A Method to Estimate Fractional Areas of Neugebauer Primary Colors

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Abstract

There are a lot of works following the Neugebauer's study on modeling the reflectance of a halftone color. In the model, there are mainly two input parameter sets: one is the reflectance of primary colors, the ink solids and their overlaps; the other is the area of each primary colors, that is, the 'fractional areas of Neugebauer primary colors' (NPAs). For the later, Demichel's dot overlap law is widely used to estimate NPAs. However, the underline assumption of Demichel's law, that halftone dots of each ink are printed in a random arrangement, is not strictly true for most halftone patterns.

In order to get more accurate estimation, a novel method is proposed to calculate NPAs by employing additional non Neugebauer primary color samples. It shows a significant improvement on estimating the reflectance of a halftone color. The average error (ΔE^*_{ab}) is reduced from 4.7 to 1.82, and the maximum error from 11.53 to 6.55.

Introduction

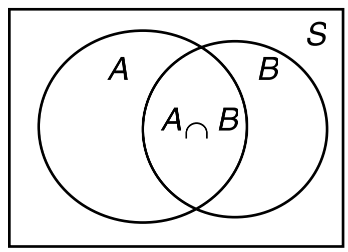
Halftone printing technique is widespreadly used today in graphic arts and desktop publishing to implement color printing. It develops colors by varying the areas covered by three or more inks. To get correct output, its corresponding printer driving signals must be determined systematically. Therefore, a lot of studies taken to derived their relationship. The most important one is Neugebauer equations[1,2]. It thinks the reflectance of a halftone color as summation of the reflectance of Neugebauer primary colors (NPC) weighted by it's corresponding fractional area (i.e. fractional area of Neugebauer primary color, NPA). On account of lateral scattering of light within the paper, Yule, Nielsen, and Stephen Viggiano modify the original equation. It is known as "spectral Yule-Nielsen modified Neugebauer model"[3] :

$$R(\lambda)_{cmyk}^{1/n} = \sum_{i=1}^{k} A_i R_i(\lambda)^{1/n} \tag{1}$$

where

$R(\lambda)_{cmyk}$ = the reflectance of the halftone color
n = the Yule-Nielsen value
k = The number of NPCs, 8 for 3-ink process; 16 for 4-ink process
A_i = the fractional area of each NPC
$R_i(\lambda)$ = the spectral reflectance of the each NPC

Usually the Demichel's law is used to estimate the relationship between the NPAs and the relative dot areas of the inks. A simple graphical representation for Demichel's law is shown in Figure 1. It can be analogous to a two-ink halftone color, where a and b are the dot area of two inks. According to the modified Neugebauer equation, the spectral reflectance of the color would be $R(\lambda)^{1/n} = (1-a)(1-b)R_{s-(A\cup B)} + a(1-b)R_A + (1-a)b \cdot R_B + a \cdot b \cdot R_{A\cap B}$. Following the same way, we may derive the spectral reflectance for four-ink process color. The 16 NPAs are shown in equation (2). Although it is said that the law used with the spectral Yule-Nielsen modified



$P(S)=1.0$
$P(A)=a$
$P(B)=b$
$P(A\cap B)=a\cdot b$

Figure 1. The illustration of the Demichel's law for a two-color case

$$\begin{aligned}
A_1 &\equiv A_w = (1-a_c)(1-a_m)(1-a_y)(1-a_k)\\
A_2 &\equiv A_c = a_c(1-a_m)(1-a_y)(1-a_k)\\
A_3 &\equiv A_m = (1-a_c)a_m(1-a_y)(1-a_k)\\
A_4 &\equiv A_y = (1-a_c)(1-a_m)a_y(1-a_k)\\
A_5 &\equiv A_r = (1-a_c)a_m a_y(1-a_k)\\
A_6 &\equiv A_g = a_c(1-a_m)a_y(1-a_k)\\
A_7 &\equiv A_b = a_c a_m(1-a_y)(1-a_k)\\
A_8 &\equiv A_{3p} = a_c a_m a_y(1-a_k)\\
A_9 &\equiv A_k = (1-a_c)(1-a_m)(1-a_y)a_k\\
A_{10} &\equiv A_{ck} = a_c(1-a_m)(1-a_y)a_k\\
A_{11} &\equiv A_{mk} = (1-a_c)a_m(1-a_y)a_k\\
A_{12} &\equiv A_{yk} = (1-a_c)(1-a_m)a_y a_k\\
A_{13} &\equiv A_{rk} = (1-a_c)a_m a_y a_k\\
A_{14} &\equiv A_{gk} = a_c(1-a_m)a_y a_k\\
A_{15} &\equiv A_{bk} = a_c a_m(1-a_y)a_k\\
A_{16} &\equiv A_{4p} = a_c a_m a_y a_k
\end{aligned} \tag{2}$$

