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Approximating Planckian Black-body Lights using Wien's Approximation

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Abstract

Planck's law and Wien's approximation of this law are both widely used to calculate the spectral radiation of a black-body based on its color temperature. The Wien approximation is a slightly simpler equation and has the advantage that the logarithm of a Wien light can be written as a linear sum of two basis vectors plus an offset (a fact that is exploited in some computer vision algorithms).

In this paper, we show that the Wien formulation can, in general, be used to approximate Planckian lights assuming there is a mapping function taking Planckian to corrected Wien temperature. Significantly, we show that a correction function $f()$ exists and for the range of color temperatures of interest the Wien spectrum calculated for $f(T)$ has a very similar shape to the actual spectrum of a Planckian light with temperature T. We find that defining $f()$ as a polynomial-type function models to a good extent the relationship between the color temperatures of Planckians and their closest Wien-Planckians lights both in terms of the angular error between their two respective spectral functions and their projections to u'v' coordinates.

Introduction

The spectral density of a black-body radiation is described by Planck's law, and is based on a single parameter, the color temperature (measured in Kelvin) [16]. The higher this temperature the cooler the color of the light: moving from 3000 to 7000 to 10000 Kelvin the corresponding Planckian lights appears yellowish, whitish and bluish. In chromaticity space, the Planckian locus can be obtained by connecting the chromaticity points of a series of Planckian lights with increasing color temperature. For brevity, we will sometimes refer to a color temperature in Kelvin and, equivalently, using the abbreviation 'K' e.g. 3000 Kelvin and 3000K denote the same color temperature.

Wien's approximation of black-body radiation [19, 22] is an alternative formula for calculating the spectral density of black-body radiation. While the two formulae are not the same they - more or less - generate the same (very similar) spectra for low color temperatures (e.g. less than 4000 Kelvin). But, the generated spectra are different the higher the temperatures become. The Wien approximation is slightly simpler than Planck's formula and has the advantage that the logarithm of all Wien spectra can be modelled as a linear sum of two basis vector in log-space [2]. This result is exploited in several computer vision papers [10, 1, 6, 5, 4, 8, 9, 13, 14, 2]. More recently, the Wien approximation was used as the basis for the development of the theory of Locus Filters[2]. A colored filter is a 'locus filter' if and only if for all lights on the Wien locus the filtered light also lies on the locus.

Figure 1 shows (zoomed in) the Planckian locus in blue and Wien locus (obtained using Wien's approximation) in red in the *uv* chromaticity diagram. Color temperatures vary from 20000K to the left to 4000K to the rigth. One can observe that the two loci are quite similar especially for low color temperatures. However, that does not mean that a Planckian light and a Wien with the same color temperature corresponds to the same point in the uv diagram. The black and green crosses shown in Figure 1 are respectively the uv chromaticity coordinates of Wien-approximation and actual 12000 Kelvin Planckian lights. Significantly, the distance between these two light in u,v is 0.004 which corresponds to the just noticeable distance [20, 21], the lights 'look like' they have different colors to an observer.

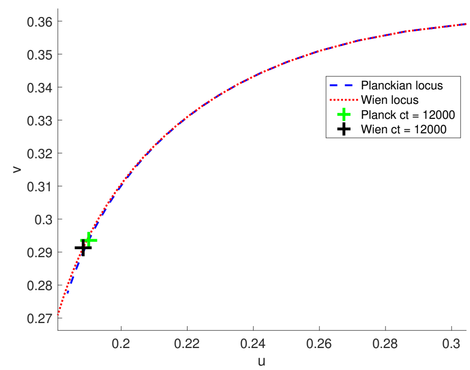

Figure 1: A zoom in to the Planckian locus and Wien locus in the u,v chromaticity diagram.

In Figure 2, we plot a 12000K Planckian spectrum and the Wien-approximation for the same temperature. It is evident that these two spectra are different from one another.

In a practical scenario, a black-body radiator - interpreted as a light spectrum that might illuminate a scene - is actually parameterised by 2 numbers. There is the color temperature T which controls the shape of the spectrum (and which is the only free parameter in Planck's equation) and the intensity of the light which is accounted for by a second scalar parameter k. This extra degree of freedom is a key consideration when considering the similarity or otherwise of Wien-approximate and actual Planckian lights i.e. we can scale any Wien approximation to better approximate a Planckian light.

