Color Dithering Back to the Roberts' Modulation

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Abstract

Dithering technologies for digital halftoning originate from Roberts' modulation by pseudo-noise. At present, Roberts modulation is not used because of its randomness. However, blue noise dithering methods are now being paid attention to the fastness of algorithms and error diffusion-like image qualities. In this paper, the visual noises and the gray scale reproducibilities by dithering are analyzed back to the Roberts and an improved pseudo-noise is discussed.

Introduction

Dithering technologies for digital halftoning originate from Roberts' modulation[1] by pseudo-noise. Roberts' modulation was firstly applied for PCM coding of TV signal at lower rate of 2-4 bits/pixel, where smoothed gray scales were recovered by adding the pseudo-random noise before quantizing. Nowadays, Roberts' modulation is not used because of its randomness. After Roberts, Limb[2] and Bayer[3] introduced a systematic noises into dithering instead of random noise as well known as ordered dither methods. Ordered dither technique has been mostly used for digital halftoning of printers or facsimiles, but is minor at present and error diffusion(ED)[4-5] is major. Recently, blue noise mask[6] is being paid attention to the fastness of algorithms and ED like image qualities. The reason why blue noise is acceptable depends upon its higher frequency spectral distribution. In this paper, the basic gray scale reproducibility by pseudo-random dithering is re-examined and the dithering noise is analyzed back to the origin of Roberts. An improved pseudo-random dither is applied for the color images with stepped gray scales and compared with the uniform random noise.

Analysis of Roberts' Modulation

Figure 1 shows a schematic model of Roberts' modulation when applied to printer with stepped gray scales. A modulated signal y by adding a dithering noise d to input x, is given by

$$y=x+d \; ; \; 1> x >0 \tag{1}$$

When the dithered signal y is fed into a printer with m-bits discontinuous gray scales of step width Δ, the printer output level z is assigned to one of the 2^m levels denoted by Gaussian notation as follows:

$$z=z_n=[y] =n\Delta; \; n = 0 \sim (2^m - 1) \tag{2}$$

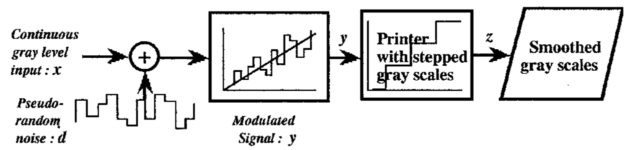


Figure 1. Roberts' Dithering Model

The mean square error E for z vs. x is defined by

$$E=\int\int(z-x)^2p(x,z)dxdz \tag{3}$$

where, p(x,z) means joint probability density function for x and z. Assuming the uniform distribution of x value, E is calculated as

$$E=A\int\int\{(z-z_m)^2+(z_m-x)^2\}p(z|x)dxdz=G+V \tag{4}$$

Here, $p(z|x)$ denotes the conditional probability density function, and

$$z_m=\int zp(z|x)dz\text{: mean level of } z \tag{5}$$

$$G=A\int\int(z-z_m)^2\, p(z|x)dxdz \text{ : gray scale error} \tag{6}$$

$$V=A\int\int(z_m-x)^2\, p(z|x)dxdz \text{ : visual noise power} \tag{7}$$

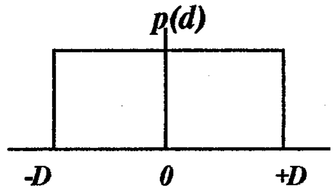
A is normalized to give $E=1$ for $d=0$ (no dithering) as the following.

$$A=12(2^m-1)^2 \tag{8}$$

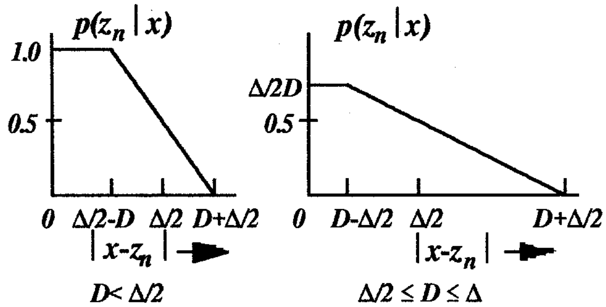
Gray Scale Error

Here the following three typical dithering sources have been examined.
(a)Sine wave,
(b)Gaussian random noise,
(c)Uniform random noise

As expected, uniform random noise resulted in the minimum error for the gray scale reproduction. In the case of uniform random dither, the probability density $p(d)$ of dither is constant in the range of $-D \le d \le +D$ as illustrated in Figure 2(a). D denotes the maximum amplitude of dither. The conditional probability density $p(z_n|x)$ of printer output level is given by the function of displacement $|z_n\text{-}x|$ from dithering center as illustrated in Figure 2(b).



