A Spectral-based Color Vision Deficiency Model Compatible with Dichromacy and Anomalous Trichromacy

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Abstract

This paper proposes a spectral-based CVD (*Color Vision Deficiency*) *model compatible with both of dichromacy and anomalous trichromacy. The spectral projection model based on Matrix-R extracts the lost spectra as a difference in the fundamentals between the normal and the color deficient. The lost spectra are re-used for image daltonization by an optimal spectral shift to maximize the spectral visibility or minimize the visual gap from the normal. The model rationally improves the scene visibility after daltonization. The proposed algorithm is designed based on the original key ideas of*

- *1) Acquisition of fundamental C*LMS* (*spectra visible to the normal*) *from sRGB camera image by a pseudo-inverse projection without expensive spectral image.*
- *2) Foundation of projection matrix-RCVD onto dichromatic and anomalous trichromatic spectral spaces by combining the cone responses in the table by DeMarco&Pokorny& Smith.*
- *3) Extraction of fundamental C*CVD* (*spectra visible to the dichromat or anomalous trichromat*) *by operating the matrix-RCVD on the fundamental C*LMS.*
- *4) Introduction of complete OCS* (*Opponent-Color Space*) *to keep the perfect achromatic grayness in the opponent-color stage.*
- *5) Estimation of lost spectra C*CVD as the difference between visible spectra* C^* *LMS to the normal and* C^* *CVD to the dichromacy or anomalous trichromacy.*
- *6) Color blindness correction* (*daltonization*) *by reviving the lost spectra C*CVD with the optimal spectral shift into the visible waveband.*

Introduction

Typical Color Vision Deficiency (CVD) is classified to dichromacy and anomalous trichromacy. Though dichromacy is considered to be caused by the lack (empty space) or absence (empty cone) or substitution of one of the LMS photopigments, the cone substitution is a dominant opinion [9]. However, in reality, anomalous trichromacy is the majority. It's explained by a spectral shift in the cones caused by arrangement of exon DNA in X-chromosome. The first CVD simulator is modeled by

Brettel-Vie'not-Mollon [1][2][3] to find the corresponding color pairs between the reduced dichromatic 2D plane and the normal 3D color gamut. The troublesome corresponding pair procedure is simplified by a systematic color transform approach by Capilla et al [4] and advanced to 2-step model including opponent-color stage by Pardo and Sharma [5]. The color-blind simulators based on Brettel are widely accepted, but any of them didn't step into the spectral analysis on what spectra are visible or invisible and how they are lost. The author proposed a spectral-based dichromatic CVD model [6][7] based on Matrix-*R* theory. The model clarified for the first time what spectra are visible or invisible to the dichromats and which parts are lost from the fundamentals in normal vision. The lost spectral analysis first interpreted the reason behind the *red-green opponent-color* blindness.

The anomalous trichromatic vision is modeled by a simple spectral shift by Yang et al [8] or Machado et al [9] but without dealing spectral losses. This paper proposes a versatile model compatible with both of dichromacy and anomalous trichromacy. The paper presents the spectral daltonization algorithm for correcting the color blindness, which is designed to maximize the visibility for the confusing colors and minimize the visual spectral gap from the normals based on the mathematical evaluation criterion.

Spectral Projection to CVD Space

Visible, Invisible & Lost Spectra based on Matrix-R

Based on *Matrix-R* theory by J. B. Cohen [10], *a* spectral input *C* is decomposed into the *fundamental* C_{LMS}^* and the *metameric black B* through the projection matrix *RLMS* as

$$
\mathbf{C} = \mathbf{C}_{LMS}^* + \mathbf{B}, \ \mathbf{C}_{LMS}^* = \mathbf{R}_{LMS} \mathbf{C}, \ \mathbf{B} = (\mathbf{I} - \mathbf{R}_{LMS}) \mathbf{C}
$$

$$
\mathbf{R}_{LMS} = \mathbf{A}_{LMS} (\mathbf{A}_{LMS}^t \mathbf{A}_{LMS})^{-1} \mathbf{A}_{LMS}^t \ \text{for} \ \mathbf{A}_{LMS} = [l(\lambda) \ m(\lambda) \ s(\lambda)]^{(1)}.
$$

The fundamental C_{LMS}^* denotes the visible spectra to normal vision,

while **B**is by-passed and lost as an invisible spectrum with zero tristimulus value. While, the spectra visible to the color deficient are much more lost through the matrix *RCVD* corresponding to each type of CVD as shown in Figure 1 (in case of *Protanopia*).

