# A Grid-Based Method for Predicting the Behaviour of Colour Printers 

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#### Abstract

A new grid-based method is proposed for predicting the behaviour of colour printers. The method takes into account the varying density of dots as well as the light diffusion in the paper by defining at different intensity levels different colorimetric values for the printed ink as well as for the paper white. Since the model integrates both the varying density of partly overlapping ink dots and the light diffusion in the underlying substrate, the obtained predictions are more accurate than those obtained with surface based colour prediction methods described in the literature.


## Introduction

Printer designers are faced with many design issues and trade-offs. How will a given lack of precision in the registration between different colour layers affect the colour gamut? Can the different colour shifts induced by such a lack of precision be quantified for various regions of the printer's gamut? Answers to these questions also depend on the selected halftoning algorithm, on the way the different colour layers are combined and on specific characteristics of the inks and the paper (transparency coefficients of the inks, roughness and porosity of the paper, etc.)

The aim of this contribution is to present a new gridbased method for estimating the behaviour of various printing parameters on the resulting colour image quality and accuracy. In this contribution, we apply this new grid-based estimation method in order to analyse the effect of a constant phase deviation between successive colour layers.

Previous surface coverage analysis methods for predicting colours are generally based on improvements of the Neugebauer model. Kanamori and Kotera ${ }^{4}$ analyse possible distributions of coloured inks within the screen element and deduce boundaries for the corresponding gamuts. Honjo, Taguchi and Suzuki ${ }^{3}$ introduce additional colour components for each printed ink in order to improve the prediction of the resulting colour from respective dot coverages. Daligault and Archinard ${ }^{2}$ develop a grid-based prediction model using n -modified Neugebauer equations.

Most previous grid-based prediction methods assume that a single printed dot can be approximated by a circular disk of equivalent radius and that the printed
dot has the same colorimetric value as the corresponding solid ink layer. While this assumption has been shown to be effective for tone correction in the context of eletrophotographic printers ${ }^{6}$, our experience as well as the experience of other authors ${ }^{2}$ shows that, in the case of ink jet printers, the region of overlap between two neighbouring printed dots contains the same ink but at a higher concentration.

The method we propose is still based on the approximation of a single printed dot of a given basic colour by circular disks of equivalent radii. But, in order to take into account the contribution of overlapping dot parts and of the diffusion of light in the substrate ${ }^{7}$, we assume that we have different colorimetric values for the ink and for the surrounding white, depending on the intensity value to be reproduced. By taking pictures with a microscope, we observed that the colour of the white space changes slightly according to the size and colour of the printed screen dot. This observation confirms the theories about the diffusion of light in the paper ${ }^{9}$.

The colour prediction method requires generating for each single printed dot a solid disk on a high-resolution grid having a resolution $20 \times 20$ times higher than the output device's resolution. One separate grid layer is associated with each separate basic colour layer. Surfaces which result from an overlap of two or more grid layers correspond to the combination of two or more basic colours.

For a given constant intensity region, the colour of a multilayer halftoned patch is obtained by computing the respective surfaces of each of the basic colours, of the white, and of each of the colour combinations. The predicted resulting colour is a combination of the contributing colours and colour combinations, weighted by their respective surfaces. Instead of just considering one colorimetric value per colour layer, the colorimetric value we will consider depends on the intensity of the corresponding source image.

Predicting a colour deviation due for example to a solid phase shift between colour layers consists in generating, for a selected halftoning algorithm, the highresolution colour dot distribution grid and computing the resulting colour estimation both with and without phase shift.

In this paper, we will show the details of this improved grid-based prediction method and compare predicted and measured colorimetric values for a clus-tered-dot halftoning algorithm and for a set of colour
values located in the blue range. We also give the prediction performances for the red and green hue domains.

## The Grid-based Surface Coverage Prediction Method

Previous methods ${ }^{6}$ have shown that in the case of electrophotography, single printed dots can be approximated by circular disks of uniform radii and rasterized on a discrete grid. The computation of the equivalent discrete surface coverage then gives a good approximation of the real surface coverage measured by a densitometer.

