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Weighted Geometric Mean (WGM) method: A new chromatic adaptation model

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Abstract

The geometric mean has been suggested to be the fundamental mathematical relationship that governs peripheral sensory adaptation. This paper proposes the WGM model, an advanced chromatic adaptation model based on a weighted geometric mean approach that can anticipate incomplete adaptation as it moves along the Planckian or Daylight locus. Compared with two other chromatic adaptation models (CAT16 and vK20), the WGM model shows more accuracy in predicting previous visual data.

Introduction

Chromatic adaptation is the most significant feature of the human visual system to understand and model color appearance. Luo mentioned that various chromatic adaptation transforms (CATs) have been derived from fitting a corresponding color data set, and the majority of the CATs include three steps of calculation (figure 1) [1].

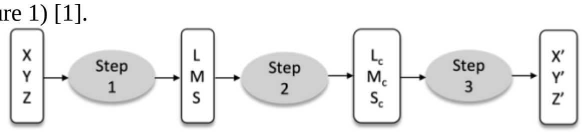

Figure 1. Three steps included in a CAT.

Step 1: Any physiologically plausible CAT model must act on signals approximating the cone responses that can be accurately converted from CIE tristimulus values (XYZ) via a 3x3 matrix, like M_{16} [2].

Step 2: This step converts the cone response (L M S), under the test illuminant ($L_nM_nS_n$), into the adapted cone responses ($L_cM_cS_c$) by using the CAT, often based on von Kries scaling of the cone responses.

Step 3: This step converts the adapted cone response ($L_cM_cS_c$) back to the tristimulus value using the inverse of the 3x3 matrix used in step 1.

For more than a century, Step 2 or the CAT itself has been investigated by many color scientists. In 1902, von Kries published his hypothesis on chromatic adaptation as a conceptual extension of Grassmann's laws of additive color mixture across two viewing conditions [3]. Although chromatic adaptation affects the three types of cone responses differently, the von Kries model implies that the relative sensitivity of each of the three cone mechanisms remains unchanged. According to this hypothesis, adaptation in the three cone types is independent and inversely related to the response to the adapting stimulus. The von Kries hypothesis can be summarized in Eq. 1, where LMS are the initial cone response, $L_nM_nS_n$ are cone responses to the adapting stimulus, and $L_aM_aS_a$ are the post-adaptation cone signals [4].

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \begin{bmatrix} \frac{1}{L_n} & 0 & 0 \\ 0 & \frac{1}{M_n} & 0 \\ 0 & 0 & \frac{1}{S_n} \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix} \quad (1)$$

Chromatic adaptation is not always complete [5]. Therefore, a D factor for the degree of adaptation has been developed to expand the von Kries model to account for incomplete chromatic adaptation, embedded in most accurate general-purpose CAT models, such as CAT02 and CAT16. The equation (Eq. 2) is expressed as follows:

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \begin{bmatrix} \frac{1}{DL_n + (1-D)L_r} & 0 & 0 \\ 0 & \frac{1}{DM_n + (1-D)M_r} & 0 \\ 0 & 0 & \frac{1}{DS_n + (1-D)S_r} \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix} \quad (2)$$

$L_rM_rS_r$ refer to the responses to the reference illuminant. In CAT02 and CAT16, the reference illuminant is equal-energy (EE) illuminant (L=M=S=100). However, various data sets [5][6][7] illustrate that the line segment connecting the adapting stimulus to the achromatic appearing stimulus does not project toward EE. In other words, EE as the reference point is not strictly valid. Fairchild proposed a new chromatic adaptation model (vk20) that takes sky blue at 15000K as a new reference point. vK20 relies on three chromaticities and three-D values. Besides, $L_rM_rS_r$ and $L_nM_nS_n$, the vK20 includes previous adapting illuminant $L_pM_pS_p$ in the model [3]. The vK20 formulation is given as Eq. 3, and the sum of D_n, D_r, and D_p should be 1.0. As shown in figure 2, all line segments are approximately projected to sky blue(15000K).

