Color Filter Design For Multiple Illuminants and Detectors

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Abstract

This paper presents one approach to designing color filters for a colorimeter that uses multiple internal illuminants and multiple filtered-detectors. The internal illuminants and optical detectors are fixed items in the colorimeter. The color filters are designed using simplex search, Vora's measure of goodness, and linear minimum mean square estimation. Radiance and reflectance data sets were used to test the performance of the designed color filters. Design experiments for solely photometric, solely radiometric, and combination colorimeters gave average ΔE_{Lab} errors below 0.6, 2.3, and 1.5 respectively.

1. Introduction and Problem Description

The standard colorimeter described in [10, 2, 5] uses one illuminant, one detector, and three color filters. The color filters are designed to be an approximation to a linear combination of the CIE color matching functions. From three measurements, estimates of CIE XYZ and CIELab values are made. Errors in the filter approximations to the CIE color matching functions will result in errors in the color values.

The colorimeter proposed in this paper will use multiple illuminants, detectors, and color filters to improve estimates of tristimulus values. By making measurements with different combinations of internal illuminants, optical detectors, and correctly designed color filters, the number of useful measurements and the range space of the measurement are increased, and the sensitivity to noise and filter errors are decreased. If the human visual subspace (HVSS)[5] is nearly contained in the range space of the measurements, then accurate estimates of CIE tristimulus values can be found from the measured data using linear minimum mean squared (LMMSE) estimation matrices[1].

This work describes a method of designing color filters that maximizes the distance between the HVSS and the range space of the measurements. In Section 2, mathematical notation and photometric and radiometric measurement matrices are introduced. Section 3 describes the design experiments carried out for task specific colorimeters. One color filter set was designed for a photometric colorimeter; one for a radiometric colorimeter; and one for a combination colorimeter. The three filter sets were based on five LED illuminants. A final performance measurement was done to test a new production device with eight LEDs. All design experiments were performed using sampled data, and all equations are expressed using vector space notation[5].

2. Approach

The CIE tristimulus values are defined in matrix/vector notation by $\mathbf{t} = \mathbf{A}^T\mathbf{L}\mathbf{r}$ where $\mathbf{A}$ is the $N \times 3$ color matching matrix of CIE color matching functions, $\mathbf{L}$ is the $N \times N$ diagonal matrix for the viewing illuminant with $N \times 1$ radiance spectrum $\mathbf{l}$, and $\mathbf{r}$ is the $\mathrm{N} \times 1$ reflectance spectrum of the object being viewed. The color matching matrix under illuminant $\mathbf{L}$ is defined as $\mathbf{A}_\mathrm{L} = \mathbf{LA}$.[5]

The measurement for a single internal illuminant is given by $\mathbf{c} = \mathbf{M}^T\mathbf{D}\mathbf{L}_\mathbf{s}\mathbf{r}$ where $\mathbf{M}$ is the $N \times N_f$ matrix of N_f color filter transmittance spectra, $\mathbf{D}$ is the $N \times N$ diagonal matrix of sensitivity spectrum $\mathbf{d}_\mathrm{s}$ for the detector, and $\mathbf{L}_\mathrm{s}$ is the $N \times N$ diagonal matrix of radiance spectrum $\mathbf{l}_\mathrm{s}$ for the internal illuminant[5]. All data were sampled for a visible wavelength range of 390-730nm at 2nm intervals; thus $N = 171$.

Suppose there are N_f unknown color filters, N_d known optical detectors and N_t known internal illuminants. The colorimeter makes $K = N_f N_d N_t$ photometric measurements and $P = N_f N_d$ radiometric measurements. If the detectors have spectral sensitivities $\mathbf{d}_i$, the internal illuminants have radiance spectra $\mathbf{l}_{sj}$, and color filter transmittance spectra $\mathbf{m}_k$, then the photometric measurement matrix which characterizes the K spectral sensitivities may be written as