Figure 1 What color spectra are visible and lost to the color vision deficient?

Projection Matrices onto Spectral CVD Spaces

In this paper, typical four types of color deficiencies are simulated with *LMS* cone spectral sensitivities for normal and anormalous trichromat [11] (DeMarco&Pokorny& Smith, 1992). The projection matrix *RCVD* for each color deficient is created from a set of dichromatic or anomalous trichromatic cone responses in Figure 2 as given by Eq. (2). The symbols *L, M, and S* denote the cone responses for normal vision with the spectral sensitivities of $l(\lambda \Box, \mathcal{A} \Box, \text{ and } s(\lambda \Box, \text{ while } L_a \text{ and } M_a \text{ mean those for anomalous})$ *L* and *M* cones with the spectral sensitivities of $l_a(\lambda \Box$ and $a(\lambda \Box$.

 $R_{\text{CVD}} = A_{\text{CVD}} (A_{\text{CVD}}^t A_{\text{CVD}}^{-1})^{-1} A_{\text{CVD}}^t$: $= R_p$ or R_p or R_{aP} or R_{aP} A_{CVD} := A_p *or* A_p *or* A_{ap} *or* A_{ap} $A_P = \frac{m(\lambda) s(\lambda)}{f}$ *or protanopia* $A_D = L(\lambda)$ s(λ) for deuteranopia $A_{aP} = \left[l_a(\lambda) \; m(\lambda) \; s(\lambda) \right]$ for protanomaly $A_{aD} = \left[l(\lambda) \, m_a \, s(\lambda) \right]$ for deuteranomaly (2)

Figure 2 Matrix-RDEF projection operators onto spectral CVD space.

Simulation Model

Versatile CVD Model for Dichromat and Anomalous Trichromat

 The proposed model simulates the CVD appearance including the spectral-based daltonization and gray balance process in OCS. The model unifies the dichromatic and anomalous trichromatic vision by just selecting the matrix *RCVD*. First, the *fundamental C* LMS*, visible spectra to normal, are captured with sRGB camera and projected to the *fundamental* C^* _{*CVD*} visible to the deficient. Then, taking the difference in C^*_{LMS} and C^*_{CVD} , the lost spectra AC^* *CVD* are calculated. Next, the lost spectra AC^* *CVD* are re-used to daltonize the confusing color's visibility by the optimal spectral shift. Finally, the simulated CVD *fundamental* image is displayed on sRGB monitor through the inverse transforms of (spectral to *LMS*), (*LMS* to *OCS*) and (*OCS* to *sRGB*). The achromatic grayness is kept in the *OCS process* inserted in the pathway of Figure 3.

Getting Fundamental from sRGB Camera Image

Since the *fundamental* C^* _{*LMS*} (spectra visible to normal) carries the tristimulus value T_{LMS} as same as the input spectrum C , it's exactly recovered by the *pseudo-inverse transform* [12](Kotera, 1996) from *TLMS*. Hence, the actual scene spectrum visible to normal vision is obtained from *sRGB* image without using expensive spectral camera as follows.

$$
C_{LMS}^* = P_{INV} T_{LMS} , where P_{INV} = A_{LMS} (A_{LMS}^t A_{LMS})^{-1}
$$

= $P_{INV} (M_{XYZ \rightarrow LMS}) (M_{sRGB \rightarrow XYZ}) sRGB_{IMG}$ (3)
where, $M_{P \rightarrow Q}$ denotes P to Q transform matrix

Where, *sRGBIMG* denotes the linearized (de-gamma) *sRGB* image from coded non-linear *sR'G'B'*.

Figure 3. Spectral CVD model compatible with dichromacy and anomalous trichromacy (*case: Deuteranomaly*)*.*

Lost and Visible Spectra to Color Vision Deficient

The fundamental C^*_{CVD} (spectra visible to CVD) is obtained by operating the matrix *RCVD* on the trichromatic fundamental C^* *LMS*. Now the lost spectra AC^* *CVD* from the normal vision is given by taking the difference between C^* _{*LMS*} and C^* _{*CVD*} as

$$
C_{CVD}^{*} = R_{CVD}C = R_{CVD}C_{LMS}^{*}
$$

\n
$$
\Delta C_{CVD}^{*} = C_{LMS}^{*} - C_{CVD}^{*} = (R_{LMS} - R_{CVD})C_{LMS}^{*}
$$
 (4)

Revive Lost Spectra for Daltonizing CVD Visibility

Though the lost spectra ΔC^* *CVD* are invisible if left as it is, we can revive it for daltonizing the confusing color visibility by shifting its distribution into the visible wavelength region. ΔC^* _{CVD} is shifted by λ_{SHT} in a manner of rotate-left and added to the original *fundamental C* LMS*.

