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A revisit of the MacAdam colour discrimination ellipses

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Abstract

MacAdam colour discrimination data have been the fundamental basis for the definition of just noticeable colour difference. Lighting stimuli and products are compared in regard to these ellipses data. The current study examines the validity of the MacAdam ellipses and investigates the best tool to evaluate colour difference of light stimuli. Pairs of chromaticity colour difference were processed with the aim to be simulated as light sources on a display and over 23000 assessments were accumulated by a group of 20 observers. Experiments were based on the ratio method and all pairs were examined against two different backgrounds; a black background and a grey background with the same specifications as the one employed in the MacAdam experiment. Different colour difference metrics and colour spaces were investigated in terms of ellipse fitting and colour difference performance.

Background

MacAdam conducted a series of experiments for his research in visual sensitivities based on a self-luminous visual colorimeter [1, 2]. A colour matching experiment was designed based on an explicit colorimeter. Twenty-five target colour centres were composed of layers of different colour films while they were illuminated by a single light source lamp with the characteristics of illuminant C. Many colour matching assessments were acquired around the selected colour centres. Colour matching was conducted along different lines/directions in xy chromaticity diagram. The viewing field consisted of a 2^o circle against a 42^o uniform background. The circle was separated into target and matching field by a biprism so these were viewed monocularly. However, the data were based only on a single observer and the stimuli were of high luminance; which is not applicable in many applications. Additionally, data analysis was based on the standard deviations of the colour matching assessments. Finally, the viewing conditions do not fit with the normal binocular vision and the matching process was not based on the trichromatic primaries.

Yet, the colour matching assessments resulted in the MacAdam discrimination ellipses which are used to visually represent the perceptible colour difference in the xy chromaticity diagram. For the lighting industry, MacAdam ellipses are still used as reference to evaluate lighting products in terms of steps of the MacAdam ellipses. A typical example is with the standards for specification of chromaticity such as the ANSI C78.376 for fluorescent lamps and ANSI C78.377 for solid state lighting products [3, 4]. However, these standards define chromaticity coordinates and correlated colour temperature of white light stimuli. In the fast-evolving lighting industry, products of LED technology include the production of a variety of hue colours; for which a concrete evaluation tool has not been specified.

In an earlier study by the authors, evaluation of colour difference metrics for white light sources based on the specifications of the ANSI C78.377 standard was performed by assessing colour difference [5]. The results were very promising regarding the use of the u'v' chromaticity diagram and CIELUV formula for the evaluation of colour difference of lighting products. Moreover, one of the findings was that the MacAdam ellipses in close proximity to these data did not agree well. Therefore, in the current study, a set compiled of white light stimuli from the ANSI C78.377 standard and coloured light stimuli from the MacAdam experiment were used for the acquisition of a unique dataset of colour difference assessments of lighting stimuli.

Experimental

A colour difference experiment was designed in order to assess the differences within pairs of chromaticity difference. A custom-made software was used to represent the stimuli on the display. Colour management and stimuli specification had been applied beforehand and all colorimetric values were consistently monitored though the experiments so as to maintain accuracy of reproduction. Sampling was processed within the u'v' chromaticity diagram; where 21 points were chosen in semi-circular manner surrounding the colour centres ranging from 0^o to 180^o. The difference between colour centres and samples was defined in terms of Euclidean distance in the u'v' chromaticity diagram. Equal difference of Δu'v' 0.007 units was applied to all colour centres.

The experimental colour centres were 18 centres from the MacAdam dataset and 8 neutral centres from the ANSI C78.377 standard. The colour centres were processed into two different luminance levels; 48 cd/m^2 and 18.5 cd/m^2. The former luminance level corresponds to the original luminance used by MacAdam for the test field, while the latter was an adjusted decreased value so as to obtain some more reproducible stimuli within the colour gamut of the display. In Figure 1, the colour centres selected from the MacAdam's ones are represented by the diamond symbol. Additionally, two extra colour centres in the blue and green areas were added so as to cover most of the important hue areas of the chromaticity diagram.

The experiments were carried out in dark room conditions and fixed viewing distance of 50 cm. The pairs were assessed by a group of 20 observers against both black and neutral grey background. The former background was used in an attempt to simulate the stimuli as light sources viewed at night. For the same reason, each stimulus was represented by circular patches of 4 degree approximately, so as to resemble light sources in their shape. The latter background has the same characteristics as the surround field of the MacAdam experiment; i.e. chromaticity of illuminant C and luminance of 24 cd/m^2. Furthermore, Cui *et al* and Berns found that the grey background can simulate well the surface colour viewing condition on displays [6-8]. Therefore, the data acquired from different types of stimuli arrangement can be compared.