In the first phase of our research, we tried to find for each of the different halftoning algorithms the equivalent radius of one printed dot. The considered halftoning algorithms are $4 \times 4$ clustered dither ( 33 grey levels) 8 , Bayer dispersed-dot dither ${ }^{1}$ and rotated dispersed-dot dither ${ }^{5}$. Figure 1 shows predicted and measured luminances for clustered-dot dither, Bayer and for rotated dispersed-dot dither. The measured samples were printed by a HP DJ-560C with black ink on CANON Bubble-Jet Paper. The analysis of the samples under the microscope has shown that the printer uses a hexagonal grid. For each halftoning algorithm a different dot radius was used to match the measurements: $r=1.18$ for clustered-dot, $r=0.98$ for Bayer dispersed-dot and $r=1.04$ for rotated dispersed-dot dither. The measuring unit is the distance between two dot centres located on the same line.


Figure 1. Predicted and measured CIE-LAB luminance as a function of the input intensity level for clustered-dot dithering, Bayer and rotated dispersed-dot dithering with black ink

Figure 1 shows that a single radius and a single colorimetric value for each colour layer do not provide a sufficiently close approximation of an ink-jet printer's printing behaviour. This is due to the fact that at dark levels, when printed dots overlap, the overlapped surface part has a greater density than a surface covered only by a single printed dot. The contribution of overlapped printed dot parts can be simulated by considering different colorimetric values at different intensities for the same layer and for a specific halftoning algorithm. Since it is not possible to measure the colorimetric values of printed dots directly at different intensity levels, we propose a method so as to obtain them by indirect means.

In the model we propose, it is necessary to accurately estimate the surface ratio between a certain combination of printed dots of a given colour layer and white. It is also necessary to estimate the resulting colorimetric value of the ink in CIE-XYZ space. When considering a patch printed at a certain intensity level with a single basic colour, the following equation, derived from the additivity of colours printed side by side is used to create a relationship between the unknown colour $C=$ $X Y Z$ of the ink of a printed dot, the printed to white surface ratio $S$ and the surrounding paper white $C_{s w}=X Y Z_{s w}$ :

$$
\begin{equation*}
S \bullet X Y Z+(1-S) \bullet X Y Z_{s w}=X Y Z_{s w} \tag{1}
\end{equation*}
$$

where the surrounding paper white is given by and the measured colorimetric value of the patch is given by $X Y Z_{m}$.
${ }^{m^{*}}$ We are making the assumption that the xy chromaticities (or, in equivalent terms, the ratios of $\mathrm{X}: \mathrm{Y}: \mathrm{Z}$ ) of dots of the same colour layer printed at different intensity values have only small variations.


Figure 2. Blank paper white $C_{p w}$, measured patch $C_{m}$, solid ink value $C_{\text {solid }}$ and computed ink colorimetric value $C_{u}$ and surrounding white value $C_{w}$.

Therefore, the estimation procedure consists in first measuring patches printed at various intensities of a single basic colour layer. Then, for each of the measured patches, an equivalent colorimetric value of the printed ink dots and of the surrounding white is computed as follows (Figure 2). The white point given by the blank paper white $C_{p w}=X Y Z_{p w}$ and the measured point $C_{m}$ $=X Y Z_{m}$ define a straight line which points approximately in the direction of the measured solid printed basic colour patch $C_{\text {solid }}=. X Y Z_{\text {solid. }}$. According to our assumption, the colorimetric value of the basic colour layers printed at different intensities must lie near the straight line $\overline{(0,0,0) C_{\text {solid }}}$. We therefore compute point $C_{u}$ on line $\overline{C_{p w} C_{m}}$ and point $C v$ on line $\overline{(0,0,0) C_{\text {solid }}}$., which define the smallest distance between the two lines. Point $C_{u}$ in CIE-XYZ space defines the colorimetric value of the printed ink dot. According to (Equation 1), point $C_{u}$ and
the given surface ratio $S$ between printed and surrounding white parts define the colorimetric value of the white part $C_{s w}$. The given surface ratio $S$ is determined by the arrangement of pixels which belong to the printed screen dot and by a single equivalent dot radius $r$. The equivalent dot radii for each of the colour layers are chosen in order to have the best fit between measured and predicted colorimetric values.