Thus the daltonized *fundamental DALC* LMS* is given by

$$
{\scriptscriptstyle DAL} C^*{\scriptscriptstyle LMS}(\lambda) = C^*_{\scriptscriptstyle LMS}(\lambda) + \Delta C^*_{\scriptscriptstyle CVD}(\lambda - \lambda_{\scriptscriptstyle SHT})
$$
 (5)

The optimal shift wavelength λ_{OPT} is determined to maximize the total evaluation function $\Psi_{OPT}(\lambda_{SHT})$. The function $\Psi_{OPT}(\lambda_{SHT})$ is defined by combining the spectral fitness function $\Psi_{FIT}(\lambda_{SHT})$ and the spectral difference function $\Psi_{\text{DIF}}(\lambda_{\text{SHT}})$ to and from the normal vision as follows [7] (Kotera, 2012).

$$
\lambda_{OPT} = \lambda_{SHT} \text{ for } \Psi_{OPT}(\lambda_{OPT}) = \max_{\lambda_{SHT}=0}^{\lambda_{max}-\lambda_{min}} \{ \Psi_{OPT}(\lambda_{SHT}) \}
$$
\n
$$
\Psi_{OPT}(\lambda_{SHT}) = w\Psi_{FIT}(\lambda_{SHT}) + (1-w)\{1-\Psi_{DIF}(\lambda_{SHT}) \}
$$
\n
$$
\Psi_{FIT}(\lambda_{SHT}) = \sum_{j=1}^{J} \left\| \left\{ \Delta \mathbf{C}_{CVD}^*(\lambda - \lambda_{SHT}), g_j \right\} \right\|^2
$$
\n
$$
\Psi_{DIF}(\lambda_{SHT}) = \sum_{j=1}^{J} \left\| \left\{ \Delta \mathbf{C}_{DIF}^*(\lambda - \lambda_{SHT}), g_j \right\} \right\|^2
$$
\n(7)

where,
$$
\Delta \mathbf{C}_{\text{DIF}}^*(\lambda) = (\mathbf{R}_{\text{LMS}} - \mathbf{R}_{\text{CVD}})_{\text{DAL}} \mathbf{C}_{\text{LMS}}^*(\lambda)
$$

Here, $\Psi_{FIT}(\lambda_{SHT})$ and $\Psi_{DIF}(\lambda_{SHT})$ are estimated by summing up the power spectra (squared norm) for all the pixels g_j ($j=1,\sim J$) in *sRGB* image as given by Eq. (7).

The weighting factor w is adjusted to maximize the visibility for the confusing colors $(w=1)$ or minimize the visual spectral gap (*w*=0) from the normals when the daltonized image is viewed by the color deficient.

Very fortunately, in many cases for dichromats, the function $\Psi_{FIT}(\lambda_{SHT})$ takes its maximum value around $\lambda_{SHT} = \lambda_{OPT}$ and simultaneously the function $\Psi_{\text{DIF}}(\lambda_{\text{SHT}})$ goes down to a quasiminimum point around the same wavelength.

The proposed algorithm dramatically improved the image visibility and naturalness after daltonization with default weight *w*=0.5 much better than that by the most popular simulator Vischeck (Brettel model).

(See Figure 4: sample "wild strawberries" for *Deuteranopia*).

Achromatic Grayness in Opponent-Color Process

Following the cone response in **stage-1**, the *LMS* stimulus is encoded to a luminance-chrominance YC_RC_B signals in the next opponent-color **stage-2**. Since the opponent-color system is thought to work for the color deficient as well as the normal, the *YCRCB* signals should keep the achromatic grayness. Because a simple anomalous trichromatic simulation model [8](S. Yang et al, 2008) only with **stage-1** causes a coloring problem for the neutral gray input, the *opponent-color process* in **stage-2** is *a key to guarantee the achromatic grayness* for all of CVDs as pointed by [9] (Machado et al, 2009). But, the OCS (Ingling and Tsou, 1977) used in the Machado's model doesn't always keep the achromatic grayness for *CB* (*Y-B*)channel due to imperfect *LMS* to $\gamma C_R C_B$ transform matrix. Unlike the Machado's OCS, a complete orthonormal OCS [13] (Kotera, 2014) is introduced.