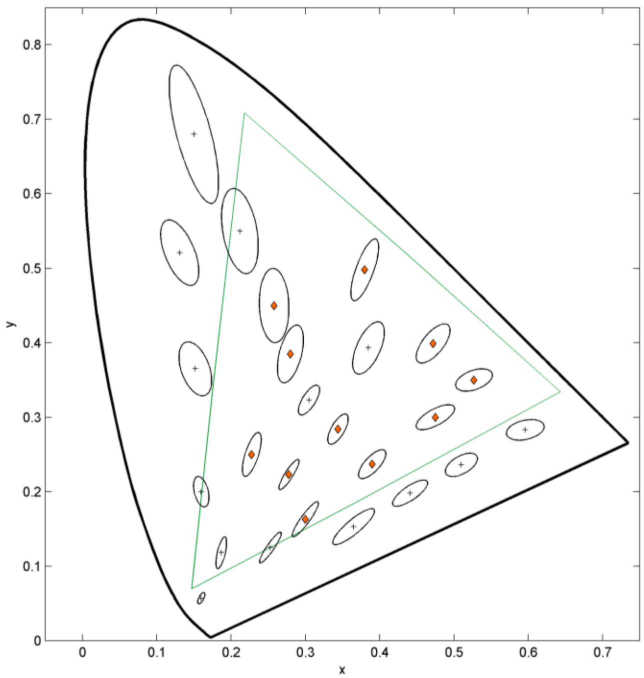

Figure 1. MacAdam discrimination ellipses along with the selected colour centers for the current study marked with the diamond symbol; display gamut by green triangle.

During the experiments, two pairs of stimuli were presented. The reference and testing pairs were presented in two rows; but their location within these rows was randomly interchanged in both dimensions in order to avoid any automatic replies and eye fatigue. The reference pair consisted of two fixed green colour stimuli close to one of the MacAdam's colour centres. It was set with fixed chromaticity and lightness difference ΔL* of 6 units. It was chosen as such to represent the colour difference clearly and remained constant through all phases. The experimental arrangement of the stimuli is illustrated in Figure 2. The observers were asked to assess the colour difference of each testing chromaticity pair against the standard reference pair which was described as having colour difference of one visual unit. Using ratio scaling method, the observers evaluated whether the testing pair had a larger or smaller colour difference than the reference pair; and assigned a ratio larger or smaller than one accordingly. A scroll bar was provided in the software interface so as to record each assessment.

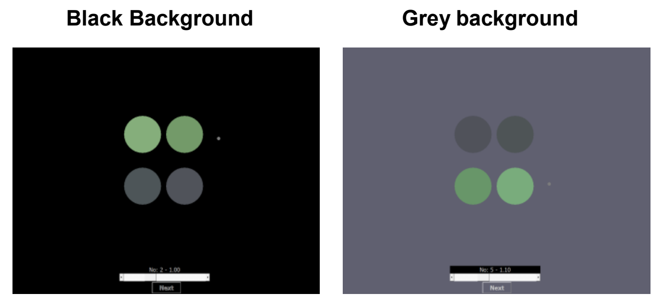

Figure 2. Experimental stimuli arrangement for black and grey background.

Data Analysis

For great deal of the statistical analysis of this study, the STRESS (Standardized Residual Sum of Squares) measure was used. STRESS is a multidimensional scaling technique that was adapted by Garcia et al. in order to investigate the relationship between measured and perceived colour difference[9].The formula can be adjusted to test the discrepancy between any two datasets in the form of a goodness-of-fit measure. The Equation 1 shows the adapted version of the formula for testing colour difference formulae. The multiplication by the factor 100 is added to produce results in percentage. Respectively, any dataset can be used in the place of the factors ΔV and ΔE shown in the equation. STRESS can illustrate the statistical significance of the data and has sensitivity to outliers. Perfect agreement between the examined datasets is given by a STRESS value of zero.

$$STRESS = 100\sqrt{\frac{\sum(\Delta V_i - f\Delta E_i)^2}{\sum \Delta V_i^2}} \quad (1)$$

where $f = \frac{\sum \Delta E_i \Delta V_i}{\sum \Delta E_i^2}$ is a scaling factor

Observer uncertainty

During the experiments, two of the colour centres and their pairs against both backgrounds were assessed twice by the observers, so as to estimate the reliability of the group. The STRESS measure was applied for the investigation of inter- and intra-observer variability. The results of this analysis are presented in Figure 3. The STRESS values are low enough to justify the group of observers as reliable. Furthermore, the values are within the same range with results of other studies such as by Luo et al. [5]. Finally, it can be seen that the repeatability of each individual's assessment for the same pair is significantly accurate.