Equivalent colorimetric values for the printed dots and for the surrounding white spaces have been computed for cyan and magenta, at different intensity levels. Table 1 gives corresponding printed dot and sur-rounding white CIE-XYZ colorimetric values. The samples were printed with a HP DJ-560C on CANON Bubble-Jet Paper using the $4 \times 4$ clustered dither ${ }^{8}$ halftoning algorithm.

The computation of equivalent colorimetric values for combined colours such as Blue, Red and Green can be obtained in a similar way. In order to compute equivalent colorimetric values for example for blue printed dots at various intensity levels obtained by superposing cyan and magenta dots, we first print a sample and analyse the superposition of the dots under the microscope. It allows us to quantify the systematic shift between cyan and magenta dots. According to this observation, we generate in high-resolution grids the cyan and magenta layers required to produce the desired blue. The corresponding patch is printed and its colorimetric value $C_{m}$ is measured. According to the additivity law of colours printed side by side and assuming that the surrounding paper white is proportionally influenced by the Blue, Cyan and Magenta colours we have the relation

Table 1. Equivalent colorimetric values for the Cyan and Magenta process colours and for their respective surrounding White.

| $\begin{gathered} \hline \text { Intensity } \\ \mathrm{c}, \mathrm{~m} \\ \hline \hline \end{gathered}$ | $\mathrm{S}_{\text {cyan }}(\mathrm{c})$ | $\mathrm{C}_{\text {cyan }}(\mathrm{c})$ | $\mathrm{C}_{\text {sw }}(\mathrm{c})$ | $\mathrm{S}_{\text {max }}(\mathrm{m})$ | $\mathrm{C}_{\text {max }}$ (m) | $\mathrm{C}_{\text {sw }}(\mathrm{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1. | $\begin{aligned} & \hline \hline 17.4345, \\ & 27.1474, \\ & 74.875 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \hline 115.56, \\ & 119.149, \\ & 117.908 \end{aligned}$ | 1. | $\begin{aligned} & \hline \hline 38.2067, \\ & 17.4017, \\ & 61.4551 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \hline 84.878, \\ & 85.0611, \\ & 100.704 \end{aligned}$ |
| 31 | 0.966484 | $\begin{aligned} & 17.4558, \\ & 27.8869, \\ & 76.0336 \end{aligned}$ | $\begin{aligned} & 115.56, \\ & 119.149, \\ & 117.908 \end{aligned}$ | 0.966484 | $\begin{aligned} & 39.3373, \\ & 18.1228, \\ & 63.7838 \end{aligned}$ | $\begin{aligned} & 84.878, \\ & 85.0611, \\ & 100.704 \end{aligned}$ |
| 63 | 0.908047 | $\begin{aligned} & \hline 17.518, \\ & 28.8443, \\ & 77.5869 \end{aligned}$ | $\begin{aligned} & \hline 96.5077, \\ & 101.411, \\ & 109.751 \\ & \hline \end{aligned}$ | 0.908047 | $\begin{aligned} & 40.6759, \\ & 18.9175, \\ & 66.3894 \end{aligned}$ | $\begin{aligned} & 91.5012, \\ & 94.7185, \\ & 106.175 \\ & \hline \end{aligned}$ |
| 95 | 0.810391 | $\begin{aligned} & 17.5819, \\ & 29.8686, \\ & 79.2269 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 87.6939, \\ & 93.4081, \\ & 106.32 \\ & \hline \end{aligned}$ | 0.810391 | $\begin{aligned} & \hline 42.4873, \\ & 20.0235, \\ & 69.9803 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 90.0973, \\ & 92.4724, \\ & 105.283 \\ & \hline \end{aligned}$ |
| 127 | 0.712422 | $\begin{aligned} & \hline 17.7139, \\ & 30.9295, \\ & 81.0412 \\ & \hline \end{aligned}$ | $\begin{aligned} & 89.6094, \\ & 95.2128, \\ & 107.189 \\ & \hline \end{aligned}$ | 0.712422 | $\begin{aligned} & 43.9898, \\ & 20.9654, \\ & 73.0081 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 88.9258, \\ & 90.5268, \\ & 104.646 \\ & \hline \end{aligned}$ |
| 159 | 0.592578 | $\begin{aligned} & 17.7792, \\ & 32.0646, \\ & 82.8083 \\ & \hline \end{aligned}$ | $\begin{aligned} & 91.3982, \\ & 96.8661, \\ & 107.927 \\ & \hline \end{aligned}$ | 0.592578 | $\begin{aligned} & 45.3559, \\ & 21.7752, \\ & 75.6446 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 91.4871, \\ & 94.3966, \\ & 106.572 \\ & \hline \end{aligned}$ |
| 191 | 0.494766 | $\begin{aligned} & \hline 17.8315, \\ & 33.8054, \\ & 85.3725 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 95.3937, \\ & 100.381, \\ & 109.287 \\ & \hline \end{aligned}$ | 0.494766 | $\begin{aligned} & \hline 47.5659, \\ & 23.063, \\ & 79.8489 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 93.574, \\ & 97.6107, \\ & 108.059 \\ & \hline \end{aligned}$ |
| 223 | 0.248672 | $\begin{aligned} & \hline 17.5991, \\ & 34.2342, \\ & 85.4681 \\ & \hline \end{aligned}$ | $\begin{aligned} & 94.1999, \\ & 99.3646, \\ & 108.921 \\ & \hline \end{aligned}$ | 0.248672 | $\begin{aligned} & 48.0452, \\ & 23.3831, \\ & 80.8524 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 92.9924, \\ & 96.6477, \\ & 107.735 \\ & \hline \end{aligned}$ |

