

# A new diffusion computational model predicts both the positive and the negative short afterimage effects

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## Abstract

The goal of this research was to develop a compound computational model able to predict the “opposite” effects of the “color dove illusion” [1], [2] and the “filling in the afterimage after the image” effect [3]. Until now, only one study [4], based on a previous model, FACADE [5], had attempted to model the negative filling-in effect. However, this model cannot predict the positive effect and was not designed for this purpose. Our proposed model is based on the diffusion equation with boundary conditions that take the location of the remaining edges into account and assumes that both illusions derive from the same visual mechanism. Using the model, we were able, for the first time, to obtain accurate perceptual predictions of the core properties of both the positive and negative effects. The suggested model supports the idea that both “conflicting” phenomena stem from the same visual mechanism. In addition, the model was able to predict additional “conflicting” phenomena from different locations of remaining edges, or the direction of the chromatic gradient.

## Introduction

This study concerns two non-classical afterimage illusions i.e., the color dove illusion [1], [2] and the filling-in the afterimage after the image effect [3]. Both involve a filling-in process of both surface and edges, and both can be derived from a narrow spatial area and relatively short induction time.

In the color dove illusion [1], [2], the inducing stimulus is a shape surrounded by a colored area or strip (red in Figure 1, first row). After the chromatic inducing stimulus is removed, an outline contour matching the original inducing stimulus is presented (Figure 1, second row). This gives rise to the perception of an afterimage (Figure 1, third row) filled with a color similar to that in the inducing stimulus [2] although with a

weaker intensity of color. Such an effect has also been reported with objects of different shapes [6]. Since the perceived color inside the shape is similar to that presented in the inducing stimulus, we, henceforth, refer to this illusion as a “positive effect”, (Figure 1, first column).

In contrast, in the “Filling in the Afterimage after the image” [3] illusion, the inducing stimulus is a chromatic shape that can be composed of two or more colors. After the chromatic inducing stimulus is removed, an outline contour matching one of the colors is presented. The complementary afterimage color perceived depends on the shape and the location of the drawn outline contour [3], (Figure 1, second column). Since the perceived color inside the contour in the perceived afterimage is complementary to the color of the inducing stimulus, we, henceforth, refer to this illusion as a “negative effect”.

The negative effect is not a simple variation of the “classical” negative afterimage [7]. The “classical” effect is a visual phenomenon in which, after a relatively long (20-30 seconds) stimulus, the observer perceives the opposite chromaticity (complementary color) when the stimulus is removed [7]. It should be noted that the color in the classical afterimage is perceived only in the retinotopic area that was induced. In contrast, in both the described illusions (the positive and the negative effects), the color is perceived via a filling-in process in areas that have not been induced or adapted previously. This filling-in property, which is not required for the classical afterimage effect, is in addition to the temporal and spatial properties of the classical after image.

Anstis and colleagues. [8] found experimentally that the classical after image requires a relatively long exposure time and a large spatial area of induction in order to fill in a small region of complementary color. In contrast, the negative and positive effects of the two illusions discussed earlier, appear after a short exposure (500 msec, [2], [3]) and can be induced by a small spatial area.

## Previous mechanisms and models

Only one previous model has been reported to describe the non-classical effect, and this model considered only the negative effect [4], [5]. The original FACADE (Form And Color And Depth) model [5] describes two main visual processing systems: a boundary contour system (BCS), which processes boundary or edge information, and a feature contour system (FCS), which uses information from the BCS to control the spreading (filling-in) of surface properties such as color and brightness. According to the FACADE model, the filling-in stage requires FCS networks to diffuse signals containing feature information regarding color and brightness across the surface, while boundaries in the BCS block the spreading.

Francis [4] modified the FACADE model of Grossberg and Mingolla [1] in order to address the negative effect of van Lier’s illusion. This variant of the FACADE model [4] was able to predict the negative afterimage illusion described by van Lier et al. [3]. Francis’s implementation caused the remaining edges to trap the perceived color and block it from further spread across the empty surface.