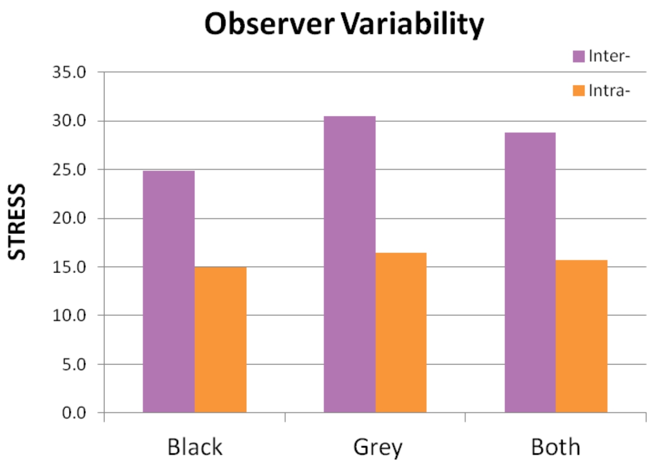

Figure 3. STRESS values for observer inter- and intra- observer variability.

Colour Discrimination Ellipses

Geometrically, ellipses are defined by the following parameters: the semi-major axes (A); the semi-minor axes (B); the ratio between the length of the semi-major and the semi-minor axes (A/B); and the angle (θ) from the horizontal axis. These parameters are important in order to describe the shape, size and orientation of the ellipses within the colour spaces and chromaticity diagrams. The ellipses illustrate the perceived colour difference within the space for which they have been calculated. Therefore, their shape, size and orientation give information about the performance of the colour space and respective colour difference metric, as well as characteristics across the different chromatic areas. In a perfectly uniform colour space, perceived differences should be represented by equal constant distances along all directions in order to represent the same value of colour difference. In ellipse terms, that is constant lengths of semi-major axes and a value of unity for the ratio between A/B across the space.

$$\Delta E^2 = g_{11}\Delta a^{*2} + 2g_{12}\Delta a^* \Delta b^* + g_{22}\Delta b^{*2} \qquad (2)$$

In this study, ellipses were fitted in four colour spaces; CIELAB, CAM02-UCS, u'v' and xy chromaticity diagrams. Ellipse's coefficients g_{11}, g_{12} and g_{22} as given in Equation 2 were optimised until the minimum discrepancy in STRESS values between the computed and perceived colour difference was obtained. The equation was adapted to the values of the testing colour difference metric accordingly. The mean ratios between A and B are given in Table 1. For a perfect colour space, the ratio for all ellipses should equal to one, i.e. to be a circle. The mean ratio from all regions represents the performance of local uniformity of the colour space tested. The values showed that the u'v' chromaticity diagram had ratio close to one, indicating the best local uniformity of space among the four tested.

Table 1. Mean ratio between the semi-major and semi-minor axes (A/B)

Mean ratio A/B	CIELAB	u'v'	xy	CAM02-UCS
Grey Background All centers	1.65	1.24	1.72	1.53
Black Background All centers	1.58	1.25	1.60	1.53
Grey Background Colour centers	1.68	1.24	1.70	1.57
Black Background Colour centers	1.65	1.25	1.59	1.58
Grey Background White centers	1.58	1.24	1.77	1.47
Black Background White centers	1.43	1.26	1.61	1.42

Discussion

Comparing experimental ellipses with the MacAdam ellipses

The coloured centres from the MacAdam experiment were compared with the current ellipses against the grey background with the similar specifications as the original MacAdam experiment. The visualization of the results of this comparison is seen in Figure 4. The present ellipses agree in a certain extent with the MacAdam ellipses. They have similar orientation, which radiate from the blue region of the diagram towards the dominant wavelengths. However, they have large discrepancy in shapes, as the current ellipses are more similar among themselves and tend to be more circular.

Comparing experimental ellipses of different luminance

Out of the 16 MacAdam colour centres, 5 were treated at the original luminance of 48 cd/m^2 , while the rest 11 were processed at the reduced luminance of 18.5 cd/m^2. The 5 colour centres of higher luminance are also included in the reduced luminance set. For the neutral white colourcentres, two of them were included in both luminance levels. In Figure 5, the discrimination ellipses of the colour centres that were assessed in both luminance levels are illustrated for the grey background. As it is observed, the ellipses for the MacAdam coloured centres are larger for the higher luminance level of 48 cd/m^2. The same effect was less visible for the colour centres assessed against the black background. This finding agrees well with the ellipse factor equation (ESF) derived by Luo *et al.* from their extended research in colour discrimination ellipses [10]. The ESF describes how the ellipse size icreases in size with the increase of luminance Y; even though, the trend was clear for luminance Y under 50.