In this paper, we ask the following question: given a Planckian light with temperature T, denoted $E^P(\lambda,T)$, what is the closest Wien approximation, $kE^W(\lambda,f(T))$, where $f(T)$ maps Planckian temperature to a *corrected* Wien temperature. We are interested in finding $f()$ and scalars k such that $E^P(\lambda,T) \approx kE^W(\lambda,f(T))$, for any T

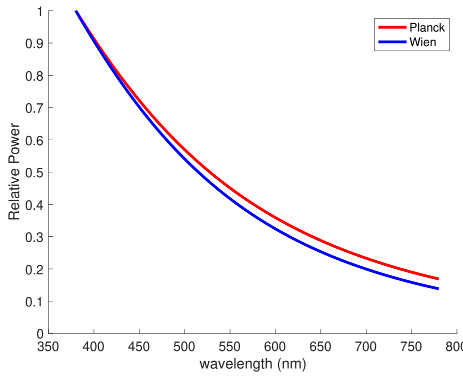

Figure 2: The spectral power distribution of a Planckian light with a temperature of 12000K and a Wien approximated light with the same temperature. Both spectra have normalised maximum power set to 1.

Broadly, we will present results that show that, recast in this way, the Wien-approximation actually models Planckian lights rather well. Further, we find a simple analytic formula for the correction function $f()$. The practical implications of our work are discussed.

Background

A Planckian black-body illuminant E^P is a function of color temperature T and wavelength λ:

$$E^P(\lambda, T) = c_1\lambda^{-5}(e^{\frac{c_2}{T\lambda}} - 1)^{-1}, \tag{1}$$

where c_1 and c_2 are constants equal to $3.74183 \times 10^{-16} Wm^2$ and $1.4388 \times 10^{-2} mK$, respectively. Here, we will also allow an intensity change which is mediated by the scalar k.

$$E^P(\lambda, T) = kc_1\lambda^{-5}(e^{\frac{c_2}{T\lambda}} - 1)^{-1}, \tag{2}$$

In the range of typical lights (2000K to 20000K), a simpler approximate form of Planck's equation - called Wien's approximation [19] - can be used to describe black-body illuminations since, across the visible spectrum both functions generate similar spectra. These Wien-Planckian lights are written as:

$$E^W(\lambda, T) = kc_1\lambda^{-5}e^{-\frac{c_2}{T\lambda}} \tag{3}$$

The constants c_1 and c_2 are as defined for Equation 1 and we, again, allow the scalar k to model intensity change.

It is well known that for color temperatures less than 4000 Kelvin the Wien approximation generates a spectrum with a very similar shape to the actual Planckian Black-body radiator. Let us quantify how similar Wien lights are to actual Planckians in terms of the shape of spectra in the range of the visible spectrum. To do this we will adopt discrete approximation of the lights. Each light spectrum will be represented as a 81-component vector (corresponding to the spectral power of the lights from 380 to 780 Nanometres at a 5 Nanometre sampling).

Now given a Planckian and Wien-approximation vectors, denoted, $\underline{E}^P$ and $\underline{E}^W$, we can calculate the angle between the vectors (as a measure of similarity that is independent of intensity). We define the angular error in the usual way as:

$$AngularError(\underline{E}^P, \underline{E}^E) = acos(\frac{\underline{E}^P.\underline{E}^W}{|\underline{E}^P||\underline{E}^W|}) \tag{4}$$

where '.' denotes the vector dot-product, $||.||$ is the vector magnitude and *acos* is the inverse cosine. The angular error, by construction, is independent of the magnitude of the vectors and compares the shape of the underlying spectra.

Figure 3 plots the relationship between angular errors between Spectral Power Distributions (SPDs) of Planckians and Wien lights for increasing color temperature. Here, we show the Planckian temperature range (denoted T^P in the Figure) of interest (up to 1,000,000 Kelvin or equivalently 10^6 Kelvin).