Legend:
$\mathrm{S}_{\text {cyan }}(\mathrm{c})$ : Modelled surface coverage of cyan (according to dot radius and halftoning algorithm)
$\mathrm{C}_{\text {cyan }}^{\text {cyan }}(\mathrm{c})$ : CIE-XYZ value of the inked cyan screen dot as a function of cyan intensity level $c$
$\mathrm{C}_{\mathrm{sw}}$ (c): CIE-XYZ value of the surrounding white space as a function of cyan intensity level $c$
$\mathrm{S}_{\text {mag }}(\mathrm{m})$ : Modelled surface coverage of magenta
$\mathrm{C}_{\text {mag }}(\mathrm{m})$ : CIE-XYZ value of the inked magenta screen dot as a function of magenta intensity $m$
$\mathrm{C}_{\mathrm{sw}}(\mathrm{m})$ : CIE-XYZ value of the surrounding white space as a function of magenta intensity level $m$
Equivalent cyan radius: $\mathrm{r}_{\text {cyan }}=0.98$
Equivalent magenta radius: $r_{\text {magenta }}=0.98$
Measured paper white $C_{p w}=(95.2791,100.282,109.251)$

$$
\begin{align*}
& C_{m}=S_{\text {blue }} \cdot C_{\text {blue }}+S_{\text {mag }} \cdot C_{\text {mag }}+S_{c y a n} \cdot C_{\text {cyan }}+ \\
& \left(1-S_{\text {blue }}-S_{\text {mag }}-S_{\text {cyan }}\right) \cdot \\
& \frac{\left(S_{\text {blue }} \cdot C_{s w}(b)+S_{\text {mag }} \cdot C_{s w}(m)+S_{c y a n} \cdot C_{s w}(c)\right)}{\left(S_{\text {blue }}+S_{\text {mag }}+S_{c y a n}\right)} \tag{2}
\end{align*}
$$

where $S_{\text {blue }}$ is the relative surface of Blue, $S_{m a g}$ the relative surface of Magenta, $S_{\text {cyan }}$ the relative surface of Cyan, $C_{m}$ the measured colorimetric value of the patch, $C_{s w}(i)$ the surrounding paper white associated with the colour layer $i$ at the corresponding intensity, and $C_{m a g}$, $C_{\text {cyan }}$ the previously computed colorimetric values for Magenta and Cyan at their current intensity levels (values interpolated from Table 1). $C_{\text {blue }}$ is the unknown colorimetric value of the blue ink.

