https://doi.org/10.2352/CIC.2022.30.1.02

Why Achromatic Response is not a Good Measure of Brightness

Luke Hellwig[1], Dale Stolitzka[2], Mark D. Fairchild[1]
[1]Munsell Color Science Laboratory, Rochester Institute of Technology; Rochester, New York, USA
[2]Samsung Display America Lab; San Jose, California, USA

Abstract

Luminance underestimates the brightness of chromatic visual stimuli. This phenomenon, known as the Helmholtz-Kohlrausch effect, is due to the different experimental methods—heterochromatic flicker photometry (luminance) and direct brightness matching (brightness)—from which these measures are derived. This paper probes the relationship between luminance and brightness through a psychophysical experiment that uses slowly oscillating visual stimuli and compares the results of such an experiment to the results of flicker photometry and direct brightness matching. The results show that the dimension of our internal color space corresponding with our achromatic response to stimuli is not a scale of brightness or lightness.

Background

One common metric for the sensitivity of the human visual system in photopic conditions is luminance, which is defined by a spectral sensitivity function called $V(\lambda)$. $V(\lambda)$ was standardized in 1924 by the Commission Internationale de l'Éclairage (CIE) using results from an experimental method called heterochromatic flicker photometry [1]. In flicker photometry, the visual stimulus alternates quickly between a test patch and a reference patch (on the order of 10-30 Hz [2, 3, 4], but dependent upon the luminance of the stimuli [5]). The intensity of the reference patch is adjusted until the two patches fuse and the perception of flicker disappears. If the frequency of the flicker is properly set, then there will be a small range of intensities for which flicker fusion occurs [3]. In the case of $V(\lambda)$, flicker photometry was used with spectral test stimuli to determine the visual sensitivity to the entire visible spectrum. (For more detailed discussion of the methods and data used to derive $V(\lambda)$, see [2].) Luminance has seen been widely adopted, partially because its definition is inherently additive: the luminance of a stimulus made from different sources is equal to the sum of the luminances of all sources.

Brightness is defined as the degree to which a visual stimulus appears to reflect or emit more or less light [6]. Lightness is defined as the brightness of a stimulus relative to the brightness of an equally-illuminated white object in the scene [6]. A common method for assessing brightness is direct brightness matching [7, 8, 9, 10, 11]. In this method, two visual stimuli are shown next to each other with a small gap in between. Observers adjust the intensity of one stimulus until the two stimuli appear equally bright or light. Unlike luminance, brightness is not additive [12]: if chromatic lights (e.g., red, green, and blue lights) are combined to form an achromatic light, the sum of the brightnesses of the constituent lights will be greater than the brightness of the achromatic light. In other words, the results of heterochromatic flicker photometry and direct brightness matching do not agree with each other, and the brightness of chromatic stimuli are underestimated by their luminance. This phenomenon is known as the Helmholtz-Kohlrausch effect and has been widely studied [8, 9, 10, 13, 14, 15, 16].

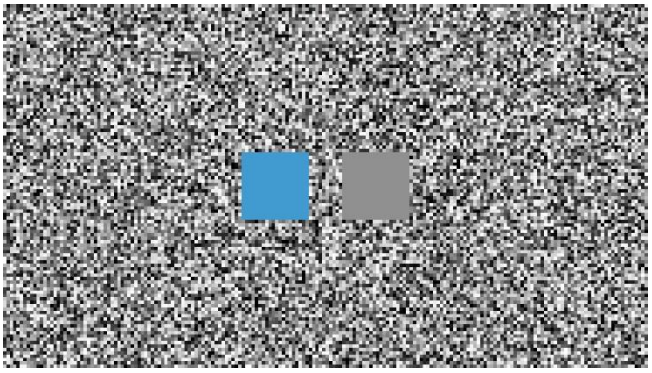

Figure 1. Screenshot of example stimulus from direct matching experiment. Observers adjusted the luminance of the achromatic patch (right) until it matched the brightness of the chromatic patch (left). The left/right orientation of the achromatic and chromatic patches was randomized for each trial. Each patch occupied approximately 2° of visual angle with a 1° gap between them. Colors are approximate.