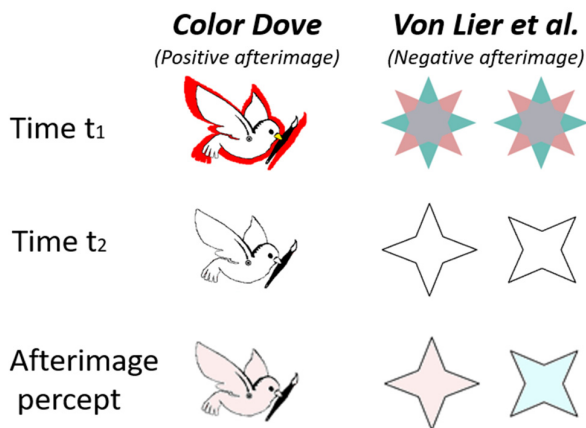


Figure 1. “The Color Dove illusion” (Positive effect) and the Von Lier et al. Illusion (Negative effect). The first row depicts the inducing stimulus, the second row depicts the drawn contours presented at time  $t_2$ , and the third row represents the resulting visual percept.

Kim and Francis [9] conducted a series of additional model simulations and psychophysical experiments to test the model predictions [4]. They tested the hypothesis that the contour traps the perceived afterimage color by adding additional contours. The model simulations predicted that additional contours would block the spread of color to the middle of the surface, Figure 4(D).

Contrary to these predictions [4] and [9], the results of psychophysical experiments [9] showed that additional contours blocked color spreading only when they overlapped with the inducer edges, but not when they were drawn away from the inducer edges, Figure 4(D).

The FACADE model, as presented by Francis [4], cannot predict the positive effect and was not designed for this purpose. This is because the model assumes that the spread of complementary color across a surface will be blocked by the drawn edge. However, in the positive effect, the colored area is outside the shape, and therefore, cannot spread inside the shape.

In this study, we present a model that is able to predict both the negative and the positive effects, and postulates that these are derived from the same mechanism. We also present the results obtained when the model was tested on additional cases beyond the two described illusions.

## Model

The following sections describe a unified computational model that can be applied to the two known separated effects, the positive “color dove illusion” and the negative “filling-in afterimage after the image” illusion. We suggest here, that these two different phenomena are induced by both spatial and temporal properties and are produced by the same mechanism. The model considers three factors: the inducing color, the remaining edges, and the diffusion process that is blocked by these edges.

### Inducing area – the driving force

The rationale behind the suggested model is the assumption that the driving force of this mechanism is the inducing stimulus. This is based on the observation that the disappearance of a chromatic or achromatic area is accompanied by the appearance of the complementary color in the same spatial location, albeit with weaker saturation, (the classical Negative after image effect). The perceived color is enhanced by the existence of a remaining edge.

At the same time, the inducer color (at reduced saturation) is perceived in the adjacent area (positive effect). This effect may be strengthened if there is an edge or border around the adjacent area. In addition, preliminary experimental psychophysical results indicate that the remaining edges play an important role in determining the perceived color [6], [10], [11]. It is also assumed that the same edges can function as borders for a “filling in” process [5]. In this report, we focus mainly on a subgroup of these effects (and possibly a subgroup mechanism) namely those relating to the afterimage that follows the inducing stimulus. These include the positive and negative effects involving a spatial-temporal chromatic gradient (chromatic edges).

The chromatic gradient is computed in the model as the Laplace operator comprising the sum of the partial second-order derivatives of the image. It can be expressed as a Different of Gaussian (DOG) filter or by the Gaussian derivative model [12]. These models have often been used to represent the spatial structure of the visual receptive fields in the retina or LGN (lateral geniculate nucleus) [13].

### Filling-in as a diffusion process

The next stage in the suggested mechanism and model is the filling in process. This process is envisioned as a diffusion process, which can be described by the diffusion equation (1), [14]. The biological justification for this assumption is that the diffusion or filling-in process takes place in an array of closely related neurons, via a cascade of local connections. In this way, contiguous neurons can easily pass signals between the compartment membranes [15]. The diffusion equation is:

$$\frac{\partial I_p(x, y, t)}{\partial t} = D \nabla^2 I_p(x, y, t) \quad (1)$$

where  $I(x, y, t)$  denotes the image in a space-time location  $(x, y, t)$ , and  $D$  is the diffusion coefficient. The time course of the perceived image is very fast [2], [3]; therefore, for the sake of simplicity we can ignore the fast dynamic stages of the diffusion equation. We therefore compute only the steady-state of the diffusion process. Consequently, the diffusion equation (1) is deduced to the Laplace equation (2).

