

A Revision of CIECAM02 and its CAT and UCS

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Abstract

Various attempts have been made to overcome the mathematical problems of CIECAM02. All focused on modifications keeping the original structure of CIECAM02, and resulted in some loss of accuracy in predicting experimental visual results. The current paper proposes to merge the separate adaptation to the chromaticity and luminance of the illuminant by using a new space rather than the two different spaces proposed in the original CIECAM02. The new space, defined by a new matrix M_{16} , is a cone-like space. From the new structure and space a new colour appearance model, named CAM16, has been derived. At the same time, a new chromatic adaptation transform, named CAT16, and a new uniform colour space, named CAM16-UCS, have been also developed. Performance tests showed that CAM16 and its associated CAT16 and CAM16-UCS not only solved all the mathematical problems reported for CIECAM02, but also performed as well as or even better than the original CIECAM02 model and its associated chromatic adaptation transform CAT02 and color space CAM02-UCS. These performance tests included predictions of experimental data currently available as visual colour appearance datasets, corresponding colour datasets and colour difference datasets. Furthermore, CAM16 is simpler than CIECAM02.

INTRODUCTION

Ever since the recommendation of CIECAM02 colour appearance model (CAM) by CIE TC 8-01, it has been widely used in scientific research and industrial applications. However, problems were found with CIECAM02. For example, in some circumstances unexpected computational failures occurred in lightness computation:

$$J = 100(A / A_w)^{cz} \quad (1)$$

At this stage we want to make it clear that all symbols used in this paper have the same meaning as in the CIE document [1]. Li and Luo [2] showed that A_w is positive for all popular illuminants. However, the achromatic signal A , having the expression:

$$A = [2R'_a + G'_a + (1/20)B'_a - 0.305]N_{bb} \quad (2)$$

can be negative, and raising the negative ratio in the bracket of Eq. (1) to the non-integer power cz is not mathematically possible. The luminance adaptation signals R'_a, G'_a, B'_a are transformed from the input tristimulus values (TSVs) via the colour and luminance adaptation of the illuminant, which can be expressed as follows:

Chromatic adaptation: Firstly, the input TSVs are transformed to the sharper sensor space. The sharper sensor response signals R, G, B are given by:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M_{02} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (3)$$

where M_{02} is the built-in CAT02 matrix [3-5], and the colour adaptation is completed using:

$$\begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix} = \Lambda(D) \begin{pmatrix} R \\ G \\ B \end{pmatrix} \quad (4)$$

where $\Lambda(D)$ is the 3 by 3 adaptation diagonal matrix:

$$\Lambda(D) = \text{diagonal}(D \frac{Y_w}{R_w} + 1 - D \quad D \frac{Y_w}{G_w} + 1 - D \quad D \frac{Y_w}{B_w} + 1 - D) \quad (5)$$

In order to complete the luminance adaptation, next the colour adapted signals R_c, G_c, B_c are transformed back to the TSVs space via the inverse transform:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = M_{02}^{-1} \begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix} \quad (6)$$

Luminance adaptation: The TSVs of the corresponding colour given by Eq. (6) are transformed to the Hunt-Pointer-Estevéz (HPE) cone space [6] for final luminance adaptation using the HPE matrix M_{HPE} [1-3]. The HPE cone response signals R', G', B' are given by:

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = M_{HPE} \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} \quad (7)$$

Finally, the luminance adaptation is completed via a nonlinear transform:

$$R'_a = \frac{\text{sign}(R') * 400(F_L |R'|/100)^{0.42}}{27.13 + (F_L |R'|/100)^{0.42}} + 0.1 \quad (8)$$

for the red channel, and similar equations for the other two channels.

Therefore, the negative sign of the achromatic signal A (Eq. (2)) comes from the two adaptations. Li et al. [7] considered modifying the CAT02 matrix to M , so that the HPE cone response signals R' , G' , B' be always positive, resulting in the constrained:

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = M_{HPE} M^{-1} \Lambda(D) M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq 0 \quad (9)$$

The CAT02 transform with the new matrix M and the modified CIECAM02 model should fit the corresponding colour datasets used for developing the CAT02 matrix [3] and the colour appearance data sets used for developing the CIECAM02 model [1,4,5] as accurately as possible. Thus, a constrained nonlinear optimization problem was established and solved numerically, resulting in a matrix named M_{CAM} [7]. The CAT02 transform with the new matrix M_{CAM} , results in a new chromatic adaptation transform named CATv1, and the CIECAM02 model with the new matrix results in a new colour appearance model, named CAMv1.

