A Complete Opponent-Color Space With Golden Vectors

Hiroaki Kotera; Kotera Imaging Laboratory, Chiba, Japan

Abstract

Oppopnent-color mechanism in the retinal ganglion cell carries the luminance-chrominance transform important to human vision. Though a variety of opponent-color spaces have been proposed, the orthonormality and the achromatic grayness in the basis function are not always guaranteed. This paper discusses a foundation of complete opponent-color space based on the concept of FCS (*Fundamental Color Space*) *derived from Matrix-R theory.*
A complete opponent-color space is constructed by [1] *choosing the Golden Vectors as an orthogonal triplet for FCS,* [2] *replacing its luminance basis by the fundamental of EE spectrum, and* [3] *orthonormalizing the basis functions with GramSchmidt method.*
The fundamental of EE spectrum is bimodal-shaped. This distinct basis makes the mathematical completeness in the opponent-color FCS possible. So far, the Golden Vectors with fundamentals for (λ_1**=455,** λ_2**=513,** λ_3**=584** *nm*) *by J. B Cohen is known to give an ideal orthogonal triplet, but is not an optimal set. The author found a new set of Golden Vectors with the fundamentals for* (λ_1**=461,** λ_2**=548,** λ_3**=617** *nm*) *as the best. A complete opponent-color FCS satisfying both orthonormality and chromatic graynesss is derived from this new Golden Vectors. The paper shows how the proposed opponent-color FCS works well to separate the opponent-color components for natural images and introduces an application to the image color segmentation.*

Introduction

Oppopnent-color mechanism in our retinal ganglion cell plays an important role in the luminance-chrominance transform for visual signal processing. Ideally, the Opponent-Color Space (here we call OCS), as a metric space, should satisfy the mathematical completeness in the orthonormality and the achromatic grayness in its basis function. However, many of existing OCSs don't always satisfy the completeness.

The Locus of Unit Monochromat (LUM) for the basis function spans a FCS (Fundamental Color Space). Though CIE1931 *xyz* color matching function is a common basis for every FCSs, it's not an orthogonal basis. Hence, MacAdam [1] orthogonalized CIE1931 *xyz* and Neugebauer applied it to the *q*-factor [2] for measuring the goodness of color scanning filters. But, the MacAdam's orthogonal *xyz* color matching function can't be an OCS basis.

The OCS model originating from Hering was succeeded by Hurvich, Guth's ATD80 [3] & ATD95, Wandell's $O_1O_2O_3$ [4] and advanced to S-CIELAB [5]. Nevertheless they are not exactly enough to be a complete OCS. For example, since Guth's ATD80 is useful but not orthogonal, Worthey [6] orthonormalized the ATD80 basis and applied it to evaluate the spectral color gamut for digital cameras. In spite of Worthey's effort, the problem that the orthonormality in basis function is not compatible with the perfect achromaticity for gray input is not solved yet but still remained.

J. B. Cohen [7] generalized the concept of FCS based on his *matrix-**R*** theory, where FCS is defined as a color space spanned by an orthonormal basis *matrix-**F*** derived from *matrix-**R***. In the previous paper [8], the author discussed the geometrical structures of typical OCS candidates created from orthonormalized *matrix-**F***. There, an insight into a possibility for constructing a complete OCS is obtained by analyzing the Cohen's "*r-C-v"* FCS [9] but not stepped into nothing more deeply.

Looking back on the above-mentioned, this paper discusses on the mathematical conditions how to choose the basis function for the complete OCS copeing with both orthonormality and achromatic grayness.

The key points lie in the selections of
[1] *an orthogonal triplet, matrix-**E**, as an initial basis for matrix-**F***
[2] *luminance basis in matrix-**E** for creating orthonrmal matrix-**F***

The paper obtained a solution by using
[1] *Golden Vectors* as a *triplet for matrix-**E***
[2] *Fundamental of EE spectrum as a luminance basis*

Fundametal Color Space based on Matrix R

Human Color Vision is characterized by *metamerism*. J. B. Cohen mathematically interpreted *metamerism* with *matrix-**R***, a projector from spectral space to HVSS (Human Visual Sub-Space). and founded the concept of FCS. FCS is spanned by a *matrix-**F*** with a selected triplet from column vectors in *matrix-**R***.