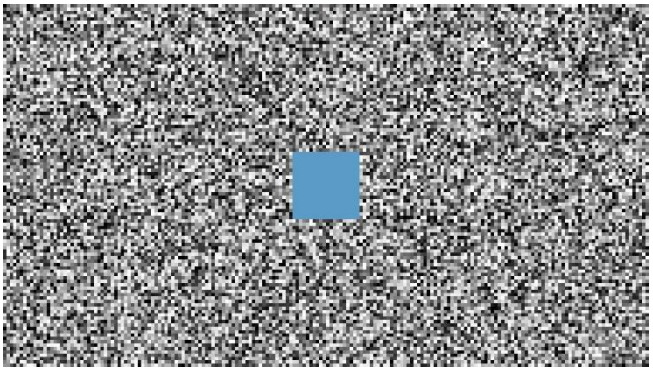

Figure 2. Screenshot of example stimulus from the flicker/temporal oscillation experiment. The patch occupied approximately 2° of visual angle. Color is approximate.

There are two fundamental explanations for the discrepancy between heterochromatic flicker photometry and direct brightness matching. Firstly, the parvocellular neural pathways that carry chromatic information are less sensitive, and perhaps completely insensitive, to the temporal frequencies at which flicker photometry is performed [4, 17]. Undoubtedly, though, they will be sensitive to the static stimuli in direct brightness matching. This suppression of chromatic information only during flicker photometry but not during direct matching could partially explain the discrepancy between the luminance and brightness of chromatic stimuli [18]. Furthermore, the two experimental methods contain different tasks: in one, to minimize flicker, and in the other, to match brightness. There is no rule of perception that states that the flicker minimization occurs when the stimuli are equally bright. In fact, task-dependency has already been reported in research that compared direct brightness matching to an alternate method called minimally distinct border [10]. In minimally distinct border experiments, a test patch and

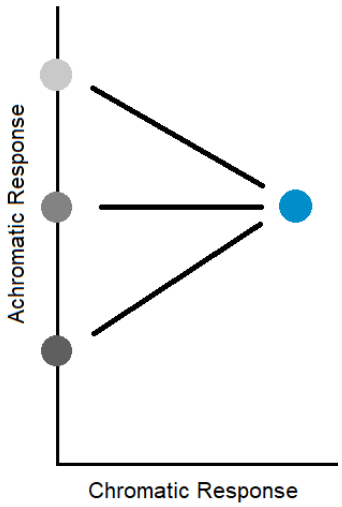

Figure 3. Schematic diagram of stimuli used for temporal oscillation experiment. The single patch stimulus (Figure 2) oscillated between the fixed chromatic endpoint and the adjustable achromatic endpoint. CAM16 lightness and chroma were used as the achromatic and chromatic response dimensions, respectively, to calculate the intermediate stimuli between the endpoints. Observers adjusted the luminance of the achromatic endpoint until a perceptual minimum was reached. This point indicates the achromatic endpoint which has the same achromatic response as the chromatic endpoint in the observer's internal perceptual color space (see Discussion).

reference patch are placed directly adjacent, and the reference patch adjusted until the border between the two is minimally visible or distinct. Such experiments appear to produce results that are more similar to luminance-like matches than the results produced by direct brightness matching, showing that task method can have a substantial effect [10, 19].