Furthermore, if $M = M_{HPE}$, it can be shown that the inequality in Eq. (9) always holds, and hence Li et al. [8] proposed that the CAT02 matrix can be replaced by the HPE matrix, resulting in a new CAM named CAMv2, and a new CAT named CATv2.

At the same time, Brill [9] found that CIECAM02 model has the so-called 'yellow-blue' and 'purple' problems, and gave a rule for correcting such problems, the so-called nesting rule. Let Ω_{CIE} be the domain enclosed by the CIE spectral locus and the purple line, and Ω_M the chromaticity domain of all TSVs giving non-negative response signals with the transformation defined by Eq. (3) with matrix M . The nesting rule can be exactly stated [10] as:

$$\Omega_{CIE} \subseteq \Omega_M \subseteq \Omega_{HPE} \quad (10)$$

It was reported that with the original CAT02 matrix, the above nesting rule is not satisfied, which is the source of the CIECAM02 problems. Recently, Li et al. [11] found that there are many matrices M satisfying the nesting rule, a special case being the matrix M equal to the HPE matrix, i.e., the two adaptations use the same HPE matrix. Furthermore, Jiang et al. [10] gave an optimum solution to the yellow-blue and purple problems with the best matrix named as M_{OPT} having the following properties: a) with the nesting rule being satisfied; b) CIECAM02 with matrix M_{OPT} has the greatest accuracy in predicting the visual results, even better than the accuracy with the CIECAM02 model using any other matrix M satisfying the nesting rule. The CAT02 transform and CIECAM02 model with the new matrix M_{OPT} form a new CAT named CATv3 and a new CAM named CAMv3.

All the above modifications (CAMv1, CAMv2, CAMv3) to CIECAM02 have two attributes in common: the two adaptations are completed in different spaces, and the CAT02 matrix is modified. It was found, as shown in Table 1, that CAMv1, CAMv2 and CAMv3 gave approximately the same accuracy as CIECAM02 in predicting the LUTCHI and Juan and Luo colour appearance datasets in terms of CV values. However, the performances of the CATv1, CATv2, and CATv3 transforms in predicting the corresponding colour datasets become much worse than the original CAT02 transform as shown in Table 2 in terms of mean and weighted mean CIELAB colour differences. Note that the CV

values were used for the development and evaluation of the CIECAM02 model as a statistical measure.

In summary, all the above modifications of CIECAM02 and CAT02 do correct the problem of negative values for the achromatic signal A (Eq. (2)) at the expense of losing accuracy in predicting the corresponding colour datasets. In addition, in the development of the optimum solution (CATv3/CAMv3) [10], it was realized that instead of the modification under the original structure, it may be better to change the structure by combining two matrixes into only one (see later). It was hoped that this change would solve the mathematical problem and at the same time improve the accuracy for predicting the visual results, as discussed in the next section.

Note that, in 2008 Gill [12] also reported an extension to the CIECAM02 model, where some of the equations are modified so that the mathematical failures of the CIECAM02 model can be avoided. The performance in predicting the visual data should be the same as the original. However, the modified version seems more complicated than the original CIECAM02 model.

Table 1. Overall mean of CV in lightness (L.), colourfulness(C.) and hue (H.) for the CAMs: CIECAM02, CAMv1, CAMv2, CAMv3 and CAM16.

	CAM02	CAMv1	CAMv2	CAMv3	CAM16
L.	14.0	14.2	14.2	14.2	14.0
C.	18.6	18.6	18.9	18.7	18.2
H.	6.9	6.8	7.0	6.9	6.6

Table 2. Weighted (W.M.) and overall mean (O.M.) CIELAB colour difference units using 21 corresponding colour datasets for CATs: CAT02, CATv1, CATv2, CATv3 and CAT16.

	CAT02	CATv1	CATv2	CATv3	CAT16
O.M.	6.3	7.4	7.5	7.3	6.3
W.M.	5.5	6.7	6.8	6.5	5.6

THE NEW COLOUR APPEARANCE MODEL, CAM16

As discussed at the end of the previous section, we have to restructure CIECAM02 model in order to solve the negative value problem, and at the same time we must keep, or even improve, the accuracy in predicting experimental visual results achieved by the original CIECAM02/CAT02 models. To be more specific, in the new model the chromatic and luminance adaptations are made in the same space, which means a different structure rather than the current CAT02 sharper sensor space and HPE cone space proposed by CIECAM02. Let the new space be defined by the matrix M mapping the tristimulus values (TSVs) to the new R , G , B space. Firstly, one can imagine from Eqs. (3-6) that if the same matrix M is used in place of both M_{02} and M_{HPE} , we have:

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix} = \Lambda(D) M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (11)$$

Obviously, when the two adaptations are completed in the same space, the first benefit is that the new model is simpler than the original. Next, it follows from Eqs. (2), (5), (8) and (11) that in order that the achromatic signal A be non-negative, the response signals R, G, B should be non-negative, i.e.,

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq 0, \quad (12)$$

since the diagonal matrix $\Lambda(D)$ (Eq. (5)) is non-negative. Besides, the nesting rule (Eq. (10)) indicated that the matrix M should be chosen to satisfy Eq. (13).

$$\Omega_{\text{CIE}} \subseteq \Omega_M \quad (13)$$

It can be further shown that the two conditions given by Eqs. (12, 13) are satisfied if the constraint defined in Eq. (14) is satisfied:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \geq 0 \quad (14)$$

Here, $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are the CIE 1931 or 1964 colour-matching functions. In addition to the above constraints, the CAT02 transform with new matrix M , and the new structure of CIECAM02 with the new matrix M must fit the corresponding colour datasets and the colour appearance datasets as accurately as possible. All these considerations lead to a constrained nonlinear optimization problem, whose numerical solution, the new matrix M , has been obtained and is given by:

$$M_{16} = \begin{pmatrix} 0.401288 & 0.650173 & -0.051461 \\ -0.250268 & 1.204414 & 0.045854 \\ -0.002079 & 0.048952 & 0.953127 \end{pmatrix} \quad (15)$$

Thus, a new CAT, named CAT16, was established from the new matrix coefficients (Eq. (15), M_{16}). When M_{16} is introduced in CIECAM02 with the new structure, a new CAM named CAM16 is established. In the next section, the performance of the new model will be investigated. In addition, the same equations used to extend from CIECAM02 to CAM02-UCS [13] were also applied to CAM16, resulting in CAM16-UCS, which performance will be also investigated.

PERFORMANCE OF CAT16, CAM16 AND CAM16-UCS

Spectral Response

Spectral responses for each of the matrices M_{02} , M_{HPE} , and M_{16} were compared. It was found that that the CAT02 matrix has negative responses for the red and green channels, while the responses from M_{HPE} and M_{16} are all non-negative. In general, the spectral responses from the matrices M_{HPE} , and M_{16} are similar. Since the spectral responses have negative parts for M_{02} , the space from the CAT02 transform was known as the sharper sensor space, while the space defined by the HPE matrix was known as the cone

space. Therefore, the CAT16 space can also be considered as a cone space.

Predicting the corresponding colour data sets

CAT16 was evaluated using the corresponding colour datasets. The overall and weighted mean results were listed in the last column of the previous Table 2. It can be seen that CAT16 performed better than CATv1, CATv2, and CATv3. Furthermore, CAT16 and CAT02 had the same overall mean difference and CAT02 is only 0.1 CIELAB colour difference unit better than the CAT16 transform for the weighted mean difference. In summary, CAT16 and CAT02 performed equally well and CAT16 performs better than all other CATs.

The Nesting Rule

The source of the CIECAM02 problem has been identified as coming from the original CAT02 matrix M_{02} , which does not satisfy the nesting rule, but the matrices M_{HPE} and M_{OPT} do. Does the matrix M_{16} satisfy the nesting rule? In fact, it can be verified that the following is true:

$$\Omega_{\text{CIE}} \subseteq \Omega_{\text{RBJK}} \subseteq \Omega_{16}$$

Here, Ω_{16} is the non-negative response region for the CAT16 and Ω_{RBJK} is the maximal saturation quadrilateral RKJB given by Fry [14]. Thus, the CAT16 not only satisfies the nesting rule, but also encloses the maximal saturation quadrilateral of Fry, which makes the CAT16 even more robust.

Performance of CAM16 when predicting the colour appearance data sets

The accuracy of CAM16 model was tested using the LUTCHI and Juan and Luo colour appearance datasets used for developing the CIECAM02 model [1,4,5], and the test results are listed in the last column of Table 1 above. The results are encouraging because, in comparison with CIECAM02, CAM16 performed equally well for the lightness attribute prediction, and even better in predictions of the colourfulness and hue composition attributes. Furthermore, the CAM16 model is better than all the proposed models CAMv1, CAMv2 and CAMv3 in predicting lightness, colourfulness and hue composition attributes. In addition, this occurred in most of the subsets.