R is a symmetric matrix whose column (or row) vector is composed of the *fundamental* $\boldsymbol{E}_\lambda$ for unit monochromat with wavelength λ. Since the freedom of ***R*** is *Rank* [***R***]=3, any selected triplet $\boldsymbol{E}=[\boldsymbol{E}_1\ \boldsymbol{E}_2\ \boldsymbol{E}_3]$ means the independent components and gives the primary color axes for FCS.

$$\begin{aligned}&\boldsymbol{R}=\boldsymbol{A}(\boldsymbol{A}^t\boldsymbol{A})^{-1}\boldsymbol{A}^t\ ;\quad \boldsymbol{A}=color\ matching\ function,\\&\boldsymbol{R}=\boldsymbol{E}(\boldsymbol{E}^t\boldsymbol{E})^{-1}\boldsymbol{E}^t\ ;\ \ \boldsymbol{E}=[\boldsymbol{E}_1\ \boldsymbol{E}_2\ \boldsymbol{E}_3]\end{aligned}\tag{1}$$

The *matrix-**R*** derived from a color matching function ***A*** is regenerated from the arbitorary selected *matrix-**E***.

Now, the basis of FCS, *matrix-**F*** is given by orthonolmalizing the selected *matrix-**E*** by *Gram Schmidt* . The *matrix-**F*** basically constitutes the basis of *matrix-**R*** as follows.

$$\begin{aligned}&\boldsymbol{F}=GramSchmidt\left[\boldsymbol{E}\right]\\&\boldsymbol{R}=\boldsymbol{F}\boldsymbol{F}^t\end{aligned}\tag{2}$$

Conditions on Perfect Opponent-Color Space

Letting a basis function ***F*** for spanning the OCS be

$$\begin{aligned}&\boldsymbol{F}=[\boldsymbol{F}_1(\lambda)\,\boldsymbol{F}_2(\lambda)\,\boldsymbol{F}_3(\lambda)]\\&\quad\boldsymbol{F}_1(\lambda)=luminance\ basis\\&\quad\boldsymbol{F}_2(\lambda)=(red\ \text{-}\ green\ )\ opponent\ \text{-}color\ basis\\&\quad\boldsymbol{F}_3(\lambda)=(yellow\ \text{-}\ blue)\ opponent\ \text{-}color\ basis\end{aligned}\tag{3}$$

The *matrix-**F*** to formulate a FCS-based complete OCS is required to satisfy the following two mathematical condtions as follows.

[Condition 1] *Orthonormality*

$$F^t F = I;\ \langle F_j \bullet F_k \rangle = \int F_j(\lambda) F_k(\lambda) d\lambda = \begin{cases} 1 \ for\ j = k \\ 0 \ for\ j \neq k \end{cases} \quad (4)$$

$$where, \langle u \bullet v \rangle \ denotes\ inner\ product\ of\ u\ and\ v$$

The [Condition 1] demands that F should be an orthonormal basis normalized in metric space to be

$$\|F_1\| = \|F_2\| = \|F_3\| = 1 \quad (5)$$

Where, (F_1, F_2 ,F_3) are $N \times 1$ column vectors sampled at discrete wavelength of $\lambda_n(n=1\sim N)$, where their signal energies are also normalized to be unity as

$$\langle F_j \bullet F_j \rangle = \|F_j\|^2 = 1 \ for\ j = 1 \sim 3 \quad (6)$$

[Condition 2] *Achromatic Grayness*

The complete OCS is required to keep neutral gray for a flat spectral input, that is,

$$\int F_2(\lambda) d\lambda = 0 \ and \ \int F_3(\lambda) d\lambda = 0 \quad (7)$$

The [Condition 2] demands that the opponent-color channels must not respond to achromatic spectral input, where both of (*red-green*) and (*yellow-blue*) signals must be zero.