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \begin{bmatrix} \frac{1}{D_nL_n + D_rL_r + D_pL_p} & 0 & 0 \\ 0 & \frac{1}{D_nM_n + D_rM_r + D_pM_p} & 0 \\ 0 & 0 & \frac{1}{D_nS_n + D_rS_r + D_pS_p} \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix} \quad (3)$$

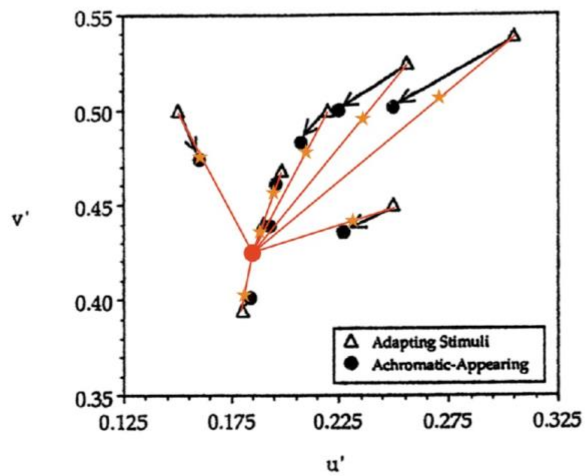

Figure 2: The 15000K reference point is shown by a red circle, orange lines indicate projections from each adapting chromaticity to that point, and orange pentagrams reflect the prediction of 70% adaptation from that reference to adapting chromaticity. This plot is originally from Fairchild (2020).

In order to better explain the terms that used in this paper, the definitions of the different cone response are summarized in table 1.

Table 1: Summary of all types of cone responses.

LMS	Cone responses to the stimulus under test illuminant.
$L_nM_nS_n$	Cone respones to test illuminant, or other selected adapting stimulus.
$L_rM_rS_r$	Cone responses to the reference illuminant. In CAM16 or CAN02, the reference illuminant is equal-energy (EE) illuminant (L_r=M_r=S_r=100). In vK20 the reference illuminant is 15000K (L_r=95.41, M_r=103.87, Sr=169.81).
$L_pM_pS_p$	Cone responses to a previous adapting illuminant or other adapting stimulus.
$L_aM_aS_a$	Post-adaptation cone signals
$L_cM_cS_c$	Adapted cone responses. (L_c=L_a*L_r, M_c=M_a*M_r, S_c=S_a*S_r)
L'M'S'	Adapted cone signal of adapting stimulus. Only mentioned in the WGM method.

It is well known that all of these widely used models (CAT02, CAT16, and vK20) are acutely based on the von Kries hypothesis, which constrains the prediction of incomplete adaptation along the line segment from adapting stimulus to a reference point. However, not only the results from figure 2 but also, for example, the data from Shen and Fairchild [6] and Zhai and Luo [7] suggest that the prediction of incomplete adaptation shouldn't move along the line segment connecting the adapting stimulus to the reference point. Instead, it should move along the curve like the Planckian or Daylight locus from adapting stimulus to reference point. Therefore, this paper proposes a better performing CAT model that can predict the incomplete adaptation moving along the curve by using the geometric mean method.

Geometric mean method

Most chromatic adaptation models, both theoretically and mathematically, can be traced back to von Kries' model. Do we really need the von Kries hypothesis? Is there any more advanced neurophysiology theory that we could apply?

A recent paper written by Wong [8] illustrates an exciting ideal that the peripheral sensory adaptation across many types of sensory adaptation is found to follow a simple mathematical relationship: the geometric mean. This relationship, illustrated in figure 3, involves the steady-state spontaneous rate (SR) prior to the introduction of stimulus, the peak response to stimulus (PR), and the subsequent new steady-state response (SS). The mathematical relationship is shown in Eq. 4.

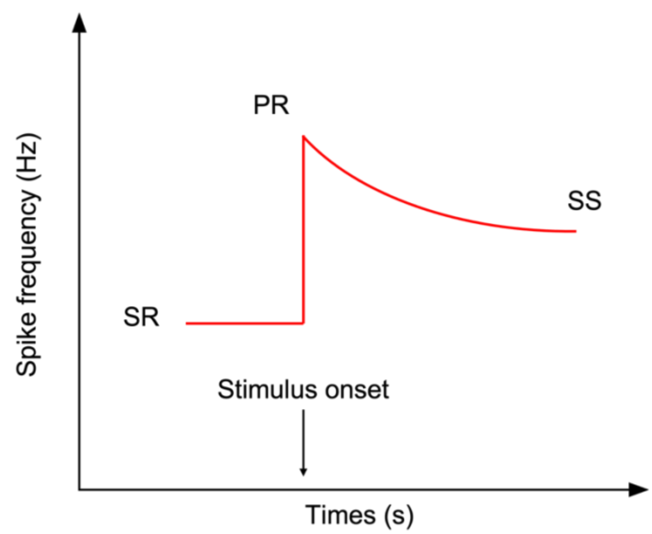

Figure 3: Peripheral sensory adaptation curve (ideal situation). This figure is recreated based on figure 1 in Wong (2021).