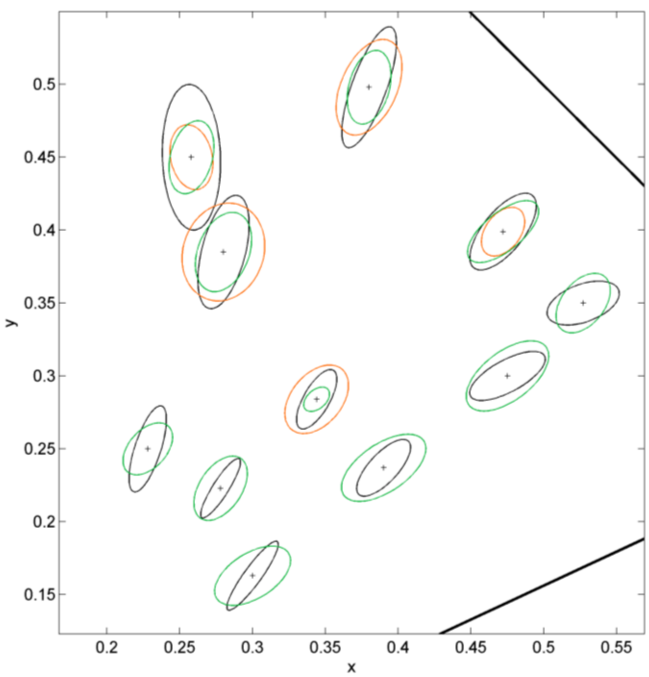

Figure 4. Experimental ellipses against MacAdam ellipses; ellipses in orange for luminance of 48 cd/m^2; ellipses in green for luminance of 18.5 cd/m^2

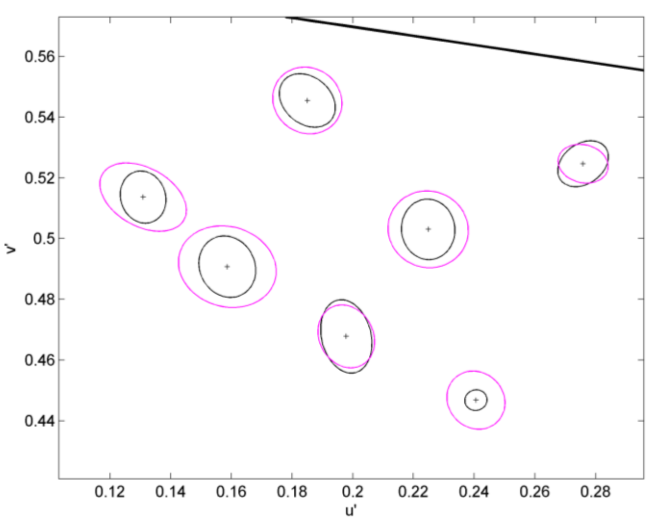

Figure 5. Colour discrimination ellipses of colour centers assesessed in both luminance levels represented in u'v' chromaticity diagram for the grey background. Ellipses in magenta are of luminance 48 cd/m^2, ellipses in black are of luminance 18.5 cd/m^2.

Testing colour difference metrics

The performance of various colour difference metrics used for the ellipse fitting was also examined. Table 3 gives the results for the four colour spaces plus the FMC-1 formula in terms of STRESS units. Originally, CIELAB and CAM02-UCS were chosen because they represent spaces for surface colours, while CIELUV and FMC-1 were chosen for representing spaces for additive light stimuli. The latter formula was specially developed to fit the MacAdam data.

It can be seen that CIELUV markedly outperformed the others. Similar findings were also found by Luo *et al* for only white light stimuli[5]. Furthermore, the results for the black background data gave more accurate predictions than the ones for the grey background. This agrees with the finding while comparing ellipse data; i.e. the sizes of ellipses are larger for the grey background. This implies that observers are more sensitive while evaluating stimuli against the black background. However, it was also found that CAM02-UCS gave very close results to CIELUV for the grey background data. The above indicate that CAM02-UCS is a better choice of space for evaluating surface colours than emissive media. Figures 6 and 7 plot the experimental ellipses of both backgrounds in CIELUV and CAM02-UCS respectively. It can be seen that CIELUV performed better than CAM02-UCS, i.e. their ellipses are close to a constant sized circle than those of CAM02-UCS's.