$$\nabla^2 I_p(x, y, t) = 0 \quad (2)$$

In order to solve the Laplace equation, we need to define (a) boundary conditions, (b) the initial values. We shall henceforth denote the inducing stimulus (the original color image) by  $I_0$ , where  $\Omega$  is an area in the image  $I_0$ , and  $\partial\Omega$  is the border of  $\Omega$ .  $I_1$  is the contour image and  $\partial\Omega_1$  is the drawn contour (the remaining boundaries, although the boundaries in  $I_1$  might be different from those in  $I_0$ , and thus,  $\partial\Omega$  is not necessarily equal to  $\partial\Omega_1$ ). The boundary condition of the perceived image (2) were chosen as the original stimulus on the contour, i.e.  $I_p|_{\partial\Omega} = I_0|_{\partial\Omega}$  (Dirichlet boundary condition), Figure 2. This was chosen to preserve the object shape, with the same chromatic color that creates the chromatic gradient. The initial state (initial values) was chosen to be achromatic and a white image was selected in order to enable the creation any perceived color (the simulations are solved iteratively).

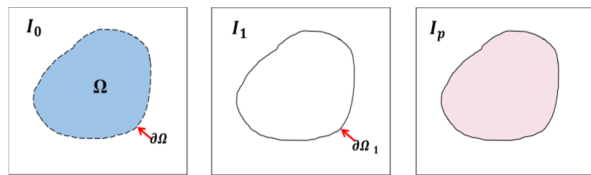


Figure 2. Left: the inducing stimulus. Center: the remaining edges. Right: the perceived image.

The model assumes that the hue of the filling in process is determined by the spatio-temporal direction of the chromatic gradient. If the chromatic edge or area disappear and the remaining edge overlaps the outer or inner chromatic edge, the perceived color will either be complementary to, or the same color as, the inducer area. The rationale for this dual effect derives from two main principles of adaptation. One is a result of adaptation of the chromatic area and the second is produced by the effect of remote areas on the central area [16]. In our case this remote effect can be extrapolated to be applicable also to edges, and not related to large inducing area only [17].

For the sake of simplicity, we calculate the diffusion Eq. (2) separately for the chromatic and achromatic zones in the original

image. The positive effect,  $I_{p,positive}$  occurs in the achromatic zones of the initial image  $I_0$ , Figure 2. The negative effect  $I_{p,negative}$  occurs in the chromatic zones of the initial image  $I_0$ , Figure 2. The existence of separate Magno, Parvo and Konio visual pathways in the visual system suggests that such separate chromatic and achromatic calculations may reflect the real biological situation [18].

### The remaining edges

The model for the visual phenomena, of the perceive image includes a role for the remaining edges, Figure 1 & Figure 2. The model suggests that these edges trap the diffused color, but only when the remaining edge  $\partial\Omega_1$  overlaps the original gradient edge, Equation (3), and. The mathematical expression of this role is:

$$\nabla^2 I_p = \nabla^2 I_0 \partial\Omega_1 \quad (3)$$

The equation is solved separately for the negative effect  $I_{p,negative}$  and for the positive effect,  $I_{p,positive}$ . (see the section above). The simulation result is calculated in (4):

$$I_p = \frac{I_{p,negative} + I_{p,positive}}{(\max\{I_{p,negative}\} + \max\{I_{p,positive}\})} \quad (4)$$

where  $\max\{I\}$  is the maximum value of all channels in the image  $I$  ( $\max\{I\}$  is a scalar).

*Implementation* - The prediction is produced by assigning the conditions as described above and applying the Gauss-Seidel method. The simulations are solved in a similar way to that reported for the ‘‘Poisson image Editing’’ [19].

### Model Simulations and predications

The simulation results are divided into three parts. The first part presents the model predictions for both negative and positive after image phenomena, [3], [1]. The second part presents the predictions of the model for two remaining edges variations as presented in previous studies [4], [9]. The third part presents the model predictions for an additional aspect of the afterimage phenomenon relating to the color perceived when the remaining edge of the image is not complete (open boundaries) and there is no border to block the diffusion process.

### Negative and positive afterimages

The model predictions for the positive [1] and the negative [3] afterimage after the image effect are presented in Figure 3(C). The first row represents the model predictions for van Lier [3] stimuli, negative effect (stars). The second row represents the model predictions for the color dove illusion [1], [2], positive effect. The predictions of the model correspond to psychophysical findings [3] and to preliminary results. Note that the model also correctly predicted the filling process of the achromatic area with respect to both negative and positive effects.