Assuming *EE* (Equal-*Energy*) white spectrum input $W_{EE}(\lambda)$ to the normal vision, the visible spectrum $_{EE}C^*$ _{*LMS*}, its *LMS* cone response T_{LMS} , and corresponding *opponent-color* signals YC_RC_B should satisfy the achromatic grayness of $(Y=1, C_R=C_B=0)$ as

Figure 4: Daltonization for correcting CVD visibility by reusing lost spectra

$$
\begin{aligned}\n\left[Y \ C_R \ C_B \right]_{EE}^t &= M_{LMS \to YCC} \cdot_{EE} T_{LMS} = \left[1 \ 0 \ 0 \right]^t \\
&= \left[L \ M \ S \right]_{EE}^t \ A_{LMS} \cdot_{EE} C_{LMS}^t(\lambda) \\
& \therefore LMS \ tristimulus \ value \ for \ EE \ white \\
&= \left[C_{LMS}^t(\lambda) \right] = R_{LMS} C_{EE}^t(\lambda) = R_{LMS} W_{EE}(\lambda) \\
& \therefore \ fundamental \ (EE \ spectrum \ visible \ to \ normal) \\
&= \left[1 \ 1 \ 1 \ \cdots \cdots \ 1 \right]^t \colon EE \ white \ spectrum\n\end{aligned} \tag{8}
$$

The linear transform matrix $M_{LMS \to YCC}$ to meet this condition is obtained by coupling *MLMS*→*XYZ* (CIECAMO2) with *MXYZ*→*^L MS* in the Complete OCS [13] (Kotera, 2014) as

$$
M_{LMS \to YCC} = (M_{XYZ \to YCC})_{Kotera} (M_{LMS \to XYZ})_{CIECAMO2}
$$

=
$$
\begin{bmatrix} 0.489 & 0.134 & 0.378 \\ 1.004 & -0.790 & -0.214 \\ 0.034 & -0.472 & 0.437 \end{bmatrix}
$$
 (9)

Indeed, *YCRCB* values to *EE* white for normal is verified to keep the perfect grayness as

$$
\begin{aligned} \left[Y \ C_R \ C_B \right]_{EE}^t &= M_{LMS/YCC} \cdot A_{LMS} W_{EE} \left(\lambda \right) \\ &= \left[1.00 \ -8.3 \times 10^{-17} \ -1.1 \times 10^{-16} \right]^t \cong \left[1 \ 0 \ 0 \right]^t \end{aligned} \tag{10}
$$

Since the spectra $_{EE}C^*$ *CVD* visible to the color deficient for EE white are lost by ΔC^* *CVD* against normal vision, its *LMS* or *YCRCB* responses deviate from $[1\ 1\ 1]^t$ or $[1\ 0\ 0]^t$ as given by

$$
_{DEF}\left[Y \ C_R \ C_B\right]_{EE}^t = M_{LMS \to YCC} \cdot_{EE} T_{DEF}
$$
\n
$$
{EE}T{DEF} = \left[L \ M \ S\right]_{E\overline{E}}^t A_{DEF} \cdot_{EE} C_{DEF}^*(\lambda)_{EE}
$$
\n
$$
C_{DEF}^*(\lambda) = R_{DEF} R_{LMS} W_{EE}(\lambda)
$$
\n(11)

Actually, the deviations from $YC_RC_B = [1 \ 0 \ 0]$ are estimated for each type of deficient as

[*Y CRCB*]*EE=*[0.912 -0.180 -0.006] (protanopia)

[*Y CRCB*]*EE*=[1.005-0.032 -0.019] (deuteranopia)

[*Y CRCB*]*EE=*[1.0050.0120.001] (protanomaly)

[*Y CRCB*]*EE=*[1.002-0.013 -0.008] (deuteranomaly)

Hence, the correction matrices to guarantee the achromatic grayness are inserted in the route between (*LMS* to *YCRCB*) and (*YCRCB* to *sRGB*) transforms in Figure 3*.*

The corrected transform matrices to meet the achromatic grayness corresponding to the normal $M_{LMS \to YCC}$ in Eq. (9) are calculated as

$$
M_{LMS\rightarrow YCC} = \n\begin{bmatrix}\n0.596 & 0.134 & 0.378 \\
1.224 & -0.790 & -0.214 \\
0.042 & -0.472 & 0.437\n\end{bmatrix},\n\begin{bmatrix}\n0.489 & 0.128 & 0.378 \\
1.004 & -0.759 & -0.214 \\
0.034 & -0.453 & 0.437\n\end{bmatrix}
$$
\n(Protanopia)