$$SS = \sqrt{PR \times SR} \quad (4)$$

In Wong's paper [8], more than 200 measurements taken from different branches of sensory adaptation are summarized and shown to be compatible with geometric mean equation. For example: (1) Auditory response in guinea pig fiber, gerbil fiber, ferrets, and saccular nerve fibers of goldfish. (2) The responses from lateral line in fish. (3) Stretch responses in crayfish and frog. (4) Response of olfactory receptor neurons in fruit flies. (5) Taste recording in fruit fly sensilla, caterpillar and blowfly. (6) Response to cooling in beetles. (7) Vision data from ON-Centre ganglion cell in the cat.

Though cones have continuous voltage responses rather than spike frequency, the geometric mean concept might still apply to cone responses directly (linear in the first stage). It is also likely that some portion of chromatic adaptation is occurring in neural cells beyond the cones that do have spike-frequency modulated responses such as those explored by Wong [8]. Therefore, in the proposed new CAT model, PR is the cone response to adapting stimulus ($L_nM_nS_n$), SR is the cone response to reference point ($L_rM_rS_r$), and the SS is the adapted cone signal of adapting stimulus (L'M'S'). See Eq. 5. The results of Fairchild [9] with the prediction by using the geometric mean method are shown in figure 4. It is surprising that such a simple model potentially works well, and prediction results (red dots in figure 4) move along the curve (either daylight curve or Planckian locus) rather than a straight line. Note that in this geometric mean model there is no factor for degree of adaptation; the geometric mean automatically computes appropriate degree of adaptation.

$$L'M'S' = \sqrt{L_nM_nS_n \times L_rM_rS_r} \quad (5)$$

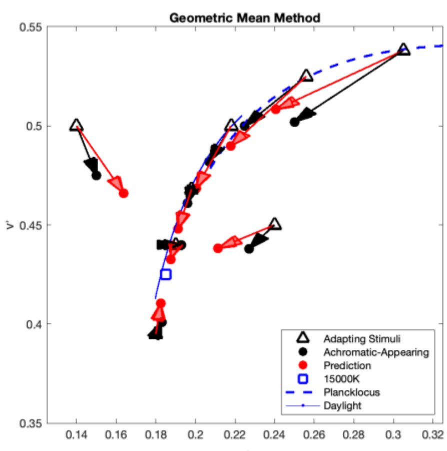

Figure 4: The blue square represents the reference point (15000K). Red arrows indicate projection from each adapting chromaticity to prediction results by the geometric mean method. Blue dotted line represents the Planckian locus, while the blue line indicates the daylight curves.

In order to better visualize the shift of neutral point across different ambient light, geometric mean method vs CAT16 (D=0.7) plot is shown in figure 5 (top). The color of adapting stimuli is represented by each square. The rectangle on the left inside each square is the neutral point predicated by CAT16 (D=0.7) while the one on the right is the neutral point predicated by geometric mean method. Chromaticity values of prediction results and adapting stimulus are shown in the bottom of figure 5 (left: CAT16 with D=0.7, right: geometric mean) This figure (both top and bottom) shows the geometric mean method roughly following the Planckian or daylight locus (color shift from yellowish to blueish and move along Planckian locus) and CAT with D=0.7 trending toward to EE (more pinkish).

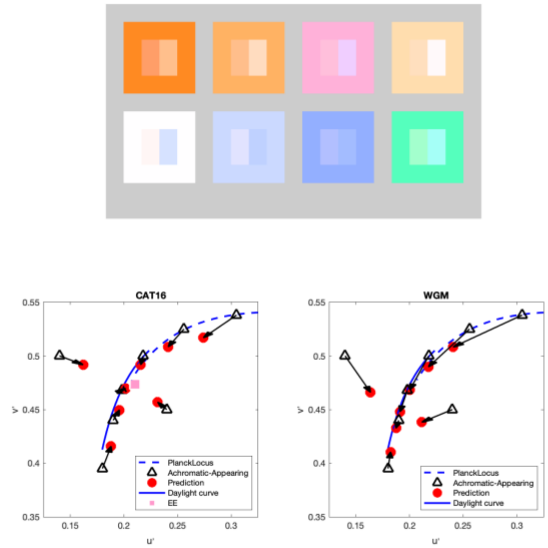

Figure 5: Top: The perceptual neutral point predicted by CAT with D=0.7 (the rectangle on the left inside each square) and geometric mean method (the rectangle on the right inside each square. The color of each square represents the adapting stimuli. Bottom: Red arrows indicate projection from each adapting chromaticity (black triangle) to prediction results (red circle) by CAT16 with D=0.7 (left) and the geometric mean method (right).