In this work, we separate the two factors described above which distinguish flicker photometry from direct brightness matching. By slowing down flicker photometry to allow both the magnocellular and parvocellular pathways to respond to the experimental stimuli [20], we can directly measure the effect of the difference between these two experimental tasks. Such work has potential implications for color spaces, such as CIELAB [21], and color appearance models, such as CIECAM16 [22, 23], which seek to predict perceptual attributes including brightness and lightness from information about the stimulus and (in the case of color appearance models) the environment in which the stimulus is viewed. Understanding the difference between methods of measuring brightness is especially relevant for those with an interest in incorporating the Helmholtz-Kohlrausch effect into such models or color spaces [15, 24, 25, 26]. Conversely, color appearance models provide a baseline within the context of experimental design to investigate how these experimental tasks operate in the perceptual domain, allowing us draw more meaningful conclusions from experimental results than would be possible using CIE XYZ tristimulus values, which have no inherent perceptual meaning.

Methods

Psychophysical experiments were run using two methods of stimulus presentation: direct matching and temporal gradient/flicker. In direct brightness matching, one chromatic patch and one achromatic patch were shown side by side against a random noise background (Figure 1). Each patch filled approximately one degree of visual angle and the two patches were separated by approximately one half of one degree of visual angle. The nine observers were instructed to use a keyboard to adjust the luminance of the achromatic patch until the two patches matched in brightness. Each judgement was repeated, resulting in eighteen total observations. The random noise background had an average CIECAM16 lightness of 50 relative to a 950 cd/m^2 D65 white point. The noise pattern was used to reduce the effect of simultaneous contrast and to prevent bias by removing a fixed reference point for observers (a uniform background) without changing their overall state of adaptation.

In the flicker/temporal gradient method, a single, one-degree patch was shown to observers against the random noise background (Figure 2). The patch oscillated continuously between the test chromatic stimulus and the achromatic stimulus (Figure 3). Observers were instructed to adjust the luminance of the achromatic stimulus to minimize their perception of flicker. Five oscillation frequencies were tested: 0.5 Hz, 1.39 Hz, 3.87 Hz, 15 Hz, and 30 Hz. Intermediate stimuli in the oscillations were evenly spaced in CIECAM16 color space between the chromatic and achromatic stimuli. The number of intermediate stimuli was determined by the oscillation frequency and the refresh rate of the monitor (60 Hz). For instance, the 1.39 Hz oscillation had 21 intermediate stimuli, whereas the 15 Hz oscillation had only one, and the 30 Hz oscillation had none.

In a follow-up experiment to test the effect of the intermediate stimuli, the intermediate stimuli were removed. In this case, the patch simply alternated between the achromatic and chromatic endpoints at each test frequency.

In an additional follow-up experiment, the intermediate stimuli were adjusted from being evenly spaced between the achromatic and chromatic endpoints to being sinusoidally spaced between the endpoints. This modification had no statistically significant effect on the results and thus is excluded from further analysis.

Experimental stimuli were displayed on an Asus ProArt PA32UCS monitor, controlled using Psychtoolbox-3 [27] and MATLAB on a Windows computer with an Nvidia Quadro P400 video card. Three chromatic test patches with CAM16 hue angles of 12, 110, and 242 were tested, roughly corresponding to red, yellow, and blue, respectively. These hues were chosen because red and blue are strongly affected by the Helmholtz-Kohlrausch effect, whereas yellow is minimally affected by it. The red, yellow, and blue patches had CIECAM16 lightness of 47.1, 50.7, and 50.7, and CIECAM16 chroma of 20.5, 26.3, and 23.2, respectively, calculated using recently-proposed corrections to CIECAM16 [28]. A dark surround and a degree of adaptation of one were used for all CIECAM16 calculations. The slight variation in lightness and chroma values were due to differences between the Rec. 2100 PQ color space [29], which was used to generate the code values for the experimental stimuli, and the performance of the Asus ProArt display in matching that standard. However, these variations do not impact the conclusions of this paper.

