Robust Color Extrapolation with Median Matrices

Nathan Moroney, Ingeborg Tastl and Melanie Gottwals
HP Labs, Palo Alto, CA, USA

Abstract

Color extrapolation is the estimation of color coordinates or transforms for values that lie beyond the sampled colors or training data. For example given a chart of measured color values and a digital image of that chart it is useful to be able to extrapolate values that are beyond the color samples provided by the chart. One option is to use linear multivariate regression based on a sampling of nearby points. This will result in a matrix transform which can be used for extrapolation. This abstract proposes the derivation of a median matrix based on a sampling of nearby points. That is given random triplets of points a closed form inverse of the first order polynomials is used to directly compute matrix elements. The final matrix is determined by the median of the individual elements. The median matrix extrapolation is shown to be more accurate than conventional multivariate regression, more robust to noise, does not require linear algebra, and can potentially be applied to streaming data.

Introduction

There are multiple cases in color image processing in which a transformation between color encodings results in a corresponding change in the domain or range of the data. One familiar instance, is a transformation between two device-referred renderings and the need for corresponding processing for any mismatch in the device gamuts. However in some cases the gamut mismatch is not the central consideration and instead it is useful to have a means to perform an isolated extrapolation from one encoding or color space to valid encodings in a second color space.[1-6] Three specific examples will be discussed in this abstract and one of the examples will be explored in detail.

In summary form, the three color extrapolation applications are color capture using test charts of varying gamut size, color extrapolation between two color orders systems with differing ranges of physical samples and a synthetic test of extrapolating from a subset gamut of a single device to a complete gamut for that device using only extrapolation.

For the example application using a known sampling of color values, for example a camera color chart, we begin by measuring the patches with a spectrophotometer or colorimeter. The resulting CIELAB values of each patch are therefore the ground truth for a colorimetric capture of that chart. Next a digital image is taken of the chart and the corresponding red, green and blue or RGB values are determined for each of the patches. For the results shown below as an example the CIELAB and RGB values are computed using the sRGB transformations.[4,7] This allows the direct computation of the values without any measurement or capture noise. The closed form forward and inverse transformations also provide a means to directly compute ground truth data or to benchmark any training data. This transform is not required but is used to explore the performance of the algorithm.

Given an array of CIELAB data and RGB data, the next step is the selection of a color of interest in the image that is not a chart color. If this color falls within the gamut of the test chart some form of interpolation can be used to estimate the corresponding CIELAB value. However, if the color of interest falls outside of the gamut of the chart it would be useful to have an accurate, robust and efficient estimation of the corresponding CIELAB value given the captured RGB value. The median matrix can be used for this extrapolation. Note that in this example we are intentionally using a colorimetric capture and processing pipeline. In this case the camera is being assumed to be used as more of a colorimetric imager. This rendering mode is not required for the extrapolation but is consistent with certain use cases and also provides a direct method to assess the performance of the extrapolation.

Median Matrix Extrapolation

The description of the median matrix extrapolation now follows. We begin by selecting the n closest colors in the chart, say 20. The specific number should be adjusted to the characteristic of the data set (linear versus non-linear) and whether a more local or global color transformation is desired. The next step is an iteration process for a given number of steps, for example 200, during which three different colors are randomly selected from the closest n points. The number of iterations needs to be large enough to approximate the results that would be obtained by using all the possible triplets of the n closest points. Computing all of triplets would obviously not be efficient, especially for large numbers of n.

This process is similar to the pairwise selection of points using the Thiel-Sen estimator for robust regression. For robust regression pairs of points are randomly selected and the corresponding slope is computed from the inverse of the two-point form of a line. The median of the computed slopes is then the estimate of the slope. The offset can then be fit given the slope or also by taking the median of the offsets. A multivariate version of the Thiel-Sen estimation process has been proposed and its performance analyzed.[7,8] However these publications do not provide the formulae for direct inverse computation of the terms in the matrix, as shown in the equations below. Likewise the previous analysis of performance is more general and not specific to the topic of color extrapolation.

Continuing with the description of the calculations, for each random selection of three points, the following first order polynomials can be used to compute a given row in a corresponding matrix relating the CIELAB and RGB data.

$$a = (bx_1) + (cy_1) + (dz_1) \quad (1)$$

$$e = (fx_1) + (gy_1) + (hz_1) \quad (2)$$

$$j = (kx_1) + (my_1) + (nz_1) \quad (3)$$

The x, y, and z data are one row of the matrix and the other parameters are color data from the chart or sampling. The above three equations can be directly inverted in closed form as follows:

$$\boldsymbol{x_1} = -\frac{-(\mathbf{dgj}) + \mathbf{chj} + \mathbf{dem} - \mathbf{a\ m} - \mathbf{cen} + \mathbf{agn}}{\mathbf{dgk} - \mathbf{chk} - \mathbf{dfm} + \mathbf{bhm} + \mathbf{cfn} - \mathbf{bgn}} \quad \textbf{(4)}$$

$$\boldsymbol{y_1} = -\frac{\mathbf{dfj} - \mathbf{bhj} - \mathbf{dek} + \mathbf{ahk} + \mathbf{ben} - \mathbf{afn}}{\mathbf{dgk} - \mathbf{chk} - \mathbf{dfm} + \mathbf{bhm} + \mathbf{cfn} - \mathbf{bgn}} \quad \textbf{(5)}$$

$$\boldsymbol{y_1} = -\frac{\mathbf{dfj} - \mathbf{bhj} - \mathbf{dek} + \mathbf{ahk} + \mathbf{ben} - \mathbf{afn}}{\mathbf{dgk} - \mathbf{chk} - \mathbf{dfm} + \mathbf{bhm} + \mathbf{cfn} - \mathbf{bgn}} \quad \textbf{(6)}$$