Weighted geometric mean (WGM) method

Like all the CAT models, a D factor for degree adaptation can be developed to expand the geometric mean method to account for more complete chromatic adaptation and differences in chromatic adaptation for various viewing conditions when cognitive adaptation mechanisms are active to various degrees. Therefore, Eq. 5 can be rewritten as Eq.6:

$$\begin{bmatrix} L' \\ M' \\ S' \end{bmatrix} = \begin{bmatrix} L_n \\ M_n \\ S_n \end{bmatrix}^D \cdot\times \begin{bmatrix} L_r \\ M_r \\ S_r \end{bmatrix}^{(1-D)} \quad (6)$$

To check the performance of the WGM model, three CAT models (CAT16, vK20, and WGM) were selected to predict the psychophysical experimental data of Fairchild [9]. The MATLAB *fmincon* function has been used to optimize the D value to minimize color difference (Euclidean distance in u'v' diagram) between prediction and experiment results. D values computed individually and totally (single D value) for each adapting stimulus. The optimized results (both individual D values for each adapting stimulus and single D value for each model) are illustrated in figure 6.

It is evident that the reference point will impact the model performance since it is the only difference between CAT16 and vK20 in this calculation (D_p = 0 assumed in vK20). In the WGM model, a 15000K reference point was also chosen based on previous results. However, another optimization method was completed by fitting both the D value (also individually for each adapting stimulus and singly by model) as well as the reference point to minimize the color difference between prediction and experiment results. In figure 6, WGM(CCT) and WGM (CCT, single) represent the results with reference point optimization. All the optimized D values are summarized in table 2. The Box plot in terms of color difference is shown in figure 6 (bottom right) indicates the prediction results of single D value in CAT16 is very large. In other words, vK20 and WGM perform better than CAT16 with single D values. This is an effective illustration of the theoretical error in CAT16, which holds that D value, as modeled in CIECAM16, is only a function of luminance level. As matter of fact, for the CIECAM16 formulation, optimal D values also depend on the chromaticity value of adapting stimuli and perhaps other factors. WGM and vK20 do not have this theoretical flaw. In addition, the best model performance also belongs to WGM with individually optimized D values.

However, more datasets need to be evaluated to confirm these results in future work.