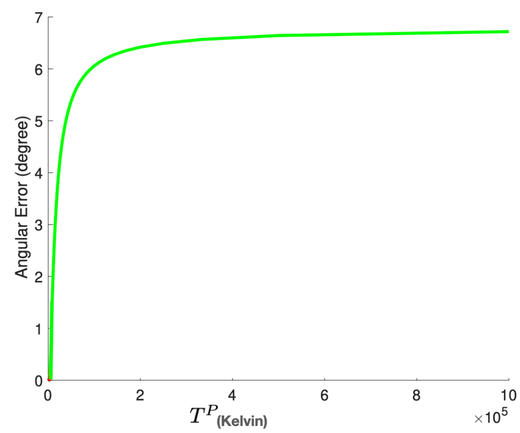

Figure 3: Angular error between Planckian and Wien lights' SPDs with the same color temperature (for temperatures in the interval [0,10^6] Kelvin)

Arguably, the temperatures < 100,000K are, practically, of most interest. We replot this range in Figure 4. We see that up to 4000 Kelvin the angular error is less than .2 degrees. Finally, returning to the 12000 Kelvin Planckian and Wien spectra plotted in Figure 2, here the angular error between the two spectra is 2.4 degrees.

Applications of the Wien-approximation

Simply stated, the Wien approximation is useful because of how the formula simplifies when logarithms are applied. Taking the logarithm of Equation 3, we see that

$$log(E^W(\lambda, T)) = log(k) + log(c_1\lambda^{-5}) - \frac{c_2}{T\lambda} \tag{5}$$

Because only k and T are variables, Equation 5 teaches that any Wien light is the sum of two scaled *basic* functions plus an offset term. Or, if we model the two components ($log(k)$ and $\frac{c_2}{T\lambda}$) as sampled 81-component vectors, we can say that a log Wien lies in a 2-dimensional basis (with a constant offset vector). To be more precise, as vectors, log-Wien lights lie on a 2-dimensional hyperplane. This two-dimensionality leads to the log RGB coordinates of lights in the log-camera RGB images also to approximately lie on a

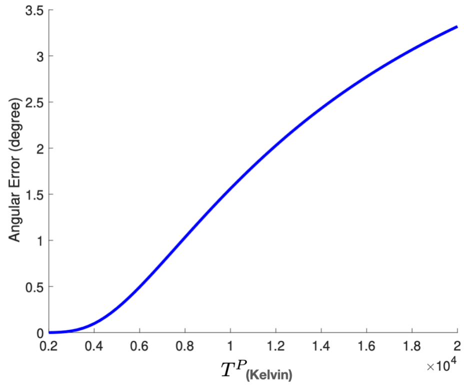

Figure 4: Angular error between Planckian and Wien lights' SPDs of the same color temperature (for temperatures in the interval [0,100,000] Kelvin)

2-dimensional hyper-plane. This in turn implies that the locus of light in a log-chromaticity diagram lies on a line.

The linearity of the Wien log-locus is used in algorithms for color constancy and illuminant invariance [10, 7, 1, 6]. Additionally, many shadow detection approaches have used Wien's approximation - as a key step - to model the light in the scene [4, 8, 9, 13, 14].

More recently, the Wien approximation has proven to be useful in Locus Filter Theory [2]. Assuming lights can be described using Wien's formula there are a unique class of filters (the Locus filters) that have the useful property that all Wien lights are mapped to other lights that are also described by the Wien formula [2]. Moreover, it was shown that the locus filters, over all other possible types of filters, were shown to uniquely have this light-to-light mapping property.

As derived, Locus Filter theory only applies to Wien approximations to Planckian lights. However, this paper, effectively, derives a nomogram that maps Wien temperature to u',v' -near-equivalent Planck temperature. Using our nomogram we will be able to locate the Wien near-equivalent temperature T^W for any Planck temperature T^p such that we have a very close spectral match to the desired Planckian. In this way Locus Filter theory can be considered - by the application of the nomogram - to apply to Planckian Lights as well as their Wien approximations, an important extension to the theory.

In [3], a modified (filter corrected) Wien Planck's Equation was shown to generate lights that were very similar to those generated by the standard Daylight equation [15].