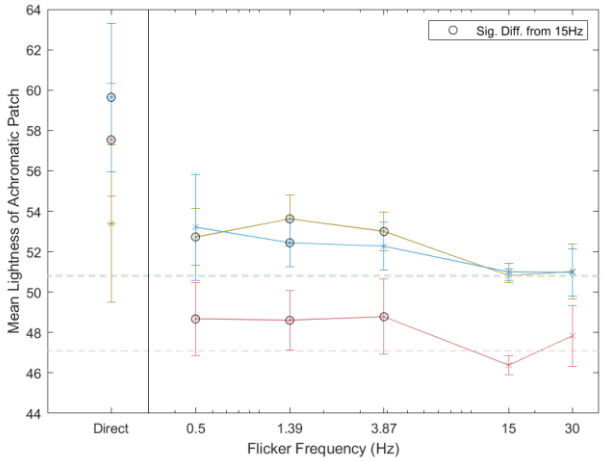

Figure 4. Results of direct matching experiment (left) and oscillation method with variable frequency (right) for the three tested hues (red, yellow, and blue). The y-axis represents the mean lightness of the achromatic patch that was adjusted by observers in each experimental condition. Dashed lines indicate the lightnesses of the fixed chromatic patches. Values that are significantly different ($p < 0.05$ on two-sample t-test with unequal variances) from the luminance-like match at 15 Hz are circled.

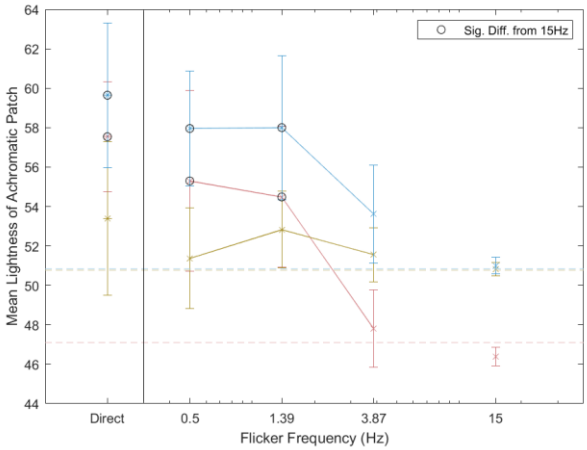

Figure 5. Results of the square-wave oscillation experiment (flicker frequencies 0.5 Hz, 1.36 Hz and 3.87 Hz) compared to the direct matching and 15 Hz results from Figure 4. The y-axis represents the mean lightness of the achromatic patch that was adjusted by observers in each experimental condition. Dashed lines indicate the lightnesses of the fixed chromatic patches. Values that are significantly different ($p < 0.05$ on two-sample t-test with unequal variances) from the luminance-like match at 15 Hz are circled.

Results

The results of the experiment are quantified by the lightness of the achromatic patch matched to the test chromatic patch by the observers in each viewing situation. The mean lightnesses of the achromatic patches are shown in Figure 4 along with estimated 95% confidence intervals. The statistical significance of the differences between mean values was calculated using Welch's two-sample *t*-test at an α level of 0.05 with equal variances not assumed.

First, the results from the 15 Hz and 30 Hz flicker observations were compared to determine which frequency best represented traditional heterochromatic flicker photometry. The mean values from the two frequencies were not significantly different for any individual hue (Table 1), but the variance of the 30 Hz observations was substantially greater. Observers reported that there was a wide range of lightnesses for which their perception of flicker disappeared at this frequency of oscillation, explaining the high variance of their responses. Thus it was decided that the 15 Hz oscillation should serve as the representative sample of the heterochromatic flicker method.

Figure 4 shows that at lower frequency oscillations, the observers chose a lighter achromatic patch to minimize their perception of flicker with the same chromatic patches. Values that were significantly different from the mean value at 15 Hz are circled in Figure 4 (*p* values in Table 1). The difference between the 15 Hz oscillation and slower frequencies could be due to the increased sensitivity of the parvocellular neural pathway—carrying color information—at lower frequencies.