Neugebauer model provides an acceptable level of accuracy for halftone patterns superimposed at a 30 degree orientation[3], the law is not accurate enough for most halftone patterns.

To alleviate the errors from inaccurate NPA law, in the following of this paper, a method to estimate NPAs is proposed. It is assumed that the NPA can be represented as the weighted summation of respective NPAs of several primary halftone patterns. And the estimated NPAs will satisfy three practical constraints. Those are (a) the sum of all NPAs should be equal to 1.0; (b) each NPA should be within the range between 0.0 and 1.0; and (c) the relative dot area of each ink depends solely on the corresponding signal.

To demonstrate the improvement by this estimation, we use the ANSI IT8.7/3 characterization chart to derive the CMYK-to-L*ab transformation and evaluate its performance. It shows a significant improvement on several proofing systems, for example on Fuji ColorArt proofing system, the average error (ΔE^*_{ab}) reduces from 4.7 to 1.82 and maximum error from 11.53 to 6.55.

Method to Estimate the Fractional Areas of Neugebauer Primary Colors (NPAs)

In order to establish the relationship between CMYK dot areas and the corresponding NPAs, some colors regularly sampled in CMYK space should be printed and measured. The NPAs for each sampled color have to be estimated firstly according to the measured spectral reflectance. Finally, a look-up-table (LUT) comprising the NPAs of all sampled colors can be built. And the transformation from dot areas to NPAs will be implemented by a interpolation method using this LUT.

In order to estimate the NPAs for each sampled color, it is assumed that the practical NPAs for a color can be approximately by some primary NPA patterns or their combinations with different weights according to their similarities. Therefore, NPA can be represented as a summation of the weighted primary NPAs, that is,

$$A_i = \sum_j r_j \cdot Comp\_A_i(j), \qquad (3)$$

where $Comp\_A_i(j)$ is the i-th NPA of the j-th NPA pattern.

The ratios r_j are estimated to minimize the color difference. And the following three practical constraints are adopted:

(a) The sum of all NPAs should be equal to 1.0, that is

$$\sum_{i=1}^{16} A_i = 1.0;$$

(b) Each NPA should be within the range between 0.0 and 1.0, $0.0 \le A \le 1.0$; and

(c) The relative dot area of each ink should depends solely on the corresponding control signal. For example,

$$\sum_{i \in \{c,g,b,3p,ck,gk,bk,4p\}} A_i$$ should be equal to the cyan dot area corresponding to C, and should not be influenced by

M,Y, or K. (4)

Our estimation method will be explained as follows.

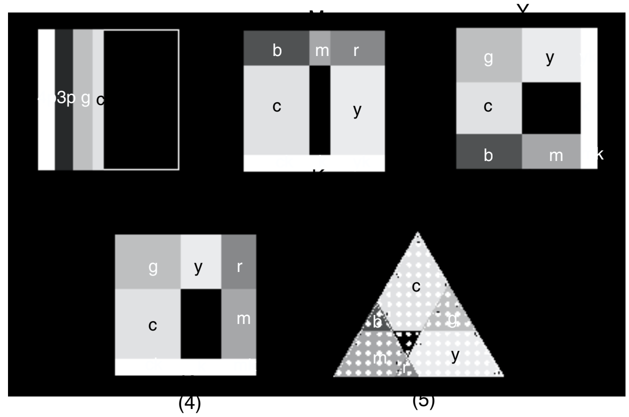


Figure 2. The first five primary halftone patterns

Six Primary Halftone Patterns

Currently, six halftone patterns are heuristically chosen as the primary patterns. Besides the pattern following Demichel's low, the other five primary patterns are shown in Figure. 2. They are represented as unit squares with each ink color growing from one side. Especially, the pattern (1) in Figure 2 with all inks growing from the same side is the same as the dot-on-dot halftone pattern. The pattern (5) is represented as a equilateral triangle with CMY ink color growing from each corner and K distributing uniformly in the triangle. According to Figure 2, the equations for calculating the NPAs from CMYK dot areas for each pattern can be established. The i-th NPA calculated from the j-th pattern is denoted as $Comp\_A_i(j)$. For examples,