In the high-resolution grid, the surface contributions of Magenta only ( $S_{m a g}$ ), Cyan only ( $S_{\text {cyan }}$ ) and Blue only ( $S_{\text {blue }}$ ) are summed up. In order to obtain the colorimetric value of the equivalent Blue patch without the nonoverlapping Magenta and Cyan parts, we need to eliminate Magenta and Cyan only contributions. Since the colorimetric values of Magenta and Cyan at the present intensities are known, the measured value $C_{m}$ can be corrected so as to represent only the mixture of Blue (regions of superimposed Magenta and Cyan) and surrounding paper white. (equation 2) is therefore modified as follows

$$
\begin{align*}
& C_{m}-\left(S_{\text {mag }} \cdot C_{\text {mag }}+S_{c y a n} \cdot C_{c y a n}+\left(1-S_{b l u e}-S_{\text {mag }}-S_{c y a n}\right)\right. \\
& \left.\cdot \frac{S_{\text {mag }} \cdot C_{s w}(m)+S_{c y a n} \cdot C_{s w}(c)}{\left(S_{\text {blue }}+S_{\text {mag }}+S_{c y a n}\right)}\right)  \tag{3}\\
& =S_{\text {blue }} \cdot C_{\text {blue }}+\frac{\left(1-S_{\text {blue }}-S_{\text {mag }}-S_{c y a n}\right) \cdot S_{\text {blue }}}{\left(S_{\text {blue }}+S_{\text {mag }}+S_{c y a n}\right)} \cdot C_{s w}(b)
\end{align*}
$$

In order to compute $C_{b l u e}$ and $C_{s w}(b)$, we have to put (equation 3) into the form of (equation 4) by dividing the left and right part by a constant value to obtain the normalized surface $0 \leq \boldsymbol{S}_{\text {blue }} \leq 1$

$$
\begin{equation*}
C_{m}^{*}=S_{\text {blue }} \cdot C_{\text {blue }}+\left(1-S_{\text {blue }}\right) \cdot C_{s w}(b) \tag{4}
\end{equation*}
$$

$$
\text { where } S_{\text {blue }}^{\prime}=\frac{S_{\text {blue }}}{\left(S_{\text {blue }}+\frac{S_{\text {blue }} \cdot\left(1-S_{\text {blue }}-S_{\text {mag }}-S_{\text {cyan }}\right)}{S_{\text {blue }}+S_{\text {mag }}+S_{\text {cyan }}}\right)}
$$

Now we apply the same method as in the previous section. We also assume that the blue values are close to the straight line $\overline{(0,0,0) C_{\text {solidblue }}}$ where $(0,0,0)$ represents the origin CsolidBlue and the measured colorimetric CIE-XYZ values of a solid printed blue patch. As before, we find a point $C u$ on a straight line $\overline{C_{p w} C_{m}^{*}}$ and a point $C v$ on line $\overline{(0,0,0) C_{\text {solidsolidblue }}}$ which minimize the distance between the two lines. Point $C_{u}$ defines the colorimetric value of the blue ink at the considered intensity
level. The relationship $\overline{C_{s w} C_{m}^{*}}: \overline{C_{s w} C_{u}}$ is the given surface ratio $S^{\prime}{ }_{b l u e}$ of printed blue and therefore allows the position of surrounding White $C_{s w}(b)$ to be computed on the $C_{p w} C_{u}$ axis. The surface ratio $S_{\text {blue }}^{\prime}$ is a function of the surface ratios $S_{i}$ which are computed by generating in a high-resolution grid circular disks with the radii $r_{c}$ and $\mathrm{r}_{m}$ corresponding to the equivalent radii of cyan and magenta. Table 2 shows equivalent colorimetric values for the Blue and for its surrounding White obtained by the method described above.