$$\mathrm{V(M)} = [\mathrm{D}_1[\mathrm{L}_{\mathrm{s}_1}\mathrm{M},\mathrm{L}_{\mathrm{s}_2}\mathrm{M},\ldots, \mathrm{L}_{\mathrm{s}N_t}\mathrm{M}], \mathrm{D}_2[\mathrm{L}_{\mathrm{s}_1}\mathrm{M}, \mathrm{L}_{\mathrm{s}_2}\mathrm{M},\ldots, L_{\mathrm{s}N_t}\mathrm{M}],\ldots\mathrm{D}_{N\mathrm{d}}[\mathrm{L}_{\mathrm{S}_1}\mathrm{M},\mathrm{L}_{\mathrm{s}_2}\mathrm{M},\ldots, \mathrm{L}_{\mathrm{SN}_t},\mathrm{M}]] \tag{1}$$

where $\mathbf{D}_i = diag(\mathbf{d}_i)$, $\mathbf{L}_{\mathbf{s}j} = diag(\mathbf{l}_{\mathbf{s}j})$, and $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2,\ldots, \mathbf{m}_{Nf}]$. Similarly, the radiometric measurement matrix that characterizes the P sensitivities is given by

$$\mathbf{W(M)} = [\mathbf{D}_1\mathbf{M}, \mathbf{D}_2\mathbf{M},\ldots, \mathbf{D}_{N_d}\mathbf{M}] \tag{2}$$

The transmittance spectra of the optimal color filter set are parameterized to reduce the number of variables needed to specify $\mathbf{M}$ and to satisfy certain physical constraints. In the design experiments, transmittance spectra of the color filter set were constrained to be Gaussian or sum of Gaussians shaped curves:

$$m_i(x) = \rho_i \frac{1}{\sqrt{2\pi\sigma_{i1}}} \exp[-\frac{(x-\mu_{i1})^2}{2\sigma_{i1}^2} + (1-\rho_i)\frac{1}{\sqrt{2\pi\sigma_{i2}}} \exp\frac{(x-\mu_{i2})^2}{2\sigma_{i2}^2} \quad (3)$$

where $\mu_{i1}, \mu_{i2}, \sigma_{i1}^2 \text{ and } \sigma_{i2}^2$ are unknown variables, ρ_i is an unknown variable between 0 and 1, and x is a wavelength in the range of 390-730 nm. The unknown transmittance color filter spectrum may be parameterized as a vector

$$\mathbf{v} = [\mu_{11},\ \sigma_{11}^2, .., \mu_{N_f 1}, \sigma_{N_f 1}^2, \rho_1, \mu_{12}, \sigma_{12}^2, .., \rho_{N_f}, \mu_{N_f 2}, \sigma_{N_f 2}^2]. \quad (4)$$

The optimal filter set is found by maximizing Vora's measures of goodness[6]. An advantage of this measure is that its analytic form works well with most optimization algorithms. If the reflectance source is illuminated by a viewing illuminant with a radiance spectrum **1**, then the photometric measure of goodness is ν ($\mathbf{A_L}$, $\mathbf{V}$($\mathbf{M}$)). The combined measure of goodness used to design the color filters is given by

$$C(\text{V},\text{W}) = [(\gamma)\nu(\text{A}_\text{L},\text{V}(\text{M})) + (1—\gamma)\nu\ (\text{A},\text{W}(\text{M}))]. \quad (5)$$

where γ is the weight between 0 and 1 which specifies the importance for photometric measure ν ($\mathbf{A_L}$, $\mathbf{V}$($\mathbf{M}$)) over radiometric measure ν ($\mathbf{A}$, $\mathbf{W}$($\mathbf{M}$)). Different photometric color filter sets were not designed for each viewing illuminant; rather, one color filter set was designed for the CIE **A** viewing illuminant and used for measurements under all other viewing illuminants.