$$Comp\_A_w(1) = 1 - \max(a_C, a_M, a_Y, a_K); \qquad (5)$$

$$Comp\_A_w(2) = \max(1 - a_C - a_Y, 0.) \times \max(1 - a_M - a_K, 0.);$$

$$Comp\_A_r(3) = \max(a_Y + a_M - 1, 0.) \times \max(1 - a_C - a_K, 0.);$$

$$Comp\_A_b(4) = \max(a_C + a_M - 1, 0.) \times \max(1 - a_Y - a_K, 0.);$$

and

$$Comp\_A_{4p}(5) = \left(\max\left(\sqrt{a_C} - 1 + \max\left(\sqrt{a_M} + \sqrt{a_Y} - 1, 0.\right) 0.\right)\right)^2 \times a_K$$

Estimate NPA Satisfying the Practical Constraints

As described above, the spectral reflectance of a CMYK halftone color may be represented as equation (6) by the corresponding NPAs of the six primary halftone patterns and the spectral reflectance of the Neugebauer primary color.

$$R(\lambda)^{1/n}_{estimated} = \sum_{i=1}^{16} A_i \cdot R_i(\lambda)^{1/n} = \sum_{i=1}^{16} \left(\sum_{j=1}^{6} r_j \cdot Comp\_A_i(j) \right) \cdot R_i(\lambda)^{1/n} \qquad (6)$$

For a CMYK halftone color, we have a measured reflectance $R(\lambda)_{estimated}$ and a estimated reflectance given by equation (6). With these information, r_j can be derived by solving the following constraint optimization problem: Find r_j, j =1~6 which minimize

$$\sum_{\lambda=400}^{700}\left(R_{measured}^{1/n}(\lambda)-\sum_{i=1}^{16}\sum_{j=1}^{6} r_j \cdot Comp\_A_i(j)R_i(\lambda)^{1/n}\right)^2 \quad (7)$$

and also satisfy the following constraint :

(a) $\sum_{i=1}^{16} A_i = 1.0$;

(b) $0.0 \le A_i \le 1.0$, $i = 1 \sim 16$; and

(c) The relative dot area of each ink should depends solely on the corresponding control signal.

Since $Comp\_A_i(j)$ are NPAs of the primary halftone patterns, the constraint (a) and (c) are equivalent to

$$\sum_{i=1}^{6} r_i = 1.0 \quad (8)$$

The former can be proved by equation (9),

Because $\sum_{i=1}^{16} Comp\_A_i(j) = 1.0$, for $i = 1 \sim 16$,

$$\sum_{i=1}^{16} A_i = \sum_{i=1}^{16}\sum_{j=1}^{6} r_j \cdot Comp\_A_i(j) =$$

$$\sum_{j=1}^{6}\left(r_j \cdot \left(\sum_{i=1}^{16} Comp\_A_i(j)\right)\right) = \sum_{j=1}^{6} r_j = 1.0 \quad (9)$$

and the latter can be explained by the following example,

Because $\sum_{i\in\{c,g,b,3p,ck,gk,bk,4p\}} Comp\_A_i(j) = a_c$ won't be influenced by M,Y, or K,

$$\sum_{i\in\{c,g,b,3p,ck,gk,bk,4p\}} A_i = \sum_{i\in\{c,g,b,3p,ck,gk,bk,4p\}}\left(\sum_{j=1}^{6} r_j \cdot Comp\_A_i(j)\right)$$

$$= \sum_{j=1}^{6} r_j \cdot \left(\sum_{i\in\{c,g,b,3p,ck,gk,bk,4p\}} Comp\_A_i(j)\right)$$

$$= \sum_{j=1}^{6} r_j \cdot a_c = a_c \cdot \sum_{j=1}^{6} r_j = a_c \quad (10)$$

also won't be influenced by M,Y, or K.