Modelling Planckians using Wien's formula

From Figures 3 and 4, we see that Wien lights are different from Planckians - in terms of angular error - and this is especially true as color temperature increases. Here we wish to find a correction function $f()$ such that for a Planckian with color temperature T, $E^P(\lambda,T)$, the Wien approximation for the corrected light, $E^W(\lambda, f(T))$ is closer than when T is used directly in Wien's formula. We'd like $|E^P(\lambda,T) - k_1 E^W(\lambda, f(T))| < |E^P(\lambda,T) - k_2 E^W(\lambda,T)|$ (where k_1 and k_2 are scalars that optimally adjust the magnitudes of the spectra). Our goal then is to find the function $f()$ such that (for all temperatures of interest):

$$E^P(\lambda,T) \approx kE^W(\lambda, f(T)), \tag{6}$$

As before, let us represent spectra as vectors in the discrete domain: a Planckian light vector is denoted $\underline{E}^P$ and its color temperature is denoted T^P. Similarly, a Wien approximated light is represented by the vector $\underline{E}^W$ and its color temperature is recorded as T^W. Suppose now that we have a set of Planckian color temperatures $\mathscr{T}^P = [T_1^P, T_2^P, ..., T_N^P]$ that correspond to Planckian lights $\Psi^P = [\underline{E}_1^P, \underline{E}_2^P, ..., \underline{E}_N^P]$.

By searching, for the ith Planckian Light we find the Wien temperature, T_i^W that minimizes:

$$\min_{T_i^W} \text{AngularError}(\underline{E}_i^P, \underline{E}_i^W) \tag{7}$$

where T_i^W are, say, integers in the interval [1667,1000000] Kelvin (the range explored in this paper). In this way we find a set of *corrected* Wien color temperatures $\mathscr{T}^W = [T_1^W, T_2^W, ..., T_N^W]$. The corresponding spectra are recorded in the set $\Psi^W = [\underline{E}_1^W, \underline{E}_2^W, ..., \underline{E}_N^W]$. In other words, we solve for the Wien color temperature that gives the closest normalized spectral radiation to each Planckian in terms of angular error between the two spectra.

Practically, the *look-up-table-type* optimisation we have just set forth will suffice to convert Planckian temperatures to Wien equivalents. But, for elegance we would like find a simple *analytic* correction function such that

$$T_i^W \approx f(T_i^P) \tag{8}$$

While T_i^P and T_i^W correspond to temperatures in a specific experiment we, in general, seek $f()$ such that

$$T^{W,c} = f(T^P) \tag{9}$$

That is the correction function $f()$ returns the corrected - the superscript ^{c} 'means' correction - temperature $T^{W,c}$ (to drive Wien's formula).

Finding the Correction Function

It is known that the difference in color temperature does not reflect the visual difference between the colors of two lights. For this reason, we choose to represent color temperatures in Mired units. The Mired color temperature [12] (micro-reciprocal-degree), is often used to measure how similar one light color is to another, and can be calculated from a color temperature T as follows:

$$M = \frac{10^6}{T} = Mired(T) \tag{10}$$

Let us define two vectors $\underline{\mathscr{T}}^{P,M}$ and $\underline{\mathscr{T}}^{W,M}$ that contain the paired Mired color temperatures our Planckian and *corrected* Wien lights respectively (where as before ^{P} and ^{W} denote dependence on Planck's and Wien's formula and the superscript ^{M} indicates we are expressing temperatures in Mired units). Using the subscript $_i$ to denote the ith colour temperature under consideration we seek an $F()$ (a correction function in Mired units) such that:

$$\mathscr{T}_i^{W,M} \approx F(\mathscr{T}_i^{P,M}) \tag{11}$$

Let us adopt the notation $P^o()$ to denote the polynomial of expansion degree ^{o} (of the function argument). When $o = 1$ $P^1(\mathscr{T}_i^{P,M}) = [\mathscr{T}_i^{P,M}\ 1]$ and when $o = 3$, $P^3(\mathscr{T}_i^{P,M}) = [\mathscr{T}_i^{P,M}\ (\mathscr{T}_i^{P,M})^2\ (\mathscr{T}_i^{P,M})^3\ 1]$ (notice we always include an offset term). According to our notation an order ^{o} expansion is a $1 \times (o+1)$ vector. Now, we solve for the optimal polynomial by finding the $(o+1) \times 1$-component coefficient vector $\underline{c}^o$ by minimizing:

$$\min_{\underline{c}^o} \sum_i ||\mathscr{T}_i^{W,M} - P^o(\mathscr{T}_i^{P,M})\underline{c}^o||^2 \tag{12}$$

The vector $\underline{c}^o$ can be found analytically by computing the Moore-Penrose inverse[11].