(a) Probability Density of Uniform Random Number



(b) Probability Function of Printer Output Level $\mathbf{Z}_n$
Figure 2. Probability Functions of Random Dither

Thus, E and G are calculated for the normalized dithering amplitude $\delta = D/\Delta$ as follows.

$$E = 1 + 4\delta^2 \tag{9}$$

$$G = \begin{cases} (1-2\delta)^2 \,;\, for \quad 0 \le \delta < \frac{1}{2} \\ 4\delta^{-2}(\delta - \frac{1}{2}\left\{(1-\delta)^2 + 2\Delta(10\delta - 9)/16\right\} \quad ; \quad for \quad \frac{1}{2} \le \delta \le 1 \end{cases} \tag{10}$$

$$V = E - G \tag{11}$$

Figure 3 shows an example of (E,G,V) calculated for uniform random dither(c). The gray scale error G is minimized for the dithering amplitude $d=\Delta/2$, and in this optimum point, the square error E reaches twice as that of without dithering. The visual noise V increases monotonously with the amplitude of d. However, the visibility of V should be estimated through human eye's filter.

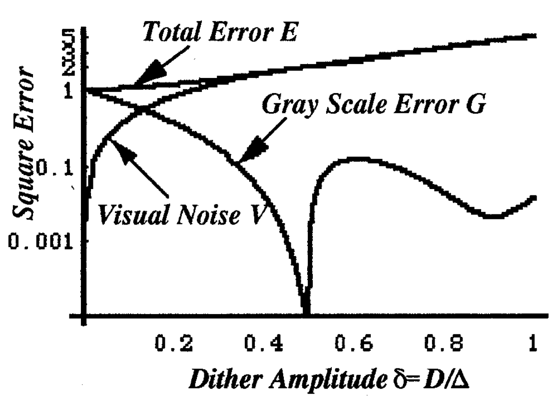
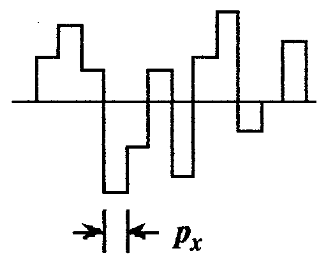


Figure 3. Gray Scale Error and Visual Noise in Random Dither

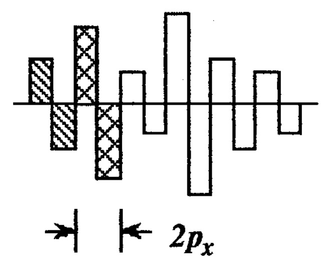
Modified Pseudo-random Noise

Roberts used a pseudo-random noise whose amplitude is distributed uniformly between *(-1/2,1/2)*. This is a white noise and has lower frequency spectrum unpleasant to human vision. We have proposed a modified pseudo-random noise[7] as illustrated in Figure 4(b).

In the conventional random dither shown in Figure 4(a), each dither pulse is generated randomly without any correlation,while in the modified random noise, the polarity of each pulse is symmetrically alternated.



(a) Uniform Random



(b) Modified Random
Figure 4. Alternating Bi-Polar Random Dither Source

Using the uniform random number sequence $0 \le r(i,j) \le 1$, these two types of dither source are described
(a)Uniform random:

$$d\,(i, j) = 2\{r([i/p_x],[j/p_y])\text{-}0.5\} \tag{12}$$

b) Modified random:

$$d(i,j) = (-1)^{[i/p_x]+[j/p_y]}\, r([i/p_x],[j/p_y]) \tag{13}$$

where, (i,j) denotes a pixel coordinate and (p_x, p_y) are the pulse intervals in x and y directions.

The modified random sequence generates alternating bi-polar pulse train with the symmetrical amplitude of $(+r,-r)$ corresponding to the pitches $(2p_x,2p_y)$.

This alternation causes the frequency shifts of dither spectrum from low band to higher band and it gives a pleasant gray scales to human vision. That is, this simple modification changes the white random noise to a kind of blue noise.

The one-dimensional power spectrum of these two dithers are given by the following equation when the shape of pulse trains is assumed to be rectangular.
(a)Uniform random:

$$W_a(f) = \{Sin(\pi p_x f)\pi/p_x f\}^2 \tag{14}$$

b)Modified random:

$$W_b(f)=W_a(f)Q(f) \tag{15}$$

$$Q(f)=1-Cos(2\pi p_x f) \tag{16}$$

Here, $Q(f)$ means a spectral forming function. By alternating the polarity of uniform random pulse symmetrically, the spectral distribution of (a) is reformed and shifted into higher band. Figure 5 illustrates the comparison of calculated 1-D power spectra and Figure 6 shows 2-D Fourier transforms obtained from computer generated random sequences. As clearly shown, the low band spectrum is shifted into high band.

This frequency shift reduces the visibility of grain noise unpleasant to human vision.