\n(Deuteranopia)

\n(12)

\n
$$
\begin{bmatrix}\n0.484 & 0.134 & 0.378 \\
0.993 & -0.790 & -0.214 \\
0.034 & -0.472 & 0.437\n\end{bmatrix},\n\begin{bmatrix}\n0.489 & 0.132 & 0.378 \\
1.004 & -0.778 & -0.214 \\
1.004 & -0.778 & -0.214 \\
0.034 & -0.464 & 0.437\n\end{bmatrix}
$$
\n(Protanomaly)

\n(Deuteranomaly)

The *YCRCB* opponent-color basis functions for each type of CVD are illustrated in Figure 5. These make it possible for the color vision deficient to view the *EE* spectrum as a complete achromatic white just as same as normal vision.

Figure 5. YC_RC_B basis functions for CVD in opponent-color stage-2

Simulated Results

Dichromatic Vision and Daltonization Effect

The simulations for the *dichromat* are compared with typical color blindness simulators including some daltonization effects in Figure 6. The corrected image denotes the CVD visual appearance viewed after daltonization. Though the simulated CVD appearances look to be much the same across the models, the proposed daltonization function clearly worked better than others for rescuing the dichromatic blindness. Though Google Chrome works well for daltonizing the *reddish color blindness* to Protanopia in Fig. 6 (a), the corrected image is much apart from the daltonized. While, proposed model works to maximize the *reddish color* visibility and minimize the visual spectral gap between the daltonized and corrected images. As clearly shown, the lost image exactly corresponds to the *reddish color blindness* as reflected on the lost spectra and the visual gap between the daltonized and corrected images is dramatically reduced after daltonization as shown in the difference image. The similar effect by the proposed model is illustrated for daltonizing the typical *red-green blindness* to *Deuteranopia* as compared with Vischeck in Figure 6 (b).

 As a daltonization technique, automatic re-coloring algorithms have been proposed to enhance the accessibility for CVDs. A re-coloring effect [14] (Huang et al, 2009) to keep the contrast between each pair of selected key colors is compared with the proposed model in Figure 6 (c). In case of "Berries", the corrected protanopia view after re-coloring is *less visible* for the *reddish fruit* than that by the proposed daltonization. While, In case of "Flower", the re-coloring effect looks to be *nice for* discriminating the *orange petal* from the greenish leaves. However the corrected bluish petal by re-coloring looks unnatural against the original orange color, while the *yellowish petal* after the proposed daltonization may be *rather natural*. The advantage of proposed model lies in its spectral-based reasonable visibility correction depending on each CVD types without any key colors. Though the content-based color appearance assessment is not easy but left behind as a future work.

Simulation for Anomalous Trichromat

The degree of anomalous trichromats are reflected on their spectral shifts from normal and sensitive to the optical density of cones [15], but hard to be standardized. The simulation results for the anomalous trichromats are compared with Yang et al [8] and Machado et al [9] in Fig. 7 (a). Both models simulate the CVD appearance with the spectral shift in LMS cone sensitivities by changing the $\Delta\lambda$ according to the degree of CVDs. The proposed model resulted in much the same visibilities as G. Machado et al in case of $\Delta \lambda = 10$ nm for *Deuteranomaly*. While the result in color

circle image by S. Yang et al looks too colorful and incorrect, because themodel doesn't include the opponent-color **stage-2**. Since the lost spectra for anomalous trichromats are smaller than for dichromats, the daltonization effect is not so distinct in most cases. A daltonized example applied for *Protanomaly* is shown in Fig.7 (b). By setting the weight *w=*0 in Eq. (6), the proposed model works to minimize the visual spectral gap between the daltonized and corrected images. In this sample, the lost spectra ΔC^* *CVD*are amplified by 3 times after optimal spectral shift and added to the original *fundamental C* LMS* for daltonization.

Figure 7. Simulation and daltonization effect for anomalous trichromat

Conclusions

A Spectral-based CVD model developed from dichromacy into anomalous trichromacy. The proposed algorithm, quite different from 2D-3D corresponding pair process based on Brettel's, can estimate the visible, invisible, and lost spectra from a usual sRGB camera image. The proposed model works compatible with both dichromat and anomalous trichromat by just selecting the projection matrix *RCVD* corresponding to the type of CVDs. The model demonstrated how the lost spectrum is revived and used for daltonizing the scene visibility much better than other methods. Though its effects are remarkable for the dichromats, but still insufficient for anomalous trichromats and needs more improvements in the future works.

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