Generating spectra using a correction function

In the range of interest, in this paper, 1667 to 1,000,000 Kelvin (1 to 600 mired units), we can use Equation 7 to find the best temperature conversion. The correction function here is a 'look up table' as there is no calculation. Rather for every integer degree of Kelvin we simply look up the optimal corrected temperature $T^{W,c}$.

$$T^{W,c} = lookup(T^P) \tag{13}$$

For a given temperature T (in Kelvin) the correction using our analytic function involved converting to and from Mired units:

$$T^{W,c} = f(T^P) = (Mired^{-1}(F(Mired(T^P))) \tag{14}$$

where $Mired()$ converts to Mired units (see Equation 10) and $F(x) = P^5(x)\underline{c}$ (we use a fifth order polynomial expansion and linearly combine the terms according to the coefficient vector solved for in Equation 12).

Whether we compute $T^{W,c}$ using Equations 13 or 14 we generated the desired approximation to a Planckian light with temperature T^P by inserting $T^{W,c}$ (for T in Equation 3).

Experiments

In order to find the function that models the relationship between Planckian color temperatures and and the Wien corrected counterparts, we use the set of Mired color temperatures ranging from 1 to 600 Mired with 1 Mired step [17], corresponding to color temperatures from to 1667 to 1000000 Kelvin with non-uniform steps. In Figure 5, we plot Mired Planckian temperatures against their Mired Wien correction temperature (or to use the notation of the last section we plot $\mathscr{T}^{P,M}$ and $\mathscr{T}^{W,M}$). For large colour temperatures (small Mired units) we also zoom in (see inset). It is evident that the relationship between the Mired temperatures are linear for large Mired units (small colour temperatures) but the graph curves for the small Mired value region (again, see inset).

Let us now consider how well a correction function $f()$ can map a Planckian colour temperature T^P to the corrected Wien temperature

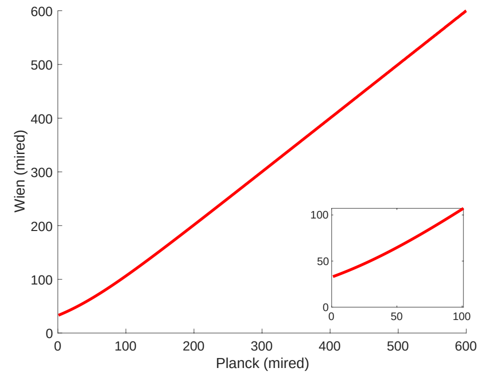

Figure 5: Planck color temperature is plotted against the Wien temperature that generates the closest approximation spectrum (when Wien's formula is used). Inset shows relationship is non linear for small Mired units (large color temperatures).

$T^{W,c}$. To assess how well a correction function works we generate the actual Planckian Light spectrum, for temperature T^P, and its Wien approximation for temperature $T^{W,c} = f(T^P)$. This calculation is carried out in the discrete domain so both spectra are represented as 81-vectors (a sampling from 380 to 780 Nanometers at steps of 5 nm). Then, to determine the closeness of the spectrum-pair we used Equation 4 to calculate the angular error. For our correction function we can use a look-up-table (see Equation 13) or different orders of Polynomial regression. Or, even we could just us the Planckian temperature T^P to drive Wien's equation (i.e. we apply no correction).

We found that the maximum error - the angle between the Planckian and Wien temeperature corrected spectra - was respectively 6.68, 2.77, 1.53 and 0.69 degrees when respectively no correction is carried out and when a 2nd, 3rd, 4th and 5th degree polynomial was used. When a lookup table is used directly, the maximum error is also 0.69. So, to 3 decimal places a 5th order polynomial has the same max error as the lookup table. Finally, we remark that, although it was not used as a constraint, the 5th order polynomial we found is a strictly monotonically increasing function (in the interval of Mired temperatures that we examined).

Figure 6, in red, plots the correction from the Planckian Temperature, $T^{P,W}$ to the corrected temperature $T^{W,c}$ (both in Mired units here). The analytic fit using the fifth order polynomial is shown in dashed green. The curves overlay each other here indicating that the analytic function works similarly to the look-up-table. Indeed, the average Mired color temperature error calculated across [1,600] Mired domain is 0.15 (the Just Noticeable Distance is 5.5 Mired [12, 18]).