Where x, y, and z are the matrix elements for a given row. The a, e, and j values are L^*_1, L^*_2 and L^*_3 coordinates. The b,c,d; f,g,k; and k,m,n triples are R_1,G_1,B_1; R_2,G_2,B_2; and R_3,G_3,B_3 coordinates. The computations for x, y and z are repeated two additional times with the a, e, and j values as a^*_1, a^*_2, and a^*_3 and finally as b^*_1, b^*_2, and b^*_3. The x, y, and z values for the set of triplets (200 in the given example) are accumulated and the median is determined per element in the matrix. These median values are then multiplied by the color of interest to determine the extrapolated CIELAB value given the input RGB value. For streaming data, the median matrix can be computed using maximum and minimum heaps.

Experiments

This algorithm allows the extrapolation of color values beyond the range of the sampled or training data. This is useful for measurement purposes, such as the mobile color sensing. It is also generally useful for other extrapolation applications and could also be used for in-range regression but this will not be considered in detail in this abstract. Note that for measurement extrapolation, only individual pixels are processed so smoothness or strict monotonicity requirements are less of an issue than for an imaging application of extrapolation.

To demonstrate the performance of the algorithm, a 75% of sRGB coarse sampling of data (where minimum RGB is 32 and maximum is 223 digital counts) three techniques were used to estimate surface gamut colors using three methods. First a weighted clamping of the distances of the nearest 20 points. The weight is the distance to the input point and as a clamping operation is not, strictly speaking, a form of extrapolation. This weighted clamp is included as a simple benchmark for a previously implemented option for video rate color extrapolation. That is it is a fast reference but not particularly accuate.

A second approach to processing of this data is a matrix based linear regression using the 20 closest points. This corresponds to the familiar matrix formulation of regression or $(X^T X)^{-1} X^T Y$. The standard linear multivariate regression provide by the R language was used in this case. Finally the median 3 by 3 matrix was used. The resulting ΔE^*_{ab} values for the test 98 points are listed in Table 1.

Table 1. Mean and standard deviation for the CIELAB 1976 color differences for three color extrapolation techniques.

Method	Mean (ΔE^*_{ab})	Standard Deviation (ΔE^*_{ab})
Weighted Clamp	17.8	7.4
Linear Regression (R)	6.1	5.2
Median 3x3	5.7	4.6

The median 3 by 3 has among the smallest mean errors and standard deviations. Adding 30% uniform white noise to the 216 point training data results in the ΔE^*_{ab} values for the 98 test points shown in Table 2.

Table 2. Mean and standard deviation CIELAB 1976 color differences for three extrapolation techniques with the addition of uniform white noise.

Method	Mean (ΔE^*_{ab})	Standard Deviation (ΔE^*_{ab})
Weighted Clamp	22.9	10.0
Linear Regression (R)	14.2	9.7
Median 3x3	6.9	7.0

With 30% noise in the training data, the median 3 by 3 has the smallest mean error and standard deviation. The mean accuracy for the linear regression using R is now over twice as large as for the previous results. The median matrix is considerably more robust than the conventional multivariate regression.

The results in visual form are shown in Figure 1 below. The left image is for the color extrapolation shown in the Table 1 and the right image is for the color extrapolation with noise shown in Table 2. The leftmost column for each image is the ideal surface gamut colors with zero error as computed using the sRGB transforms. The second column is the weighted clamp results and the third column is the multivariate linear regression. The rightmost or fourth column is the median 3x3 results. This provides a direct visualization of the ΔE^*_{ab} summary statistics shown in the above tables. The median matrix is an accurate and robust color extrapolation technique. The greater visual noise seen in the linear regression is not entirely surprising given that as outliers are included in the training data, the extrapolated results also become less consistent.