Predicting the colour discrimination datasets using CAM16-UCS

The same equations used in the CAM02-UCS [13] to extend CIECAM02 were applied to CAM16 to define the CAM16-UCS. The performance was tested using the three groups of colour discrimination data: SCD (small magnitude colour difference group), LCD (large magnitude colour difference group) and A (illuminant A group), used to develop CAM02-UCS [13]. Table 3 lists the three groups with their number of pairs and mean CIELAB colour differences, Table 4 lists the STRESS results for the CIECAM02, CAM02-UCS, CAM16 and CAM16-UCS when predicting each of the three colour discrimination groups. The STRESS index [15] is frequently used to evaluate the merit of colour difference formulae, in such a way that STRESS values close to 0 imply a good formula.

Table 3: Number of colour pairs and mean CIELAB colour differences in three groups of colour discrimination datasets: SCD, LCD and A.

	SCD	LCD	A
No. Pairs	3813	2953	1053
Mean ΔE^*_{ab}	2.0	11.1	2.9

Table 4: STRESS [15] results for different colour difference formulas for each of the three groups

	SCD	LCD	A
CIECAM02	38.5	25.2	37.7
CAM02-UCS	27.1	23.5	31.3
CAM16	37.6	25	35.2
CAM16-UCS	26.7	23.3	29.9

The results in Table 4 show that CIECAM02 and CAM02-UCS performed worse than CAM16 and CAM16-UCS, respectively. Therefore CAM16 is not only simpler than CIECAM02, but also predicts more accurately the visual results, and the associated CAM16-UCS space performs even better in predicting colour differences results.

CONCLUSIONS

In this paper, three methods, named CAMv1, CAMv2, CAMv3, to solve the problems of CIECAM02 were reviewed. These also resulted in three candidate CATs (CATv1, CATv2, and CATv3) to replace CAT02. Performance tests on the predictions of CAM and CAT visual datasets showed that, although the three CAMs performed similarly to the original CIECAM02 for colour appearance datasets, their associated three CATs predicted much less accurately than the original CAT02 using the corresponding colour datasets. It was realized that all previous proposals solved the problems concerned with the CIECAM02 model at the expense of losing the accuracy for predicting the visual experimental results.

Then, a different approach by changing the CIECAM02 structure for dealing with chromatic adaptation and cone transformations was carried out. The resultant model performed as accurately as the original. The chromatic and luminance adaptations were first made in a new 'cone' space defined by a new matrix M_{16} , which satisfies the constraint in Eq. (14). At the same time, the new model accurately predicted the corresponding colour datasets and colour appearance datasets. This resulted in a new CAT named CAT16 and a new CAM named CAM16. Further evaluation results showed that CAT16 performed marginally better than CAT02 in almost all subsets in the corresponding colour dataset. Most importantly, the CAT16 matrix satisfies the nesting rule, while the CAT02 matrix does not. When predicting the colour appearance datasets, CAM16 and CIECAM02 performed equally well in predicting the lightness data, but CAM16 performed better than CIECAM02 in predicting colourfulness and hue composition.

Furthermore, a uniform colour space CAM16-UCS, based on CAM16, is proposed using the same extension equations as CAM02-UCS. It was comprehensively tested and found to be at least equal to or better than CAM02-UCS using three groups of colour difference datasets: small magnitude, large magnitude and

illuminant A. It is encouraging that CAM16-UCS space gave similar or better performance than the CAM02-UCS space.

In summary, the long awaited remedy to the deficiencies of the CIECAM02 model has been achieved. The resultant is a simple modification of CIECAM02/CAT02/CAM02-UCS models. In many applications such as colour management, colour rendition metrics, image processing, colour inconstancy prediction, etc. CIECAM02, CAT02 and CAM02-UCS, should be replaced by CAM16, CAT16, and CAM16-UCS, respectively, with great confidence.

Finally we note that because of the D factor, CAT16 (and CAT02) does not have the transitive property when D is different from a value of unity, i.e., transforming TSVs under illuminant 1 to TSVs under illuminant 2 using the CAT, and then transforming the transformed TSVs under illuminant 2 to TSVs under illuminant 3 is different from the TSVs under illuminant 3 directly transformed from TSVs under illuminant 1. The CAT16 working in this way can be considered as one-step CAT. Recently, Li et al. [16] suggested that CAT16 can be organized as a two-step CAT, which has the nice transitive property. It was also be found that the two-step CAT and one-step CAT have roughly the same performance in predicting the visual corresponding colour datasets. This will further reported in papers [16, 17].

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