Figure 3 shows in tridimensional XYZ space the intensity dependent colorimetric values for printed screen dots Cyan, Magenta, Blue and for their corresponding surrounding White values.

When predicting colorimetric values for the cases for which a measurement was made (see Tables 1 and 2 ), by construction no error between predicted and measured colorimetric value arises.

## Grid-based Colorimetric Prediction

The colorimetric prediction of the colour of a printed surface patch can be obtained by first generating one bitmap at the printer's resolution for each of the basic colour layers, using the selected halftoning algorithm and generating the desired colour patches. These colour layer bitmaps specify whether a given single dot of a given colour layer is to be printed or not.

The information resident in the colour layer bitmaps as well as the set of equivalent radii and the observations made with the microscope are used to generate the high-resolution grids simulating the printed dots. There is one high-resolution grid (HRG) per basic colour layer. The high-resolution grids are generated by scanning the colour layer bitmaps scanline by scanline and pixel by pixel, and for each pixel to be printed, selecting its layer dependent equivalent radius $r$ and generating one disk
and

$$
\begin{equation*}
C_{m}^{*}=\frac{\left[C_{m}-\left(S_{\text {mag }} \cdot C_{\text {mag }}+S_{c y a n} \cdot C_{\text {cyan }}\right)-\left(1-S_{\text {blue }}-S_{\text {mag }}-S_{\text {cyan }}\right) \cdot \frac{S_{\text {mag }} \cdot C_{s w}(m)+S_{c y a n} \cdot C_{s w}(c)}{S_{\text {blue }}+S_{\text {mag }}+S_{c y a n}}\right]}{\left(S_{\text {blue }}+\frac{S_{\text {blue }} \cdot\left(1-S_{\text {blue }}-S_{\text {mag }}-S_{c y a n}\right)}{S_{\text {blue }}+S_{\text {mag }}+S_{c y a n}}\right)} \tag{5}
\end{equation*}
$$

Table 2. Equivalent Colorimetric Values for Blue and Surrounding White at Different Intensities

| Intensity b | $\mathrm{S}_{\text {bluc }}(\mathrm{b})$ | $\mathrm{C}_{\text {buuc }}(\mathrm{b})$ | $\mathrm{C}_{\text {sv }}(\mathrm{b})$ |
| :---: | :--- | :--- | :--- |
| 0 | 1. | $10.4541,6.2769,50.3733$ | $34.9437,33.201,68.7815$ |
| 31 | 0.966484 | $10.9919,6.57038,52.864$ | $34.9437,33.201,68.7815$ |
| 63 | 0.908047 | $11.6821,6.96828,56.133$ | $51.5564,51.4789,81.2895$ |
| 95 | 0.810391 | $12.4113,7.23675,59.0658$ | $56.5691,56.8199,85.605$ |
| 127 | 0.712422 | $13.0681,7.80715,62.8345$ | $68.607,70.2829,93.9308$ |
| 159 | 0.592578 | $13.8691,8.35957,66.9385$ | $70.7011,72.5336,96.2028$ |
| 191 | 0.494766 | $14.2363,9.51343,71.8698$ | $82.7205,86.2193,103.12$ |
| 223 | 0.248672 | $14.5594,9.48118,72.6655$ | $89.5067,93.7927,106.262$ |

## Legend:

$\mathrm{S}_{\text {blue }}(\mathrm{b})$ : Modelled surface coverage of Blue
$\mathrm{C}_{\text {blue }}$ (b): CIE-XYZ value of inked blue screen dots as a function of blue intensity level b $\mathrm{C}_{\mathrm{sw}}(\mathrm{b})$ : CIE-XYZ value of the surrounding white space as a function of blue intensity level $b$


Figure 3. Intensity dependent CIE-XYZ colorimetric values for Cyan, Magenta, Blue and for their corresponding surrounding White values
of radius $r$. Once all high-resolution grids are generated, the colorimetric evaluation procedure can begin. First a given surface of the high-resolution grids is considered, for example the surface corresponding to the surface of a single screen element. Let us assume, for the sake of simplicity and without loss of generality, that the considered colour samples have a constant intensity. In that case, for each of the basic layers and combination of layers, the colorimetric values associated with the given intensities are taken, possibly by interpolating between the colorimetric values given in Tables 1 and 2.