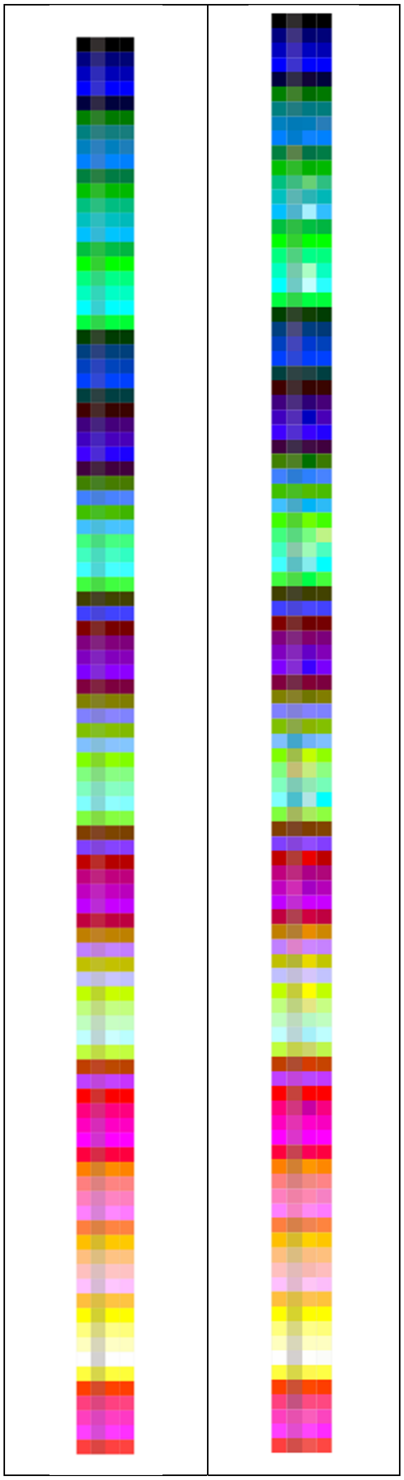

__Fig. 1.__ Color visualization of the extrapolation errors for no noise, left, and thirty percent uniform white noise, right. The leftmost column of each is ideal surface gamut color, second column is weighted clamp, third column is multivariate regression and rightmost row is column 3x3 results.

Discussion

Note that in this previous example the true relationship between RGB and Lab is a polynomial of the order of three (corresponding to the analytical transformation from sRGB to Lab) and the relationship is the same across the color space. Thus, in using a 3x3 matrix for both the multivariate regression and the median 3x3 matrix the modeling performs an approximation that can't result in a zero error. Having said that, the median 3x3 matrix achieves a useful approximation.

The general idea of this abstract can also be expanded to higher order polynomials or n by m matrices where n and m are larger than 3. Advantages and disadvantages of using higher order polynomials haven't yet been explored. Another advantage of the proposed method is the possibility to tune it either towards a more locally defined (lower number of closest points) or a more globally defined method (n is closer to the number of points defining the color transformation). In addition, these results should be compared with regression with some form of regularization, such as ridge regression, Lasso regression or with the addition of ghost points. This analysis is ongoing will be included with the final manuscript. In general regularization will also improve the regression results but requires a more complex analysis, fitting and implementation.

A second major area of research is the application of the median matrices color extrapolation to a colorimetric imaging system. This system can use 4 and 6 color printed test charts for the purpose of real-time color interpolation and extrapolation. We are actively investigating the trade-offs with using 4 color charts and the previously described color extrapolation. Finally we are looking at how this technique can be used to transform colors across color order systems. For instance given the Munsell Book of Color and the Uniform Color Scales of the Optical Society of America how the color samples at the extremes of one color order system be used to estimate the corresponding samples in the second system. Unlike the more general issue of designing a single transformation between the systems this approach could be useful for extrapolation of a smaller number of critical colors in a locally fitted manner. Results for these two applications will be part of the final paper.

Conclusions

A simple, computationally efficient and robust technique for color extrapolation was described. The calculation of a median matrix based on randomly sampling triples of data is central to this technique. The performance of the extrapolation was demonstrated for a test case in which a gamut reduced sRGB sampling of colors was extrapolated to the full gamut using several techniques. The median matrices was shown to be accurate and robust in the presence of noise. Two additional applications and topics of current investigation were also described in summary form.

References

[1] Bhachech, M., Shaw, M. and DiCarlo, J. (2006) Improved Color Table Inversion Near The Gamut Boundary, 14th IS&T/SID Color Imaging Conference, Scottsdale, AZ, 44–49.

[2] http://www-umlauf.informatik.uni-kl.de/~umlauf/Papers/extrapolation.pdf , “Natural Neighbor Extrapolation Using Ghost Points”.

[3] Tibshirani, R. "Regression shrinkage and selection via the lasso". Journal of the Royal Statistical Society, Series B 58 (1): 267–288. (1996).

[4] Marguier, Joanna, et al. "A home décor expert in your camera." Proc. Color and Imaging Conference, Society for Imaging Science and Technology, (2009).

[5] IEC 61966-2-1:1999 & Amendment 1 to IEC 61966-2-1:1999, “Multimedia Equipment – Color measurement and management - Part 2-1: Colour Management – Default RGB colour space –sRGB

[6] H.Kang: “Color Technology for Electronic Devices”, SPIE, (1997).

[7] Dang, Xin, et al. "Theil-Sen Estimators in a Multiple Linear Regression Model." (2008).

[8] Lia, Fang, Hanxiang Penga, and Fei Tanb. "The Asymptotics of the Mahalanobis Depth-Based Theil-Sen Estimators in Multiple Linear Models." (2009).