To find the optimal parameter vector **v** from Equation 4, the **fmins** function from MATLAB is used on the function 1-C(**V,W**). The photometric error, $e_p = E\{\|\mathbf{A_L}^T\text{r} - \mathbf{RV}^T\text{r}\|\}$ is minimized when

$$\mathbf{R} = (\mathbf{A_L}^T E[\text{rr}^T]\mathbf{V})(\mathbf{V}^\text{T} E[\text{rr}^T]\mathbf{V})^{-1} \quad (6)$$

where $E\{\bullet\}$ is the expected value operator. **R** is called the photometric LMMSE matrix. Likewise, the radiometric LMMSE matrix,

$$\mathbf{S} = (\mathbf{A}^T E[\text{gg}^T]\mathbf{W})(\mathbf{W}^T E[\mathbf{gg}^T]\text{W})^{-1}, \quad (7)$$

minimizes the radiometric error $e_r = E\{\|\mathbf{Ag} - \mathbf{SW}^T\text{g}\|\}$. $E[\text{gg}^T]$ and $E[\text{rr}^T]$ are the correlation matrices of radiance and reflectance spectra respectively.

3. Design Experiments and Results

In the first experiment, a photometric colorimeter was designed, i.e. $\gamma = 1$ in Equation 5. Five LEDs, whose radiance spectra are provided by Color Savvy were used as the internal illuminant set. The optical detector was assumed to be uniform, making **d** a vector of 1's. The number of color filters was four. The transmittance spectra of the color filters were constrained to be Gaussian shaped curves, making $\rho_i = 1$ for $i = 1,...,4$. The correlation matrix $E[\text{rr}^T]$ was obtained from the Dupont paint sample set[7]. The experimental results on reflective sets of photometric measurements made under D65 are shown

Table 1. An optimal Gaussian-shaped color filter set, designed under viewing illuminant A with $\gamma = 1$, is tested for its color measurement accuracy using both reflectance sets and radiance sets. The five Color Savvy LEDs were used as the internal illuminant.

Spec. ⇓	viewing illum.	ε_{mean}	ε_{max}	ΔE_{mean}	ΔE_{max}
Munsell	D65	0.0021	0.0088	0.0969	0.3609
Dupont	D65	0.0025	0.0081	0.1701	0.8286
Litho	D65	0.0021	0.0104	0.0917	0.2294
Object	D65	0.0016	0.0133	0.1180	1.0138

Table 2. The error analyses on the three radiance sets are tabulated for a radiometric colorimeter with respect to the expected radiance sets used to compute $E[\text{gg}^T]$. The original sources for R-Dupont and R-Object are given in the third column. $\gamma = 0$.

Radiometric device, $\nu = 0.9975$, $\tau = 0.9956$						
Expected Spec.	Rad. Spec. ⇓	orig. source	ε_{mean}	ε_{max}	ΔE_{mean}	ΔE_{max}
D65/Object	CRT	-	0.0078	0.1461	0.5477	2.9247
	R-Dupont	F2	0.1651	0.8682	1.1459	4.4970
	R-Object	D65	0.0024	0.0333	0.1813	1.3699
CRT	CRT	-	≈0	≈0	≈0	≈0
	R-Dupont	F2	0.8350	2.9225	1.4452	7.3561
	R-Object	D65	0.8048	17.0150	2.2750	6.6760

in Table 1. The complete results are available in [1]. The worst case reflectance measurements are made under the fluorescent viewing illuminant due to the spikes in F2 spectrum[3].

A radiometric colorimeter, where $\gamma = 0$ in Equation 5, was designed in the second experiment using a uniform detector and four Gaussian-shaped filters. The illuminant and reflectance set used to generate the radiance spectra set for the computation of $E[gg^T]$ are listed in the first column of Table 2. The radiometric colorimeter based on these filters was tested using the radiance sets consisting of CRT monitor radiances[1], the Dupont paint set[7] illuminated by the F2 fluorescent, the Object set[7] illuminated by the D65 incandescent. Note Table 2 is a subset of the results presented in [1]. The table shows that the correlation matrix for radiometric measurements can greatly effect the results of the experiment. Spectra which have unique features can cause problems. Examples of such features include the spectral peaks in F2 illuminated objects and the very low dimensionality of the CRT set (linear combinations of three color guns).

A combination colorimeter was designed with $\gamma = 0.2$ in Eq(5). A uniform detector, two Gaussian-shaped filters, and two sum-of-Gaussians shaped filters were used. The same five Color Savvy LEDs from the first experiment were used as the internal illuminants. The correlation matrix $E[rr^T]$ was obtained from the Dupont reflectance set, and $E[gg^T]$ was set equal to the $N \times N$ identity matrix I. The latter assumes maximum ignorance of the data set being measured; the expected g is independent, identically distributed with $\sigma = 1$. The results of the photometric and radiometric experiment are shown in Table 3. Table 4 shows the radiometric measurements for the combination colorimeter with respect to different expected radiance sets used for $E[gg^T]$.