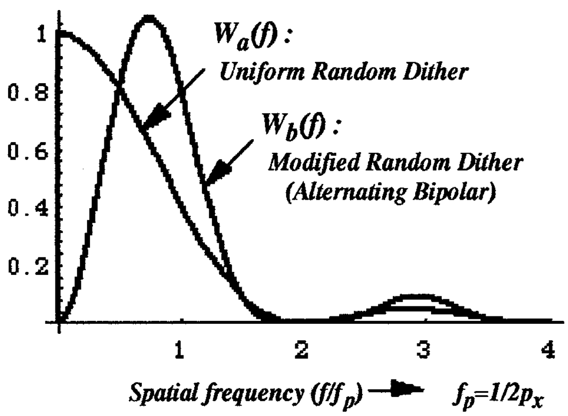


Figure 5. Comparison of Dither Spectrum Calculated

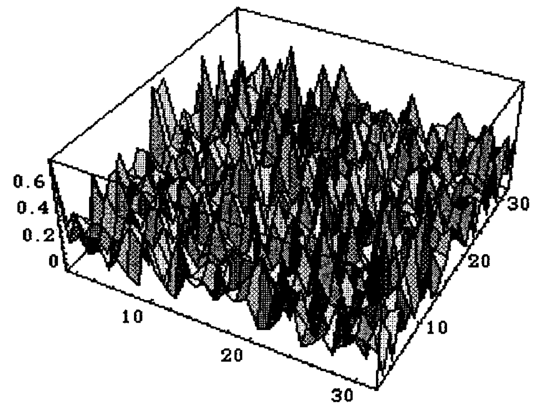
Dithering Color Image

Simulation

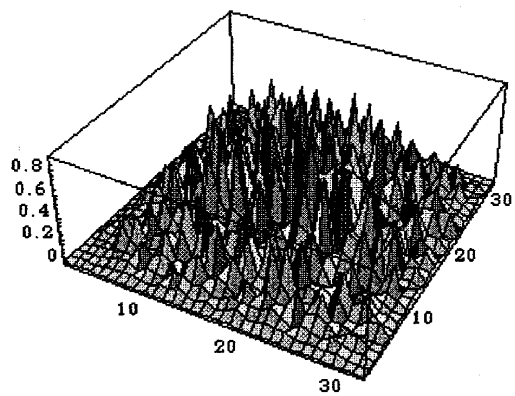
The Roberts' modulation and its modified random dither were applied for smoothing the color images with 2-4 bits stepped gray scales and the visibility of false contours has been examined by computer simulation. For example, the strong false contours of 3-bits images almost disappeared by adding the dither patterns with amplitude of $d=\Delta/2$ in the both methods. However, in the Roberts' modulation, the visual noise V was intolerable because of its low frequency noise spectra. On the other hand, in the modified alternating bi-polar random dither, the visual noise was much reduced and gave a pleasant visual impression by the spectral shift into around the alternating bi-polar frequency $(1/2p_x,1/2p_y)$.

Application to Color Printer

In practice, this modified alternating bi-polar random dither has been applied to a DOD inkjet color printer with multi-level gray scales[7]. A dot size modulation of piezo-electric DOD inkjet can make a 4~16 multi-level color images easily, but can't reproduce continuous tone. In our printer, a local jump caused by the discontinuous ink dots appeared in tonal curve and the strong false contours degraded the image quality to be unacceptable. By applying said modified random dither, these false contours could be drastically cleared away with pleasant visible noise. The similar effect could be also obtained by adding the ordered dither pattern with bi-polar-like threshold matrix. For example, a halftone screen type matrix given in Figure 7 was nice in visibility of dithering texture when applied for facial images or portraits. This inkjet printer has been used for making a variety of pictorial posters excellent in color and tone at professional print shop since 1985.



(a) Power Spectra of Uniform Random



(b) Power Spectra of Modified Random Dither (Alternating Bipolar Random)

Figure 6. Comparison of 2-D Power Spectra(Computer Generated Dither

44	16	24	36	46	18	26	38
48	0	8	54	50	2	10	58
28	32	40	20	30	34	42	22
12	60	52	4	14	62	54	6
47	19	27	39	45	17	25	37
51	3	11	59	49	1	9	57
31	35	43	23	29	33	41	21
15	63	55	7	13	61	53	5

Figure 7. Halftone Screen Type Dithering Pattern

Conclusions

A fundamental dithering effect is discussed back to the Roberts' modulation. Here the relation between gray scale error and visual noise has been clarified as a function of dither amplitude. The modified random dither has the same smoothing effect for stepped gray scales as that of Roberts and can reduce the visible noise at the same time. The classical random dither has not been taken notice because of its poor image quality, but recently blue noise mask method is being paid attention. A very simple modification of Roberts' modulation resulted in a great improvement in visible noise. This method has not been applied for bi-level halftoning so far and will be further investigated and must be improved to do so. The more simple and costless algorithms to change the white noise into blue noise are hoped to be developed.

References

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