When the pattern of the slow frequency oscillations was changed from an even gradient to simply alternating between the two endpoints, the matched lightness of the achromatic patch increases even further (Figure 5) for the red and blue patches. The difference between the two methods was statistically significant using test described above for the red and blue patches, but not for the yellow patches. An explanation for this result is given in the Discussion section.

The results of the direct matching experiment are also shown in Figure 4. For the red and blue test patches, the mean lightnesses of the matched achromatic patches were significantly greater than their value from the smooth oscillation method (Table 2).

Table 1. *p* values for Welch's two-sample t-test with equal variance not assumed for the data represented in Figure 4 compared to mean achromatic lightness for the 15 Hz oscillation. Statistically significant values are bolded.

Color	Flicker Frequency (Hz)				
	Direct	0.5	1.39	3.87	30
Red	**4.2×10^{-7}**	**0.027**	**0.011**	**0.024**	0.085
Yellow	**0.22**	**0.018**	**2.8×10^{-4}**	**4.1×10^{-4}**	0.79
Blue	**2.6×10^{-4}**	0.12	**0.036**	0.063	0.96

Table 2. *p* values for Welch's two-sample t-test with equal variance not assumed for mean results of oscillation method compared to mean results for the direct matching achromatic. Statistically significant values are bolded.

Color	Flicker Frequency (Hz)				
	0.5	1.39	3.87	15	30
Red	**1.4×10^{-5}**	**8.4×10^{-6}**	**1.8×10^{-5}**	**4.2×10^{-7}**	**2.5×10^{-6}**
Yellow	0.75	0.91	0.85	0.22	0.27
Blue	**2.6×10^{-4}**	**2.4×10^{-4}**	**0.0012**	**0.0015**	**0.0089**

Discussion

The experimental results clearly show three distinct regimes of observer behavior corresponding with the direct matching, slow oscillation, and fast oscillation methods of stimulus presentation. The fast oscillation presentation leads to a luminance-like match solely based on the achromatic information present in the stimuli [17]. In direct brightness matching, chromatic information is incorporated into the stimulus judgement, leading chromatic stimuli to be judged as brighter in direct matching than in luminance-like matching [18]. This is known as the Helmholtz-Kohlrausch (H-K) effect and is represented in the results of our experiment. That we observed a stronger H-K effect for the red and blue stimuli than for the yellow stimulus is in agreement with other studies [15, 30]. Given that observers in the direct matching stimulus presentation were asked to match the patches in brightness, we can conclude that

patches directly matched have equal (perceived) brightness. Therefore, the patches that were matched in the fast oscillation method do not have equal brightness.

The results from the novel slow oscillation presentation require more detailed explanation. Given the increased chromatic contrast sensitivity at the low frequencies tested in this experiment, we expect the chromatic (parvocellular) channel of our visual system to respond to the oscillation, as in direct brightness matching [17, 20]. However, the observers' task is different for the slow oscillation matching than the direct brightness matching: minimizing their perception of flicker versus matching brightness.

Unlike with fast flicker, where the observers' perception of flicker could disappear when the chromatic and achromatic endpoints were matched, the perception of oscillation never disappeared as the patch oscillated between its achromatic and chromatic endpoints. Nevertheless, observers could adjust the achromatic patch to find a clear perceptual minimum in the slow oscillation. In this case, observers experienced a minimum in the perceived speed of the oscillation. The perception of a minimum in speed can be understood by conceptualizing the observers' perception of the experimental stimuli as existing in an internal color space (Figure 3). As the achromatic endpoint moves up and down the achromatic axis (as it is adjusted by the observer), it moves farther from and closer to the chromatic endpoint. Thus, the oscillation covers more or less distance in our internal color space (over the same amount of time) and appears to speed up or slow down. A minimum in perceived speed occurs when the distance between the achromatic and chromatic endpoints is at a minimum in the observers' internal color space. This minimum distance occurs when the achromatic and chromatic endpoints have an equal position along the dimension of achromatic response. Thus, unlike other psychophysical methods of luminance or brightness measurement, this slow oscillation is a direct measure of this dimension of our internal color space.