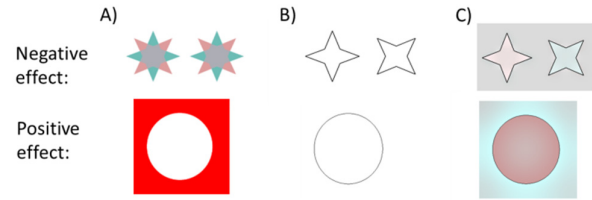


Figure 3. The model predictions of the negative (first row) and the positive (second row) afterimage effects. A) The inducing stimulus. B) The remaining edges. C) The simulation results. In the first row, the left star is perceived as having a pink color (the complementary color to cyan), while the right star is perceived in cyan (the color complementary to pink). The centers of the stars are filled with color. In the second row, the center of the circle is filled with a red color through a filling in process and the background becomes cyan as a result of a classical afterimage effect.

### The role of the remaining edges

#### Comparison to previous results

Figure 4 presents the predictions of the suggested model and Francis's predictions to von Lier [3] stars with two possibilities of drawn contours Figure 4(D) and Figure 4(C), respectively. One option is where the remaining edges overlap the chromatic gradients, which exist in the inducing stimuli. In the second option, the remaining edges do not overlap the chromatic gradients.

For the overlapping boundaries, the predictions of both models of Francis [4], [9] and our (Figure 4 first row, C-D) yield the same results, which also agree with the perceived results [9]. However for non-overlapping boundaries, the predictions of Francis's [4] (Figure 4 second row) differ (Figure 4 (C)) from those of our model (Figure 4 (D)). The psychophysical findings support our model [9], which predicts that drawn contours which do not overlap the chromatic gradient do not block the diffusion process.

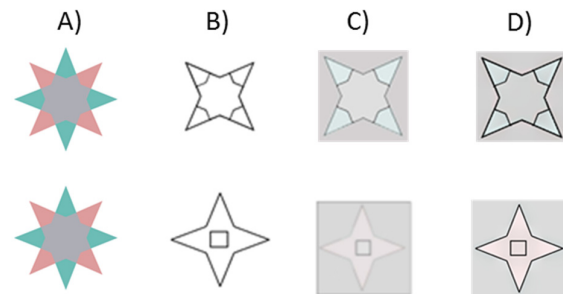


Figure 4 Comparison to previous results. A) The inducing stimulus. B) The remaining edges. C) The simulation results as in [4], [9]. D) The simulation results of the suggested model. In the first row, the inner drawn contours (B) overlap the chromatic gradients that exist in the inducing stimulus (A). In the second row, the inner drawn contours do not overlap the chromatic gradients in the inducing stimulus.

We also compare the predictions to alternating watercolors, which can be considered as related phenomena [6]. The effect of the luminosity of the induced area on the positive and negative effects is evaluated (Figure 5 and Figure 7 in: [6]). The model succeeded in predicting the general trend of the experimental results only for the positive effect, but not for the negative effect due to their experimental design [6].

### Model predictions for a new stimulus with variations of remaining edges

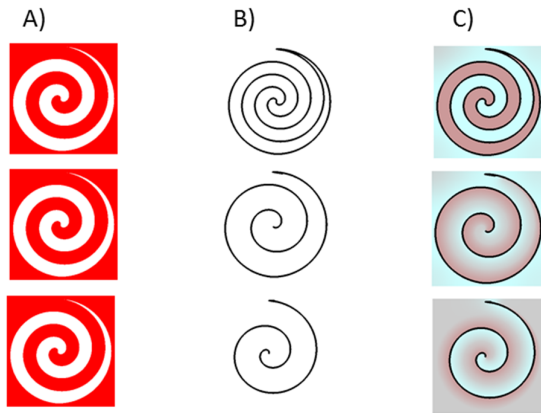


Figure 5. Model predictions for a variety of contours. A) The inducing stimulus. B) The remaining edges. C) Our model's predictions. In this figure, the inducing stimulus is the same in all the rows, column (A), however the remaining edges are different. In the first row, the drawn contour is a full spiral, as in the positive afterimage, see second row in Figure 3. In the second row, the outer edge of the spiral shape is presented and in the third row the drawn contour is the inner edge of the spiral shape, column (B). Our model predicts that in the first case the result will be similar to the positive after image as in Figure 3 (C). In the second case, the dominant color of the spiral shape will be perceived as reddish while the dominant color in the third condition will be perceived as cyan.