In the beginning, the constraint (b) is ignored. And the r_j, j= 1~6 which minimize the equation (7) and satisfy the equation (8) can be solved by using the first-order necessary condition of equality constraints[5], that is, if r_j, j= 1~6 is represented as a vector $\bar{r} \equiv (r_1, r_2, \ldots, r_6)$ and $\bar{r}^*$ is a local extreme point of f which is represented as equation (7) subject to a set of equality constraints on E^6, $h_1(\bar{r}) = 0$, $h_2(\bar{r}) = 0, \ldots, h_m(\bar{r}) = 0$ that are equation (8) in this case, there is a $\bar{\mu} \in E^m$ such that

$$\nabla f(\bar{r}^*) + \bar{\mu}^T \nabla h(\bar{r}^*) = 0. \quad (11)$$

This equation together with the m constraints give a total of 6+m equations in the 6+m variables, $\bar{r} \in E^6$ and $\bar{m} \in E^m$. Since this solution $\bar{r}^*$ satisfy equation (8), the corresponding NPAs also satisfy the constraint (a) and (c). But the constraint (b) should be checked. If this constraint is satisfied, these NPAs are the result of the estimation for this CMYK halftone color. If it isn't, the following method is used.

62 sets of boundaries constraints, $\{r_1 = 0.\}$, $\{r_2 = 0.\}$, ..., $\{r_1 = r_2 = 0.\}$, ..., and $\{r_1 = r_2 = \ldots = r_5 = 0.\}$ are adopted one by one together with the equation (8) in the above minimization problem. There is at least one set of boundary conditions which will let the estimated NPAs satisfy the constraint (b). For example, because of $\sum_{i=1}^{6} r_i = 1.0$, the boundary constraint, $\{r_1 = r_2 = \ldots = r_5 = 0.\}$ will let r_6 equal to 1. It means that the estimated NPAs are equal to the NPAs of the sixth primary pattern. Hence, the constraint (b) will be satisfied. Finally, the color differences, ΔE^*_{ab} between the measured color and the estimated color for all sets of boundary conditions which result the estimated NPAs satisfying the constraint (b) are calculated, and the NPAs with minimum ΔE^*_{ab} are chosen for this CMYK color.

Establishing the Relationship between Dot Areas and NPAs

An IT8.7/3 test chart is printed to be used for establishing the CMYK-to-L*ab. In order to establish the relationship between dot areas and NPAs, the NPAs for the colors corresponding to the following values: {K∈ 0%,20%};and C,M,Y∈ {0%,40%,70%,100%}}, {K=60%; and C,M,Y∈ {0%,40%,100%}},and {K=100%; and C,M,Y∈ {0%,100%}}, in A, B, and C parts should be estimated. Finally, a 4-dimension LUT between dot areas and NPAs is established, and the transformation from dot areas to NPAs is established by a 4-dimension interpolation method using this LUT. The 4-dimension interpolation method is implemented by the 3-dimention cubic interpolation method[6] according to the CMY values, together with the 1-dimention linear interpolation method according to the K value. For Sun Sparc Server 5, it need 15 seconds to build this LUT.

Experimental Results

One set of film screens for IT8.7/3 with ordinary angles are printed by an AGFA image setter. Three proof machines, Fujix ColorArt, Agfa Proof PRS, and Agfa Dry Proof ADL, are used to print the IT8 test charts. These charts are used to build the CMYK-to-L*ab transformation and to verify the performance of the transformation. The procedure shown in Figure 3 is adopted. 68 colors in the basic-S7 part of each chart are used to estimate the n value and 1-dimension LUTs between the input digital values—CMYK and the corresponding dot areas- a_c, a_m, a_y and a_k. The method proposed by Rolleston[5] is used here. 155 additional sample colors in A, B, and C parts are used to establish the transformation between dot areas and NPAs, and spectral Yule-Nielsen modified Neugebauer equation is used to calculate the spectral reflectance. All 746 colors in A, B, and C parts are used to evaluate the performance of the CMYK-to-L*ab transformation. An X-Rite 938 spectrophotometer with 0°/45° measuring geometry is used to measure the spectral reflectance over the range of 400 nm-700 nm at 10 nm increment. The average ΔE^*_{ab} of these colors and the maximum ΔE^*_{ab} among the color differences of these colors are calculated. The standard deviations of ΔE^*_{ab} are also calculated. The results are shown below.