This interpretation of the results of the slow oscillation matching is confirmed by the effect of switching from a colorimetrically smooth oscillation between the achromatic and chromatic endpoints (Figure 4) to a temporal square-wave pattern that simply alternated between the two endpoints without any intermediate stimuli (Figure 5). For the smooth oscillation, the mean lightnesses of the achromatic patches matched to the red and blue chromatic patches were significantly less than the lightnesses of the achromatic patches matched to the same chromatic patches via direct brightness matching (Figure 4). However, when the intermediate stimuli were removed, the lightnesses of the matched achromatic patches increased as the frequency decreased, trending towards the directly matched lightnesses. This equivalence between directly matching two patches and viewing them in alternation on a single location is coherent with the idea that the observer must move their gaze between the two patches when directly comparing, so both situations generate similar temporal patterns in the visual system. More importantly, that the square wave oscillation results match the direct matching results confirms that the difference between the smooth oscillation results and the direct matching results is not due to temporal effects in the visual system's response to the low-frequency oscillation which are not accounted for in our above explanation. Put simply, this result supports our assumption that the same neural pathways in our visual system are active during the direct matching and slow oscillation stimulus presentations and that the difference in the results is primarily due to the difference in task.

The statistically significant difference between the slow oscillation method and the direct matching method leads us to the conclusion that an achromatic color and a chromatic color with equal (perceived) brightness *do not* have the same position along the this measurable dimension of our internal color space. Instead, our results show that an achromatic color and a chromatic color are closest to each other in our internal color space when they produce a similar achromatic response in our visual system. Thus, we can conclude that this dimension in our internal color space is the achromatic response, not brightness or lightness. These results have profound implications for one-dimensional scales of achromatic response, such as the L^* dimension in CIELAB or the Q and J dimensions in CIECAM02 and CIECAM16. We have presented direct experimental proof that a one-dimensional scale of achromatic response exists in our internal color space and that such a dimension is *not* scale of brightness or lightness and *cannot* be so.

A common aim of models of the H-K effect has been to "correct" the achromatic response scale (e.g., L^*, Q, J) so that scale values of chromatic stimuli match the scale values of equally-bright achromatic stimuli. Our experiment demonstrates that brightness (and therefore lightness) are dependent on multiple dimensions and thus should not be modeled using a one-dimensional scale based on a single physical metric such as luminance. This conclusion does not mean that the one-dimensional scales are unneeded or incorrect; our experiment actually demonstrates the opposite: that such a scale does exist in our internal color space. Rather, we have demonstrated that the terms "brightness" and "lightness" should not be used to label any single dimension in color spaces, such as CIELAB or CAM16-UCS, which attempt to match our internal representation of color in three-dimensional space. "Value" could be a better term for achromatic response scales (e.g., L^*, Q, J) to avoid the misconception that colors with equal achromatic response scale values have equal brightness and lightness. "Value" comes from the achromatic scale of the Munsell color order system [31] and has a clear meaning without the connotation that colors with equal value has equal perceived brightness.

The other consequence of this experiment is to show that models of the H-K effect should be multidimensional, combining both the achromatic response dimension and a dimension or dimensions related to chromatic intensity. Examples of such scales of brightness include vector brightness in the color appearance models of Guth [16, 32] and vividness as proposed by Berns [33, 34]. Recent work by Xie psychophysically measuring the zero-grayness threshold also holds potential for developing a measure of brightness that accounts for the colorfulness of stimuli [35, 36], building off of previous work on zero-grayness and brilliance by Evans [37, 38]. Fairchild and Heckaman have discussed whether brightness and lightness should even be mapped in three-dimensional space or simply be modeled as an independent scale [39].

Research is still underway as to the correct method for combining achromatic and chromatic response dimensions to generate a two-dimensional brightness scale. Future work is also planned to use the slowly oscillating gradient method to investigate the chromatic dimension of our internal perceptual color space as was accomplished for the achromatic dimension in this study.