In Figure 7, we replot Temperature in Kelvin. Here the x-axis is the actual desired Planckian temperature. And, the y-axis is the temperature which drives Wien's equation such that the resulting spectrum is close (in terms of angular error) to the the Planckian light. Here the line in red shows the temperature conversion using the look-up-table. Notice how non-linear this curve is (compared to the linear relation in Mired units, Figure 5). In green, we show the conversion using Equation 14. The average distance between these two curves is just 15 degrees (Kelvin) which is, visually, never noticeable.

Let us now consider how close a Planckian spectrum, with temperature T^P is compared to a spectrum created using Wien's function

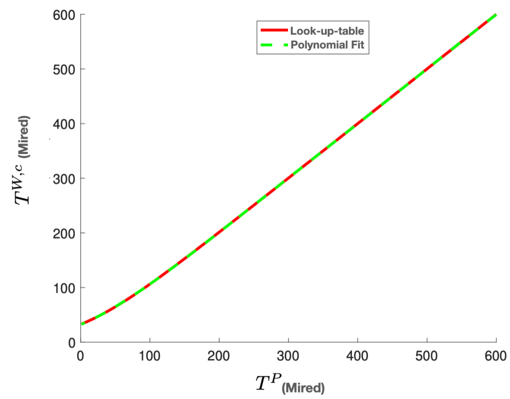

Figure 6: The relationship between Planckian Mired colour temperature and the corrected Wien temperature. Red line is a plot of the look-up-table mapping. Dashed-green plots the fit using a 5th order polynomial.

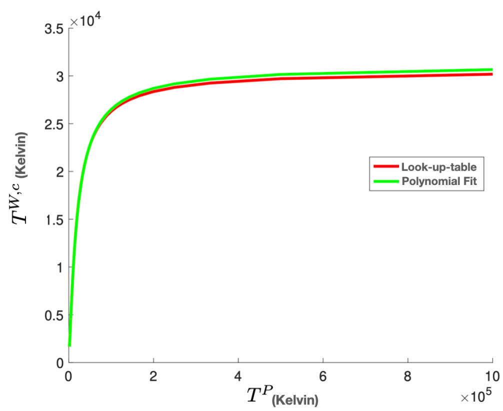

Figure 7: The relationship between Planckian colour temperature (in Kelvin) and the corrected Wien temperature. Red line is a plot of the look-up-table mapping. Green plots the fit using a 5th order polynomial

and the corrected temperature $T^{W,c} = f(T^P)$. These spectra are, as before, represented as discrete sampled vectors (at 81 uniformly distributed points from 380 to 780 Nanometres) and Equation 4 is used to calculate their angular error. The blue line in Figure 8 records the angular error for the look-up-table solution. The error associated with the analytic correction function (see Equation 14) is shown with a dashed red line. For reference if we do not use a correction function (and use T^P) as the argument to Wien's function then we generate the green error curve. It is evident that carrying out no correction results in an angular error of, in the worst case, more than 6 degrees for very high temperature Planckian lights. In contrast, the look-up-table and correction function $f()$ always deliver lights that are less than 1 degree from the desired Planckian spectrum.

To illustrate what these angular error numbers mean, let us return to our example in Figure 2. In Figure 9, we show in red the 12000K Planckian spectrum. Blue is the spectrum of light generated by Wien's formula using $T^{W,c} = 10912$ Kelvin. The dashed blue light is for

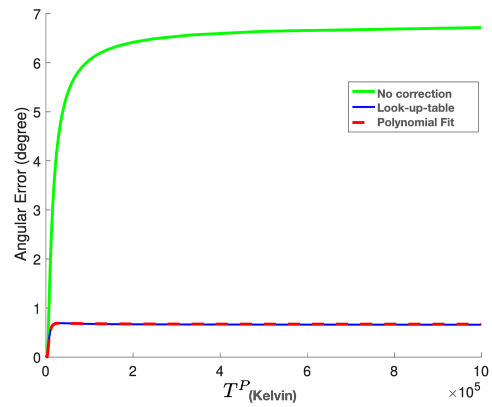

Figure 8: For Planckian color temperatures up to 1,000,000 Kelvin we plot the angular error for the best corresponding Wien approximation where a look-up-table (blue line) and analytic temperature conversion (dashed red) line is used.In green we plot the error when the Planckian colour temperature is used in Wien's equation.

the 12000K Wien formula. It is evident that by correcting the color temperature that drives Wien's function that we arrive at a much more similar spectrum. Indeed, the angular error reduces from 2.4 to 0.58 degrees.