Completeness Test for Typical OCSs

The following describes the tested results whether the typical OCS models satisfy the above-mentioned two conditions or not.

[T-1] MacAdam's Orthogonal Color Matching Function

Since the MacAdam's orthogonal color matching function is orthogonalized but not normalized, it's orthonormalized by *GramSchmidt* and we got the MacAdam's FCS basis as

$$F^t_{Mac} = \begin{bmatrix} 0.00000 & 1.00000 & 0.00000 \\ 1.57247 & -1.10246 & -0.46897 \\ 0.56196 & -0.46104 & 0.62259 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \quad (8)$$

Of course, Eq. (7) satisfied the [Condition 1], but didn't satisfy the [Condition 2], resulting in

$$\int F_2(\lambda) d\lambda = 0.000854, \ \int F_3(\lambda) d\lambda = 0.703443 \quad (9)$$

That is, the *red-green opponent-color* channel is almost good, while, the *yellow-blue* channel was insufficient.

[T-2] Guth's ATD80

Since Guth's first OCS, ATD80 was not orthogonal, it's orthonormalized by *GramSchmidt* just as J. Worthey done as

$$F^t_{Guth} = \begin{bmatrix} 0.00000 & 1.00000 & 0.00000 \\ 1.64805 & -1.17116 & -0.34894 \\ 0.26903 & -0.27423 & 0.69698 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \quad (10)$$

As well, Eq. (10) satisfied the **[Condition 1]**, but didn't satisfy the **[Condition 2]**, resulting in

$$\int F_2(\lambda) d\lambda = 0.12776, \ \int F_3(\lambda) d\lambda = 0.691744 \quad (11)$$

In this case, both *red-green* and *yellow-blue* were insufficient.

[T-3] Wandell's $O_1O_2O_3$

Typical OCS, Wandell's $O_1O_2O_3$ advanced to S-CIELAB. Since the original basis was not orthogonal, we get the orthonormal FCS basis for $O_1O_2O_3$ by *GramSchmidt* process as

$$F^t_{Wandell} = \begin{bmatrix} 0.23033 & 0.81137 & -0.04171 \\ -1.60831 & 1.30297 & 0.10387 \\ -0.26292 & 0.14448 & 0.76004 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \quad (12)$$

In this case, both *red-green* and *yellow-blue* opponent-color channels didn't satisfy the [**Condition 2**], resulting in

$$\int F_2(\lambda) d\lambda = -0.201267, \ \int F_3(\lambda) d\lambda = 0.641635 \quad (13)$$

Figure 1 shows the tested FCSs with their basis functions. Each FCS is graphically displayed as a LUM (Locus of Unit Monochromats) pointed by the 3-D vectors of $\{F_2(\lambda_n), F_3(\lambda_n), F_1(\lambda_n)\}$ sampled at discrete wavelength λ_n=380+5(n-1) *nm* (n=1~71) in the range of 380~730 nm by 5 nm step.

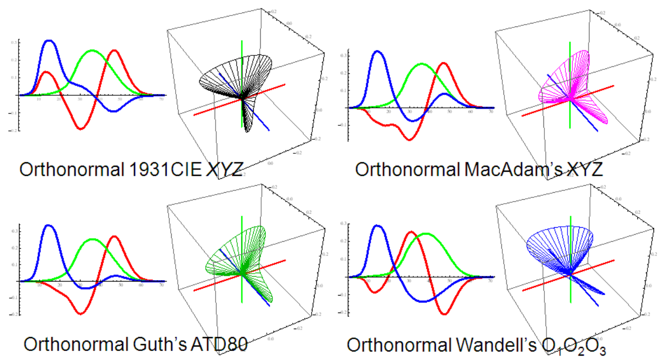


Figure 1. *Typical FCS spanned by orthonormal basis function*

[T-4] YIQ OCS

YIQ coding system is a well-known OCS used for NTSC TV signal transmission. This OCS is designed based on opponent-color process in human vision. The transform to *YIQ* basis function, *matrix*-F_{YIQ} from CIE1931 *xyz* is given by

$$F^t_{YIQ} = \begin{bmatrix} 0.000 & 1.000 & 0.000 \\ 1.389 & -0.827 & 0.453 \\ 0.938 & -1.195 & 0.233 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \quad (14)$$