Table 3. The performance of colour difference metrics in STRESS units

Background	xy	CIELAB	CIELUV	CAM02-UCS	FMC-1
Grey	36	35	23	24	26
Black	29	28	20	30	26

It should also be addressed that the outperformance of the u'v' chromaticity diagram does not relate with having the sampling process performed at the same diagram. Multiple studies have shown that CIELAB did not perform better than other tested metrics, although the data sampling was based on the same colour space [7, 8, 10, 11].

Additionally, the data were also tested using FMC-1 formula which was developed to fit the original MacAdam data [12]. The performance of the metric with the current experimental data did not predict colour difference accurately as it would be expected. This also supports the fact that the current data do not agree very well with the MacAdam data. Therefore, u'v' chromaticity diagram and CIELUV formula could be used for the evaluation of both coloured and white light stimuli as an easy and effective tool to test chromaticity differences. Further work should also be conducted to use real light luminaires.

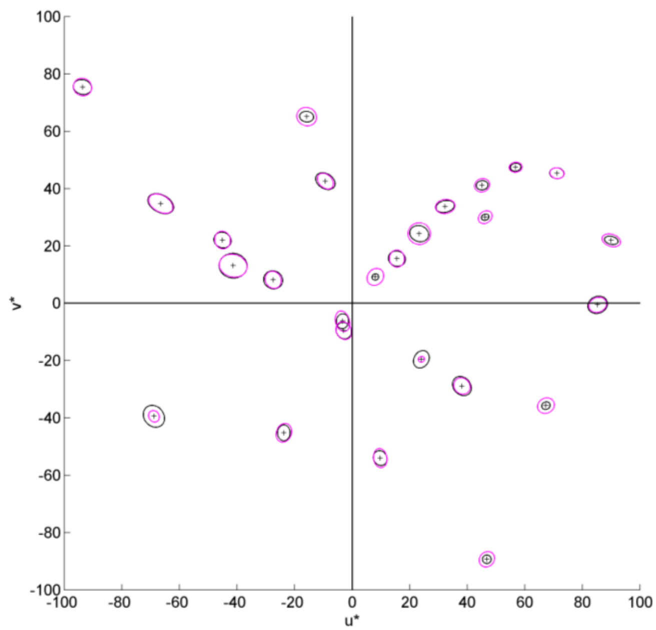

Figure 6.CIELUV colour discrimination ellipses plotted for the experimental data; ellipses in magenta are for the grey background, ellipses in black are for the black background.

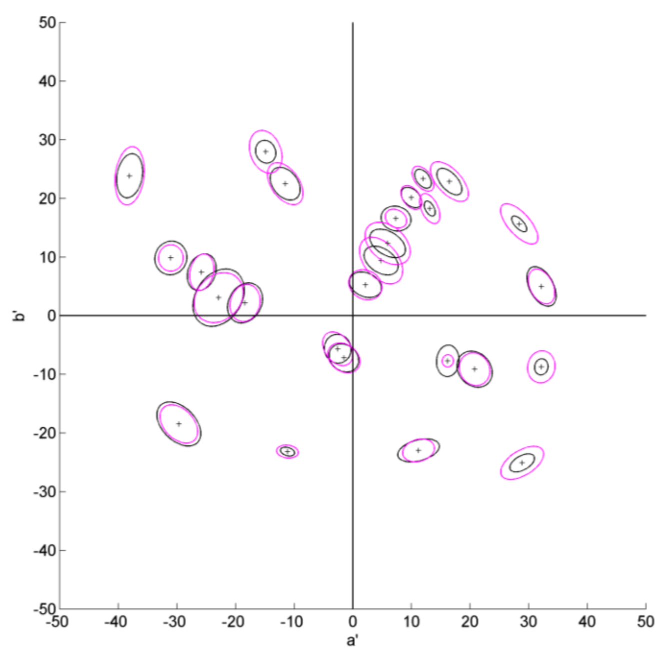

Figure 7.CAM02-UCS colour discrimination ellipses plotted for the experimental data; ellipses in magenta are for the grey background, ellipses in black are for the black background

Conclusion

An experiment to assess colour differences of chromaticity for lighting stimuli was carried out for evaluating colour differences of light stimuli. The colour centres were similar to MacAdam's original work. The results showed that the present results are different from MacAdam's in terms of colour discrimination ellipses and the colour difference metrics tested. This difference could be due to the experimental conditions used, the arrangement of samples and the number of observers. CIELUV and u'v' chromaticity diagram were found to outperform the ones tested. Therefore, it strengthens their definition as tools for evaluation of additive stimuli. However, further work is required to evaluate these results by using the real light luminaires.

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