The predicted colorimetric value is given by the colorimetric values of the basic colours and of their combinations, weighted by their respective surfaces. For example, in the case of a printer printing with the 3 layers Cyan Magenta and Yellow, we count the number of contributing elementary surface elements in each of the high-resolution grids (HRG) as shown in the following.

$$
\begin{aligned}
& N_{\text {cyan }}=\sum_{i, j}(C y a n H R G[i, j] \wedge \neg \text { MagentaHRG }[i, j] \wedge \neg \text { YellowHRG }[i, j]) \\
& N_{\text {Magenta }}=\sum_{i, j}(\neg C y a n H R G[i, j] \wedge \text { MagentaHRG }[i, j] \wedge \neg \text { YellowHRG }[i, j]) \\
& N_{\text {Yellow }}=\sum_{i, j}(\neg \text { CyanHRG }[i, j] \wedge \neg \text { MagentaHRG }[i, j] \wedge \text { YellowHRG }[i, j]) \\
& N_{\text {Red }}=\sum_{i, j}(\neg \text { CyanHRG }[i, j] \wedge \text { MagentaHRG }[i, j] \wedge \text { YellowHRG }[i, j]) \\
& N_{\text {Green }}=\sum_{i, j}(\neg \text { CyanHRG }[i, j] \wedge \neg \text { MagentaHRG }[i, j] \wedge \text { YellowHRG }[i, j]) \\
& N_{\text {Blue }}=\sum_{i, j}(\neg \text { CyanHRG }[i, j] \wedge \neg \text { MagentaHRG }[i, j] \wedge \neg \text { YellowHRG }[i, j]) \\
& N_{\text {White }}=\sum_{i, j}(\neg \text { CyanHRG }[i, j] \wedge \neg \text { MagentaHRG }[i, j] \wedge \neg \text { YellowHRG }[i, j]) \\
& N_{\text {Black }}=\sum_{i, j}(C y a n H R G[i, j] \wedge \text { MagentaHRG }[i, j] \wedge Y e l l o w H R G[i, j])
\end{aligned}
$$

The predicted colorimetric value $X Y Z$ is given by the colorimetric values of the basic colours and of their combinations for the considered intensity levels $c, m, y$ of the basic colours and for the intensity levels $r, g, b, k$ of the combined colours, weighted by their respective contributions.

$$
\begin{aligned}
\text { XYZ }= & \left(N_{\text {Cyan }} \bullet C_{\text {Cyan }}(c)+N_{\text {Magenta }} \bullet C_{\text {Magenta }}(m)+N_{\text {Yellow }} \bullet\right. \\
& C_{\text {Yellow }}(y)+N_{\text {Red }} \bullet C_{\text {Red }}(r)+N_{\text {Green }} \bullet C_{\text {Gree }}(g)+N_{\text {Blue }} \\
& \cdot C_{\text {Blue }}(b)+N_{\text {white }} \cdot C_{\text {surr Whitete }}(c, m, y, r, g, b, k)+N \\
& { }^{\text {Black }} \bullet C_{\text {black }}(k) /\left(N_{\text {cyan }}+N_{\text {magenta }}+N_{\text {yellow }}+N_{\text {red }}+\right. \\
& \left.N_{\text {green }}+N_{\text {blue }}+N_{\text {white }}+N_{\text {Black }}\right)
\end{aligned}
$$