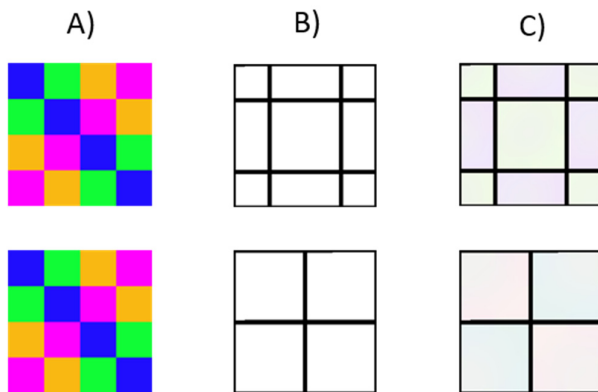


Figure 6. Model prediction for a different variation of afterimage [10]. A) The inducing stimulus. B) The remaining edges. C) The simulation results.

Until this stage, we have simulated and tested our model on previously experimented stimuli [9]. We further challenged our model with a new stimulus that has not been described or experimentally tested yet. The new stimulus (Figure 5) includes both positive and negative effects in the same stimulus, simultaneously. In addition, this stimulus enables us to test a critical property with regards to closed or open reminding edges. For this purpose, we chose a spiral stimulus since it has both inner and outer borders (in contrast to the previous shapes, Figure 3).

The model's results indicate that the dominant color perceived in the afterimage depends on the location of the remaining edges (inner or outer edge), first row and second row of Figure 5, respectively. The dominant color, predicted by our model, therefore can be either complementary or similar to that of the inducer color. Thus, the model predicts that the outer

border produces a dominant positive effect, while the inner border produces a dominant negative effect, Figure 5 (C). These predictions are supported by the preliminary psychophysical results.

### Discussion

We present a compound model that is able to predict both positive and the negative effects, i.e., both the "filling-in the afterimage after the image" illusion and the "color dove illusion" [2], [3]. The model also succeeded in predicting positive effects in other conditions that possess non-closed remaining edges. These results contradict the assumptions of a previous computational model [4], [9]. The success of the model predictions supports the idea that both positive and negative effects stem from the same visual mechanism. In addition, the model was also able to predict a recently reported predominantly negative afterimage effect related to mixed colors [10], Figure 6.

Although the FACADE model [5], used by Francis [4], successfully predicts the negative effect, this model also mistakenly predicts that a remaining edge that does not overlap the chromatic gradient edge border, will block the spread of color.

Our model predicts the opposite; if a border does not exist in the original inducing stimulus, it will not block the diffusion process, as in Figure 4. These predictions are in agreement with the psychophysical findings of Kim and colleagues [9]. Furthermore, they also agree with more recent psychophysical findings, relating to general stimuli of both positive and negative effects. Based on experimental results, Kim and colleagues [9] formulated a general rule that additional contours block color spreading when they overlap with the inducer edges, but not when they are drawn away from the inducer edges.

Our model is therefore supported by psychophysical findings relating to both negative and positive effects as well as by additional preliminary results relating to the positive effect [20]. However, it is becoming apparent that the full expression of the positive and negative effects cannot be fully explained by the direction of gradient of diffusion. (Or in other words, the location of the remaining edge in relation to the gradient edge). Additional factors might play a role in at least the degree of the effect (the degree of color saturation). For example, the intensity of the achromatic region of the induced area has been tested [6], and additional factors such as the size of the inducer and induced area, the shape curvature etc. are the subjects of ongoing experiments in our laboratory. The model predictions related to the intensity of the induced area are only successful for the positive effect, since the model computes only the chromatic gradients which overlap the remaining edges. This overlapping occurs only in the positive effect (Figure 4 in [6]). Consequently, the model, at this stage, obtains accurate perceptual predictions of the core properties of both the positive and negative effects.

Even though the present model does not permit predictions of the behavior of all the free parameters (see above) this is the first time that a model has been able to make crucial predictions relating to the location of the gradient edge and remaining edge, and their effect on the hue of the perceived color. In other words, our model succeeds in predicting apparently conflicting phenomena, i.e., those producing positive and negative effects. The model can also predict the dominant color in the spiral afterimage effect, where there is an ambiguous object with an incomplete remaining edge (Figure 5). Preliminary results indicate a lack of requirement for trapping by remaining edges in order to obtain a perceived color in the positive and negative effects.

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