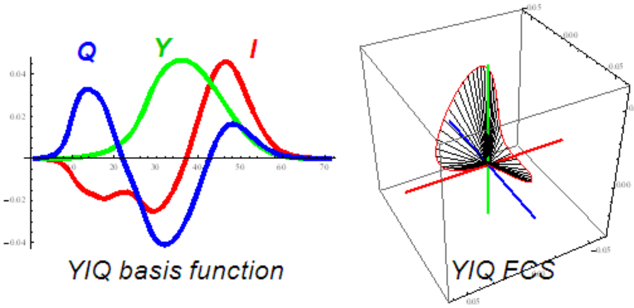
Since this *YIQ* basis function is orthogonal but not orthonormal, it doesn't satisfy the [**Condition 1**]. Though, it proved to satisfy the [**Condition 2**] as it is, without normalization. Surely Eq.(14) keeps the grayness in Eq. (7) that guarantees achromatic NTSC TV signal transmission without any coloring. It's notable that once the *YIQ* basis function is orthonormalized, the [**Condition 2**] is no longer valid as same as **[T-1] MacAdam's**.

This feature of *YIQ* is quite different from MacAdam's. *YIQ* is not an orthonormal but an orthogonal basis copeing with achromatic gyayness in the [**Condition 2**].

Now, we can say *YIQ* is an unique **"*quasi-complete*"** OCS in practice that has the orthogonality with normalized luminance as

$$\int F_1(\lambda) d\lambda = 1 \ and \ \langle F_j \bullet F_k \rangle = \int F_j(\lambda) F_k(\lambda) d\lambda = 0 \ (j \neq k) \quad (15)$$

Figure 2 shows the *YIQ* basis functions.



Fiure 2. *YIQ FCS spanned by orthogonal basis function*

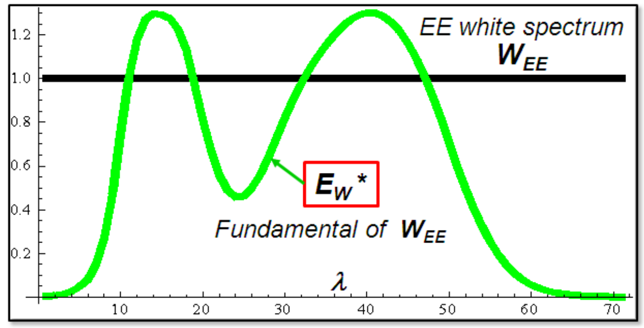


Figure 3. *Fundamental E_W* as luminance basis for complete OCS*

FCS-based Complete OCS

As clarified in **[T-4] YIQ OCS,** *YIQ* basis function is not orthonormal but just orthogonal, though it's a usefull OCS. The major objective of this paper is to find such OCS as satisfying both **[Condition 1]** and **[Condition 2]**, because orthonomality is mathematically beautiful and *worthy as a* ***metric space***. Therefore, the key is to find a *matrix-**E*** to fulfill the **[Condition 2]** even after orthonormalized by the **[Condition 1]**.

Firstly, we should choose an orthogonal triplet from *matrix-**R*** as a candidate for *matrix-**E***. Though, in general, even if chosing an orthogonal triplet as a *matrix-**E*** from *matrix-**R***, it's hard to get a *matrix-**F*** that satisfies the **[Condition 2]**.

The author intuitively found that the primary factor lies in the seletion of luminance vector E_1 and the problem is solved by using the ***fundamental*** E_W* of *EE* white spectrum W_{EE} as a vector E_1.