Because the most recent version of the Colormouse produced by Color Savvy uses eight LEDs, an updated experiment was run to determine the performance of the filter set designed for a 20/80 weighting of reflectance/radiance. The results presented in Table 5 show that the reflective errors are negligible when using this many measurements. The radiance measurements are the changed from Table 3 only by the fact that estimation method used a slightly different correlation estimate.

4. Summary and Conclusions

The results of three design experiments; the photometric colorimeter, the radiometric colorimeter, and the combination colorimeter are described. The colorimeter designed

Table 3. A sum of Gaussians optimal color filter set, designed under viewing illuminant A with $\gamma = 0.2$, is tested for its color measurement accuracy using both reflectance sets and radiance sets. The five Color Savvy LEDs were used as the internal illuminant. $E[gg^T] = I$

Spec. ⇓	(meas. type, viewing illum.)	ε_{mean}	ε_{max}	ΔE_{mean}	ΔE_{max}
Munsell	**V**, D65	5.0555e-04	0.0086	0.0801	0.3589
Dupont	**V**, D65	9.2106e-05	6.6574e-04	0.0750	0.4432
Litho	**V**, D65	0.0042	0.0079	0.3332	0.5660
Object	**V**, D65	4.6063e-04	0.0182	0.1074	0.6099
CRT	**W**	0.0020	0.0472	0.6501	2.2641
R-Dupont	**W**, F	0.2335	1.4241	1.4529	4.4123
R-Object	**W**, D65	0.0366	0.4300	0.7558	1.8866

Table 4. The error analysis on the three radiance sets are tabulated for a radiometric colorimeter. The original sources for R-Dupont and R-Object are given in the third column. $\gamma = 0.2$, and the sum of Gaussians optimal color filter set was used.

Radiometric device, $\nu = 0.9975$, $\tau = 0.9956$						
Expected Spec.	Rad. Spec. ⇓	orig. source	ε_{mea}	ε_{max}	ΔE_{mean}	ΔE_{max}
D65/Object	CRT	-	0.0035	0.0464	0.7494	3.2877
	R-Dupont	F2	0.3149	1.9202	1.9052	8.4037
	R-Object	D65-	0.0070	0.1036	0.3447	1.4068
CRT	CRT	-	≈ 0	≈ 0	≈ 0	≈ 0
	R-Dupont	F2	35.4696	126.6670	22.2008	111.9266
	R-Object	D65	0.8402	9.4696	4.3865	27.5697

Table 5. A sum of Gaussians optimal color filter set, designed under viewing illuminant A with $\gamma = 0.2$, is tested for its color measurement accuracy using both reflectance sets and radiance sets. A new set of eight Color Savvy LEDs were used as the internal illuminant. $E[gg^T]$ from Dupont under illuminant A.

Spec. ⇓	(meas. type, viewing illum.)	ε_{mea}	ε_{max}	ΔE_{mean}	ΔE_{max}
Munsell	**V**, D65	9.0786e-06	0.0000	0.0078	0.0162
Dupont	**V**, D65	1.0358e-05	0.0000	0.0089	0.0164
Litho	**V**, D65	1.6706e-05	0.0001	0.0113	0.0181
Object	**V**, D65	8.8923e-06	0.0001	0.0106	0.0345
CRT	**W**	0.0284	1.4123	1.2790	7.5217
R-Dupont	**W**, F	0.1502	0.9492	1.1173	3.7477
R-Object	**W**, D65	0.0782	1.4040	0.9868	2.7033

for only photometric and radiometric measurements outperformed the combination colorimeter as expected. However, the use of the measure of goodness and the weighting factor allowed for a combination design that makes reasonable photometric and radiometric measurements. Radiometric measurements are sensitive to the correlation matrix used in the linear minimum mean squared estimation of tristimulus values. Choosing a nonspecialized data set for the estimate of the correlation is recommended.

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