Acknowledgments

This research was funded by Samsung Semiconductor, Inc.

References

[1] CIE, "Comission internationale de l'Eclairage proceedings, 1924.," Cambridge University Press, Cambridge, 1926.

[2] L. T. Sharpe, A. Stockman, W. Jagla and H. Jägle, "A luminous efficiency function, V*(λ), for daylight adaptation," *Journal of Vision*, vol. 5, pp. 948-968, 2005.

[3] R. A. Bone and J. T. Landrum, "Heterochromatic flicker photometry," *Archives of Biochemistry and Biophysics*, vol. 430, pp. 137-142, 2004.

[4] B. B. Lee, P. R. Martin and A. Valberg, "The Physiological Basis of Heterochromatic Flicker Photometry Demonstrated in the Ganglion Cells of the Macaque Retina," *J. Physiol.*, vol. 303, pp. 323-347, 1988.

[5] G. S. Brindley, J. J. du Croz and W. A. H. Rushton, "The Flicker Fusion Frequency of the Blue-Sensitive Mechanism of Color Vision," *J. Physiol.*, vol. 183, pp. 497-500, 1966.

[6] R. W. G. Hunt, "Colour terminology," *Color Res. Appl.*, vol. 3, pp. 79-87, 1978.

[7] J. P. Comerford and P. K. Kaiser, "Luminous-efficiency functions determined by heterochromatic brightness matching".

[8] C. L. Sanders and G. Wyszecki, "Correlate for brightness in terms of CIE color matching data," in *Proc. 15th Session*, Vienna, 1963.

[9] G. Wyszecki, "Correlate for Lightness in Terms of CIE Chromaticity Coordinates and Luminous Reflectance," *J. Opt. Soc. Am.*, vol. 57, no. 254-257, 1967.

[10] G. Wagner and R. M. Boynton, "Comparison of Four Methods of Heterochromatic Photometry," *J. Opt. Soc. Am.*, vol. 62, no. 12, pp. 1508-1515, 1972.

[11] M. Ikeda and Y. Nakano, "Spectral luminous-efficiency functions obtained by direct heterochromatic brightness matching for point sources and for 2 and 10 fields," *J. Opt. Soc. Am. A*, vol. 3, no. 12, pp. 2105-2108, 1986.

[12] P. K. Kaiser and G. Wyszecki, "Additivity failures in heterochromatic brightness matching," *Color Res. Appl.*, vol. 3, no. 4, pp. 177-182, 1978.

[13] Y. Nayatani, Y. Umemura, H. Sobagaki, K. Takahama and K. Hashimoto, "Lightness perception of chromatic object colors," *Color. Res. Appl.*, vol. 16, no. 1, pp. 16-25, 1991.

[14] Y. Nayatani, Y. Gomi, M. Kamei, H. Sobagaki and K. Hashimoto, "Perceived lightness of chromatic objects including highly saturated colors," *Color Res. Appl.*, vol. 17, no. 2, pp. 127-141, 1992.

[15] M. D. Fairchild and E. Pirrotta, "Predicting the lightness of chromatic object colors using CIELAB," *Color Res. Appl.*, vol. 16, no. 6, pp. 385-393, 1991.

[16] S. L. Guth and H. R. Lodge, "Heterochromatic additivity, foveal spectral sensitivity, and a new color model," *J. Opt. Soc. Am*, vol. 63, no. 4, pp. 450-462, 1973.

[17] B. B. Lee, "Visual pathways and psychophysial channels in the primate," *J. Physiol.*, vol. 589, pp. 41-47, 2011.

[18] P. K. Kaiser and J. P. Comerford, "Flicker Photometry of Equally Bright Lights," *Vision Res.*, vol. 15, pp. 1399-1402, 1975.