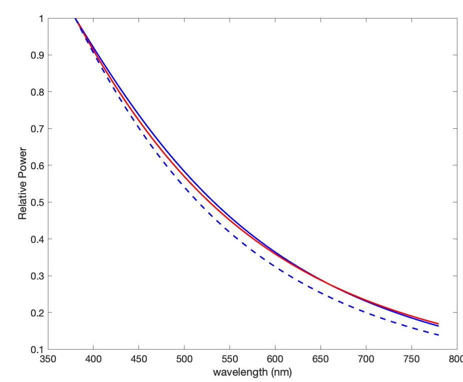

Figure 9: A 12000K Planckian is plotted as a solid red line. The dashed blue line is a 12000K Wien light and the solid blue is 10912K Wien light. Evidently the latter Wien light is closer to the Planckian All spectra are normalised to have maximum power equal to one

For Planckian lights in the interval 1667 to 1,000,000K, we report in Table 1 the mean, median, max and 95th% percentile of the angular errors between the actual and Wien approximate spectrum (using the corrected temperature). Three 'corrections' are tested. First in the column 'uncorrected' we do not correct the temperatures. Rather the Planckian color temperature is used to directly drive the Wien approximation. Then, we correct the Planckian temperature using our look-up-table, Equation 13. Our look up table function is approximated by an analytic function $f()$, see Equation 14, and the results are reported in the last column.

Additionally, we evaluated our corrected version in terms of the distance in u',v' chromaticity space, $\Delta u'v'$, between the Planckian lights set and the three sets of Wien lights (uncorrected, look-up-table and analytic function). Here the efficacy of our fit is even more evident. The mean $\Delta u'v'$ is 0 (to 3 decimal places). However, uncorrected temperatures leads to a max $\Delta u'v'$ of 0.016 (significantly above the 0.004 threshold taken to be a jnd in u'v' space [20, 21]). However, the look-up-table and analytic conversion function leads to maximum errors of 0.001 and 0.002 respectively.

	Uncorrected	LUT-corrected	$f()$-corrected
Mean	0.848	0.180	0.187
Median	0.065	0.030	0.047
Max	6.718	0.691	0.691
95th%	4.734	0.682	0.684

Table 1: The mean, median, max and 95th percentile of the angular error between Planckian lights and their equivalent Wien lights when the colour temperature is uncorrected, Look-up-table corrected and corrected with the analytic function $f()$.

	Uncorrected	LUT-corrected	$f()$-corrected
Mean	0.002	0.000	0.000
Median	0.000	0.000	0.000
Max	0.016	0.001	0.002
95th%	0.010	0.001	0.001

Table 2: The mean, median, max and 95th of $\Delta u'v'$ between Planckian lights' chromaticity coordinates and those of their equivalent Wien lights when the colour temperature is uncorrected, Look-up-table corrected and corrected with the analytic function $f()$.

Conclusion

Planck's famous equation defining a black-body radiator is tolerably well approximated by a simpler formula called Wien's approximation. However, especially for higher colour temperatures (say T $>$ 4000 Kelvin) the approximation is not as good with the delta between the actual Planckian and the Wien approximation becoming visually quite noticeable for temperatures larger than 10000K. In this paper, we have shown that we can map a Planckian temperature T^P a new converted temperature $T^{W,c} = f(T^P)$ such that the spectra generated by Wien's approximation becomes much closer to the desired Planckian.

Our correction function can be implemented as a look-up-table or as a polynomial function (operating in Mired units and then converted back to temperature in Kelvin). After application of our correction function, the generated spectra are very similar to the desired Planckian spectra. Over the range of lights with typical temperatures (1 to 600 Mired units or 1667 to 1,000,000 Kelvin) our corrected spectra plotted on the same graph as Planckian are almost coincident and the worst case Delta u'v' Euclidean chromaticity distance (between actual Planckian and corrected Wien) is much less than 1 JND.

Acknowledgments

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