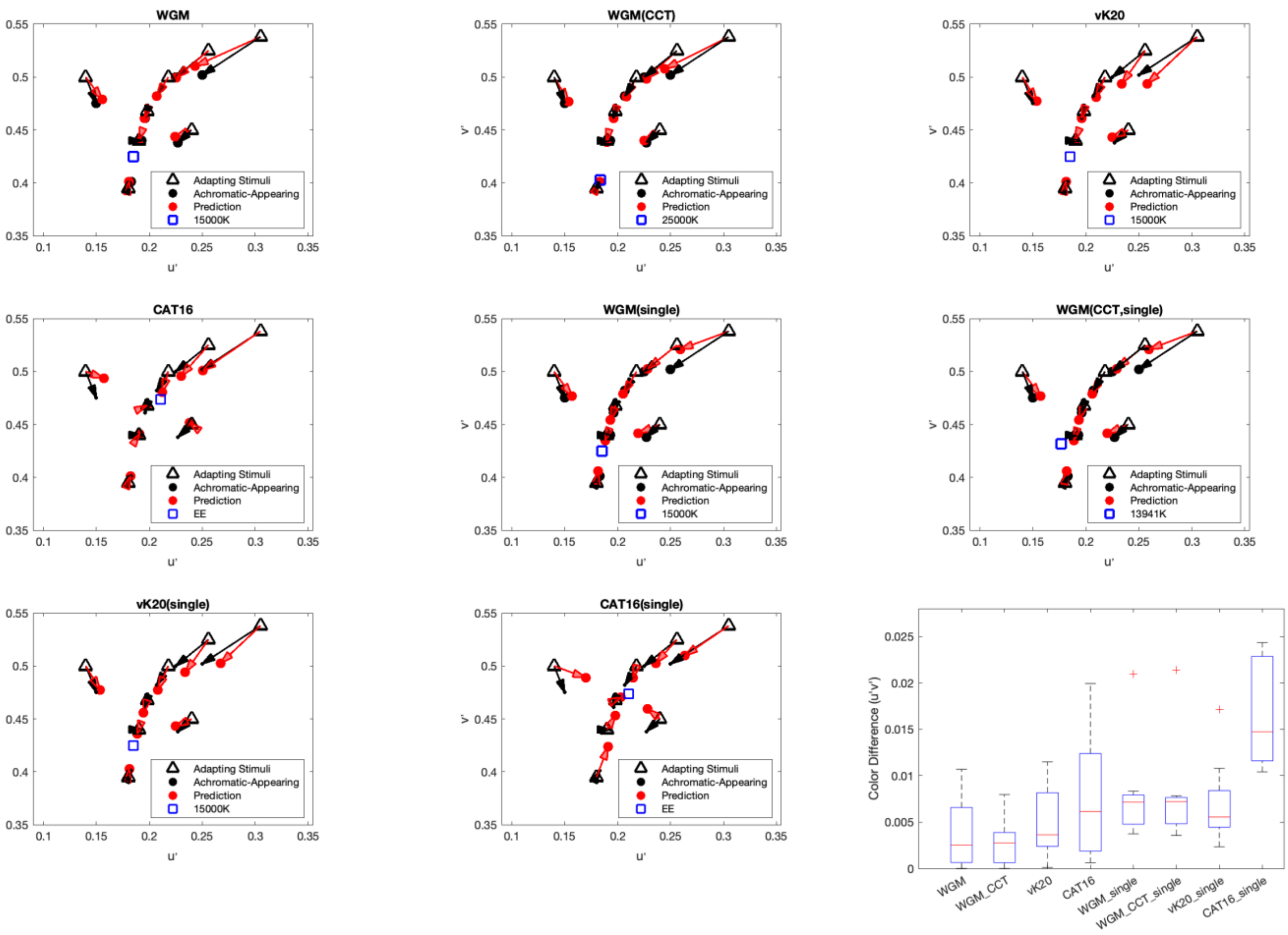

Figure 6: Prediction of Farichild's dataset [9] using optimized WGM model (WGM, WGM(CCT), WGM (single),WGM(single CCT)), vK20 (vK20,vK20(single)), and CAT16 (CAT16,CAT16(single)). The box plot (bottom right) illustrates the color difference value between prediction and experimental results.

Table 2: Optimized D values for CAT16 and WGM.

Adapting stimuli								
CAT16	0.4563	0.4526	0.9274	0.2677	1	0.9575	0.9066	0.7692
CAT16(single)	0.5896							
vK20	0.6619	0.7310	0.7333	0.7745	0.8506	1	0.7768	0.7293

vK20(single)	0.7332							
WGM	0.5217	0.6067	0.7324	0.6944	0.8213	1	0.7989	0.6752
WGM (single)	0.6446							
WGM (reference point:25000K)	0.5421	0.6406	0.7595	0.7424	0.8728	0.9541	0.2276	0.7141
WGM (single, reference point:13941K)	0.6474							

Do we still need von Kries?

Yes! Though achromatic appearance can be more accurately predicted by this advanced neurophysiology theory (WGM), we can't correlate these results directly to $L_aM_aS_a$ (post-adaptation cone signals). In other words, chromatic color under different light condtion can't be predicted. The WGM model should also be based on the von Kries hypothesis. Therefore, by changing the previous incomplete adaptation equation, the final WGM model is shown in Eqs 7:

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \begin{bmatrix} \frac{1}{L_n^D \times L_r^{1-D}} & 0 & 0 \\ 0 & \frac{1}{M_n^D \times M_r^{1-D}} & 0 \\ 0 & 0 & \frac{1}{S_n^D \times S_r^{1-D}} \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix} \quad (7)$$

Our future work is to evaluate the performance of this model based on other historical datasets, such as Hunt and Winter [10] Hurvich and Hameson [11], Ma et al. [12], Zhai and Luo [7].

Conclusions

Unlike any other von Kries-type chromatic adaptation models, WGM was able to anticipate incomplete adaptation moving along the Planckian or daylight curves and to automatically compute an appropriate degree of incomplete sensory chromatic adaptation with no need for an empirical degree of adaptation factor. Such a factor will be needed in practical applications where both sensory and cognitive adaptation mechanisms are active. The geometric mean method has been proved to be independent of sensory modality and to be satisfied across a wide range of animal species, which is the theoretical basis of WGM. In summary, Eq. 7 provides the structure of WGM. However, this model still has to be tested using more and different corresponding-colors datasets.

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