[19] P. K. Kaiser, "Minimally Distinct Border as a Preferred Psychophysical Criterion in Visual Heterochromatic Photometry," *J. Opt. Soc. Am. A*, vol. 61, pp. 966-971, 1971.

[20] D. H. Kelly and D. van Norren, "Two-band model of heterochromatic flicker," *J. Opt. Soc. Am.*, vol. 67, no. 8, pp. 1081-1091, 1977.

[21] CIE, "Colorimetry," Bureau Central de la CIE, Paris, 1986.

[22] C. Li, Z. Li, Z. Wang, Y. Xu, M. R. Luo, G. Cui, M. Melgosa, M. Brill and M. Pointer, "Comprehensive color solutions: CAM16, CAT16, and CAM16UCS," *Color Research and Applications*, vol. 42, pp. 703-718, 2017.

[23] "CIE 248:2022: The CIE 2016 Colour Appearance Model for Colour Management Systems: CIECAM16," CIE, 2022.

[24] R. W. G. Hunt, "Revised colour-appearance model for related and unrelated colours," *Color Res. Appl.*, vol. 16, no. 3, pp. 146-165, 1991.

[25] M. Kim, J.-H. Jo, Y. Park and S.-W. Lee, "Amendment of CIECAM02 with a technical extension to compensate Helmholtz-Kohlrausch effect for chromatic characterization of display devices," *Displays*, vol. 56, pp. 1-10, 2018.

[26] S. Hermans, K. A. G. Smet and Hanselaer, "Color apperance model for self-luminous stimuli," *JOSA A*, vol. 35, no. 12, pp. 2000-2009, 2018.

[27] M. Kleiner, D. Brainard and D. Pelli, "What's new in Psychtoolbox-3?".

[28] L. Hellwig and M. D. Fairchild, "Brightness, lightness, colorfulness, and chroma in CIECAM02 and CAM16," *Color Res. Appl.*, 2022.

[29] "Recommendation BT.2100-2: Image parameter values for high dynamic range television for use in production and international programme exchange," International Telecommunications Union - Radiocommunication, 2018.

[30] Y. Nayatani, "Simple estimation methods for the Helmholtz-Kohlrausch effect," *Color Res. Appl.*, vol. 22, no. 6, pp. 385-401, 1997.

[31] R. G. Kuehni, "The Early Development of the Munsell System," *Color Res. Appl.*, vol. 27, no. 1, pp. 20-27, 2002.

[32] S. L. Guth, "Model for color vision and light adaptation," *J. Opt. Soc. Am. A*, vol. 8, no. 6, pp. 976-993, 1991.

[33] R. S. Berns, "Extending CIELAB: Vividness, Depth, and Clarity," *Color Res. Appl.*, vol. 39, no. 4, pp. 322-330, 2014.

[34] L. Hellwig, D. Stolitzka, Y. Yi and M. D. Fairchild, "Brightness and Vividness of High Dynamic Range Displayed Imagery," in *SID Symposium Digest of Technical Papers*, San Jose, 2022.

[35] H. Xie and M. D. Fairchild, "G0 Revisited as Equally Bright Reference Boundary," in *Proc. IS&T 29th Color and Imaging Conf.*, 2021.

[36] M. D. Fairchild, "A Digital Test Chart for Visual Assessment of Color Appearance Scales," in *Proc. IS&T 29th Color and Imaging Conf.*, 2021.

[37] R. M. Evans, "Fluorescence and Gray Content of Surface Colors," *J. Opt. Soc. Am.*, vol. 49, no. 11, pp. 1049-1059, 1959.

[38] R. M. Evans and B. K. Swenholt, "Chromatic Strength of Colors: Dominant Wavelength and Purity," *J. Opt. Soc. Am*, vol. 57, no. 11, pp. 1319-1324, 1967.

[39] M. D. Fairchild and R. L. Heckaman, "Deriving Appearance Scales," in *Proc. IS&T 20th Color and Imaging Conf.*, Los Angeles, 2012.