Interpolated colorimetric value $C_{\text {surrWhite }}$ for the resulting surrounding white is computed as a mean value between the surrounding Whites associated with the basic and combined colour layers

$$
\begin{aligned}
C_{\text {surrWhite }}= & \left(N_{\text {cyan }} \cdot C_{s w}(c)+N_{\text {Magenta }} \bullet C_{s w}(m)+N_{\text {Yellow }} \cdot C_{\text {sw }}\right. \\
& (y)+N_{\text {red }} \bullet C_{s w}(r)+N_{\text {green }} \bullet C_{s w}(g)+N_{\text {blue }} \bullet C_{s w} \\
& \left.(b)+N_{\text {black }} \bullet C_{s w}(k)\right) /\left(N_{\text {cyan }}+N_{\text {Magenta }}+N_{\text {yellow }}+\right. \\
& N_{\text {red }}+N_{\text {green }}+N_{\text {blue }}+N_{\text {white }}+N_{\text {Black }}
\end{aligned}
$$

We have tried to measure the accuracy of the colour predictions for a $4 \times 4$ clustered dither ${ }^{8}$ halftoning algorithm and radii $r_{\text {cyan }}, r_{\text {magenta }}, r_{\text {yellow }}$ all equal to 0.98 . For this purpose, we select patches located along the bluewhite, red-white and green-white axis, located halfway between the patches which have been measured to train the system. The deviations we obtain between predicted and measured colours have a mean value of $\Delta E=1.31$ and a maximal value of $\Delta E=3.02$ in the CIE-LAB system.

## Grid-based Prediction of Colorimetric Deviations

While grid-based colour prediction is not sufficiently accurate for exact colour prediction, it is however sufficiently precise to give a qualitative estimate of the colour deviations induced by phase shifts between colour layers. We used the same halftoning algorithm and equivalent radii as in the previous section.

Figure 4 shows in the CIE-LAB space the predicted (diamond shape) and the measured (triangle) colour deviations due to a solid phase shift of horizontally half a screen element between the cyan and magenta, magenta and yellow and cyan and yellow colour layers for respectively the blue, red and green hues. This phase shift corresponds exactly to the difference between in-phase printing and counter-phase printing (flamenco).


Figure 4. Predicted and measured colour differences due to a solid phase shift between magenta and cyan, magenta and yellow and cyan and yellow colour layers.

The mean respectively maximal measured deviations due to phase shifts are in the CIE-LAB system $\Delta E_{\text {mean }}=$ 6.17 , respectively $\Delta E_{\max }=21.73$ with a standard deviation $\sigma=5.21$. The mean, respectively the maximal colour deviation between measured and predicted values in counter phase for the considered samples is $\Delta E=3.08$, respectively $\Delta E=6.05$ with a standard deviation $\sigma=$ 1.61 in the CIE-LAB system. As figure 4 shows, the predictions are correct for both the blue and the red hues. In the green hue however, the colorimetric deviations due to phase shifts are smaller than the prediction errors and no accurate prediction can be made.

## Conclusions

We have presented a new grid-based method for estimating the colorimetric behaviour of printers under colour phase shifts. The method departs from previous grid-based methods by the fact that a single printed dot is no longer modelled by a disk of fixed radius having a single colorimetric value. In order to take into account the ink density variations due to overlapped ink surfaces and the influence of the printed screen dot on its surrounding white space due to light diffusion in the substrate, the grid-based method we propose computes a set of equivalent colorimetric values for the printed screen dot and for the surrounding white space. These equivalent colorimetric values are computed from measured surface patches printed at different intensity levels.

This new method has proven to give good qualitative colorimetric predictions for the significant colorimetric variations in the blue and red ranges originating from solid phase shifts between colour layers. The authors have applied the same method for predicting the effect of solid phase shifts for colours located on the green-white axis. Since colour deviations between in phase and in counterphase printing in the green tones are smaller than the prediction errors, predictions cannot be made, especially in the green mid-tones.

The authors are aware that this grid-based prediction method only represents a first step and needs to be further improved. It seems that not only the tristimulus values of the printed dot depend on corresponding input image pixel intensities but that also the equivalent radius of a printed dot should be varied according to input image pixel intensities.

Finally, the authors intend to analyse whether an improved version of the proposed printed dot modelization technique could be used to develop a highly accurate colour prediction method defining an output device behaviour precisely. Such a predictive method could replace table-based calibration methods requiring large sets of measured colour patches.

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