Now, a complete OCS is founded as follows.

[Step1] Choose an orthogonal triplet from *matrix-**R*** as a candidate for *matrix-**E***

[Step2] Replace E_1 by E_W* and get a new *matrix-**E**=* E_{Opt}

$$E_W{}^* = RW_{EE}\,,\ E = E_{Opt} = \left[\,E_W{}^*\ E_2\ E_3\,\right] \qquad (16)$$

[Step3] Get the basis *matrix-**F**=* F_{Opt} by orthogonalizing E_{Opt}

$$F_{Opt} = GramSchmid\ t\left[E_{Opt}\right] = \left[F_{O1}\ F_{O2}\ F_{O3}\right] \qquad (17)$$

Since the luminance basis $F_{O1}(\lambda)$ is derived from the ***fundamental*** E_W* by gathering all the monochromatic *fundamentals* included in *EE* white spectrum, both opponent-color basises $F_{O2}(\lambda)$ and $F_{O3}(\lambda)$ are orthogonal to $F_{O1}(\lambda)$ and satisfy the **[Condition 2]** as

$$\int F_{O2}(\lambda)d\lambda = \int F_{O3}(\lambda)d\lambda \cong 0 \qquad (18)$$

Therefore, F_{Opt} constitutes a *FCS* basis for complete OCS.

Figure 3 illustrates the *fundamental* E_W* led to a complete OCS solution, which has the unique ***double-peaked*** shape different form the usual single-peaked luminance function.

Complete OCS Using Golden Vectors

Complete OCS with Cohen's Golden Vectors

As a triplet with the best orthogonality among the columin vectors in Matrix-***R***, the set of *fundamentals* for **(λ_1=455, λ_2=513, λ_3=584)** nm is known as ***Golden Vectors*** 7). This set becomes a promising candidate for the initial *matrix-**E***. A solution to the complete OCS is obtained by the following steps.

[Step1]: The *matrix-**E*** is initially set to ***E***=[E_{513} E_{584} E_{455}].

[Step2] Replace the vector E_{513} corresponding to luminance by the *fundamental* E_W*and get a new *matrix* E_{Opt}=[E_W* E_{584} E_{455}].

[Step3]: Get the basis *matrix-**F**=* F_{Opt} by orthogonalizing E_{Opt} as

$$F_{Opt} = GramSchmid\ t\left[E_W{}^*\ E_{O2}\ E_{O3}\right] = \left[F_{O1}\ F_{O2}\ F_{O3}\right]$$

Note that *GramSchmidt* should be applied by choosing E_W* as the first vector for starting the orthonormalization.

Complete OCS with New Precision Golden Vectors

Although the Cohen's ***Golden Vectors*** are almost perfect in their mutual orthogonality, the choice of monochromats' spectra seems to be little to shorter wavelengths. The author re-examined if any other optimal triplet better than Cohen's is there or not. The precision search procedure is as follows.

[1] Generate high precision *matrix-**R*** with Eq. (1). Use CIE *xyz* color matching function with resolution $\Delta\lambda$=1nm at λ_n=380+*n*-1 (*n*=1~351) in the range of 380~730 nm. The *matrix-**R*** is 2-D array of 351×351.

$$R = A(A^t A)^{-1} A^t\,, \qquad (19)$$

where ***A*** deotes CIE color matching function with $\Delta\lambda$=1 nm.

[2] Normalize E_n in *matrix* ***R***=[E_n] as

$$\hat{E}_n = E_n / \left|E_n\right|\ ;\ n = 1 \sim 351 \quad , \qquad (20)$$

where E_n and $\left|E_n\right|$ denote *n-th* column vector and its norm.

This operation is placed to suppress the calculation errors in the next Eq. (21), because E_n takes very small value near the both ends of visible spectral range of human vision.

[3] Choose an arbitorary triplet ***E***=[E_i E_j E_k] (*i*=1~351, *j*=1~351, *k*=1~351) from the normalized [E_n].