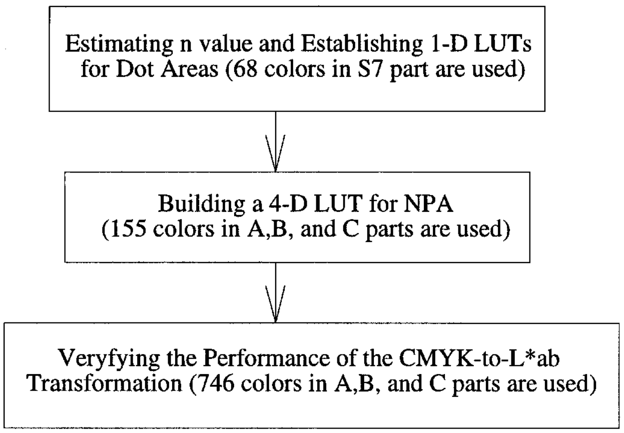


Figure 3. The Procedure of the Experiment

Table 1. The Comparison between the Performance of the Traditional Method (Demichel's Law) and the New Method

		Fuji Colo Art	Agfa Proof PRS	Agfa Dry Proof ADI
	n value	1.33	2.13	1.99
4D NPA LUT	avg ΔE^*_{ab}	1.82	1.81	1.99
	max ΔE^*_{ab}	6.55	6.2	7.76
	std of ΔE^*_{ab}	1.1	1.07	1.24
Demichel's Law	avg ΔE^*_{ab}	4.7	4.65	4.72
	max ΔE^*_{ab}	11.53	11.1	9.82
	std of ΔE^*_{ab}	2.19	1.95	2.2

The results show that the performance of the new method is significantly better than the performance of the Demichel's law.

Discussion and Conclusion

The results of this new approach for ColorArt have been checked, and it is found that the maximum ΔE^*_{ab} occurs for (C=M=Y=100%,K=60%). However, the NPAs for this color are correctly estimated as $A_{4P} = a_k$ (for K=60%), $A_{3P} = 1.0 - A_{4P}$ and $A_{others} = 0.0$. We used a camera with a magnifying lens to view this color, and found that some part of the color is split off the paper in printing process. This should be the reason resulting the color difference. This checkout convince us that the results of the new approach have been reduced to the variation extent of the printing process.

Besides the three practical constraints used in the new approach, there are some other practical constraints. For example, for some given C and Y, $a_g \equiv \sum_{i \in \{g,3p,gk,4p\}} A_i$ should not be influenced by M, or K. The relationship between a_g and M for C=70%, Y=30% and K=0% is plotted in Figure 4. It shows that a_g is not a constant for different M. However, we won't use these constraints besides the three. The reason is that these constraints won't be real if there is some extent of variation in the printing process.

In this study, we proposed a method to estimate the relationship between NPAs and dot areas based on six primary halftone patterns. By using the first-order necessary condition of equality constraints, the estimated NPAs will satisfy three practical constraints. This new approach can reduced the transformation error to the variation extent of the printing process.

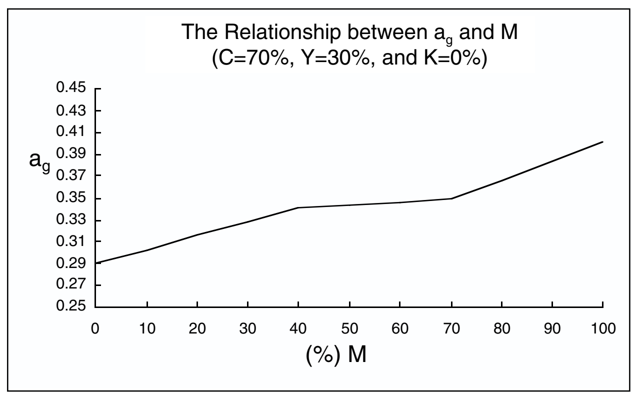


Figure 4. The relationship between estimated a_g and M for C=70%, Y=30% and K=0%

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