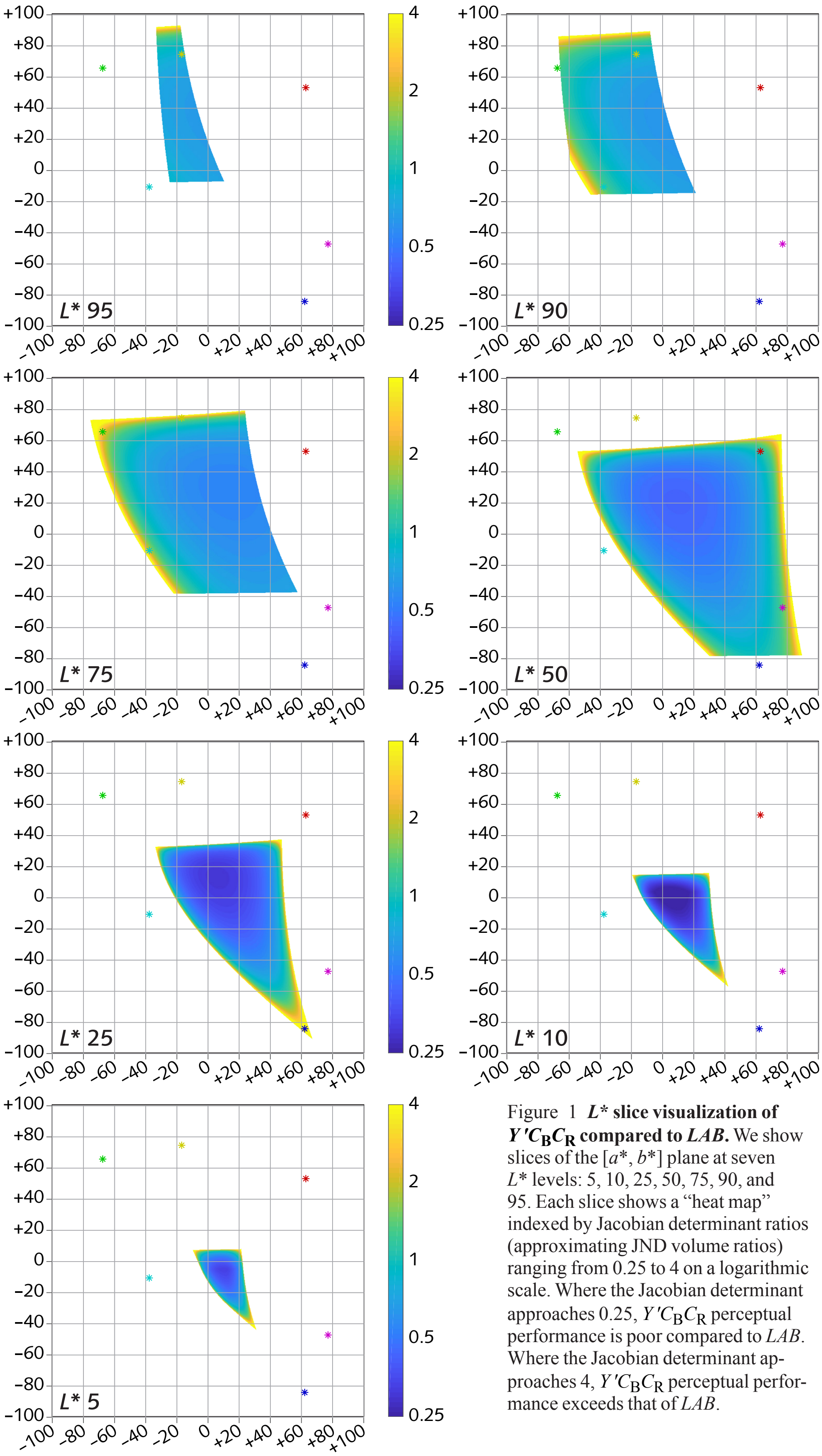

Figure 1 ***L** slice visualization of** $\boldsymbol{Y'C_BC_R}$ **compared to *LAB*.** We show slices of the [*a**, *b**] plane at seven *L** levels: 5, 10, 25, 50, 75, 90, and 95. Each slice shows a "heat map" indexed by Jacobian determinant ratios (approximating JND volume ratios) ranging from 0.25 to 4 on a logarithmic scale. Where the Jacobian determinant approaches 0.25, $Y'C_BC_R$ perceptual performance is poor compared to *LAB*. Where the Jacobian determinant approaches 4, $Y'C_BC_R$ perceptual performance exceeds that of *LAB*.

Table 1 **Algorithm steps** are summarized.

1	Specify the details and constraints of the test case. • Example details and constraints include the color triplets for black, diffuse white, and peak white.
2	Calculate the scale factor for the Jacobian determinant values. • The scale can be the ratio of the color gamut volumes of $S1$ and $S2$ for the test case. • A simpler approach is to defer this step and later normalize the results for each specific color sample triplet.
3	Select and construct the color dataset for visualization of the resulting scaled Jacobian determinant values. • An example color dataset is a slice of $L^*a^*b^*$ volume at a specific L^* for the test case.
4	Transform this dataset to tristimuli (XYZ).
5	Transform this dataset to the 3D coordinate system $S1$. • An example $S1$ is $L^*a^*b^*$ consistent with your testcase.
6	Copy this dataset six times and offset each by $\pm 0.5\varepsilon$ in each of the three dimensions.
7	Transform these seven (six plus original) datasets to tristimuli (XYZ).
8	Transform these seven datasets to the 3D coordinate system $S2$. • An example $S2$ is BT.1886 primaries and EOTF and BT.709 $Y'C_BC_R$ weights to form $Y'C_BC_R$.
9	Calculate three differences (one for each dimension) between the $\pm 0.5\varepsilon$ $S2$ datasets.
10	Calculate the Jacobian determinant values for each valid color triplet using three $S2$ differences and three $S1$ ε values. • This valid color triplet check applies to 14 sets (seven for $S1$ and seven for $S2$).
11	Scale the Jacobian determinant values.
12	Map the scaled Jacobian determinant values to the visualization representation (typically either $S1$ or $S2$).
13	Construct the visualization of these scaled Jacobian determinant values.