[4] Evaluate the orthogonality between the triplet with sum of inner products reflecting the directional cosine given by

$$\Theta(i,j,k) = \left|\left\langle \hat{E}_i \bullet \hat{E}_j \right\rangle\right| + \left|\left\langle \hat{E}_j \bullet \hat{E}_k \right\rangle\right| + \left|\left\langle \hat{E}_k \bullet \hat{E}_i \right\rangle\right| \qquad (21)$$

[5] Search the optimal set of (i, j, k)=***Opt***(i, j, k) to minimize $\Theta(i,j,k)$ for all possible combinations of ${}_nC_3$=7145775.

$$Opt(i,j,k) = \min_{i,j,k=1}^{351}\left[\Theta(i,j,k)\right]\ for\ i < j < k \qquad (22)$$

However, unfortunately, this search for optimal solution couldn't converge to the minimum in a short period but took a long time when using the high resolution of $\Delta\lambda$=1nm. Then, firstly, Eq. (22) was executed with the resolution of $\Delta\lambda$=5nm and got a coarse solution ***Opt***(i, j, k) giving the triplet of **(λ_1=465, λ_2=550, λ_3=610)** nm. Next, the finer search with $\Delta\lambda$=1nm was performed in the near range of (**465**±15, **550**±15, **610**±15) nm. As a result, the optimal triplet of **(λ_1=461, λ_2=548, λ_3=617)** nm was obtained as a ***new Golden Vectors***.

Now, the proposed complete OCS is obtained with this ***new Golden Vectors*** by just replacing the *luminance* vector E_{548} in *matrix-**E*** by the ***fundamental*** E_W* as same as with Cohen's ***Golden Vectors***.

Figure 4 illustrates how the complete OCSs with (a) Cohen's and (b) Kotera's ***Golden Vectors*** are constructed. Here we call these complete OCS as ***Golden-EE*** FCS.

As clearly shown, the two ***Golden-EE*** FCSs have the tilted primary color axes each other and span the similar but little bit different their own FCSs. As compared with Cohen's ***Golden-EE*** FCS, Kotera's ***Golden-EE*** FCS has the explicit opponent-color responses in its basis functions as shown in the *matrix-**F***.

Although the both ***Golden-EE*** FCSs satisfy the conditions for complete OCS, strictly speaking, the orthogonality and the achromatic grayness in the basis function F_{Opt} by Kotera's ***Golden Vectors*** is mathematically perfect and superior to that by Cohen's as shown in Table 1.

Table 1. Improvement in completeness by new Golden Vectors

Cohen's Golden Vectors (λ_1=455, λ_2=513, λ_3=584 nm)		
Orthogonality: $\langle F_1 \bullet F_2\rangle$ - 9.98x10^{-16}	**Orthogonality: $\langle F_2 \bullet F_3\rangle$ - 1.26x10^{-15}**	**Orthogonality: $\langle F_3 \bullet F_1\rangle$ - 1.63x10^{-15}**
Luminance normality: $\int F_1(\lambda)d\lambda \cong 1.0$	**Grayness: $\int F_2(\lambda)d\lambda \cong$ -1.17x10^{-15}**	**Grayness : $\int F_3(\lambda)d\lambda \cong$ -1.36x10^{-15}**
Kotera's new Golden Vectors (λ_1=461, λ_2=548, λ_3=617 nm)		
Orthogonality : $\langle F_1 \bullet F_2\rangle$ 3.16x10^{-16}	**Orthogonality:$\langle F_2 \bullet F_3\rangle$ 2.66x10^{-16}**	**Orthogonality: $\langle F_3 \bullet F_1\rangle$ - 8.54x10^{-17}**
Luminance normality: $\int F_1(\lambda)d\lambda \cong 1.0$	**Grayness: $\int F_2(\lambda)d\lambda \cong$ 1.21x10^{-16}**	**Grayness: $\int F_3(\lambda)d\lambda \cong$ - 5.85x10^{-17}**

Completeness Test by Spectral Decomposition

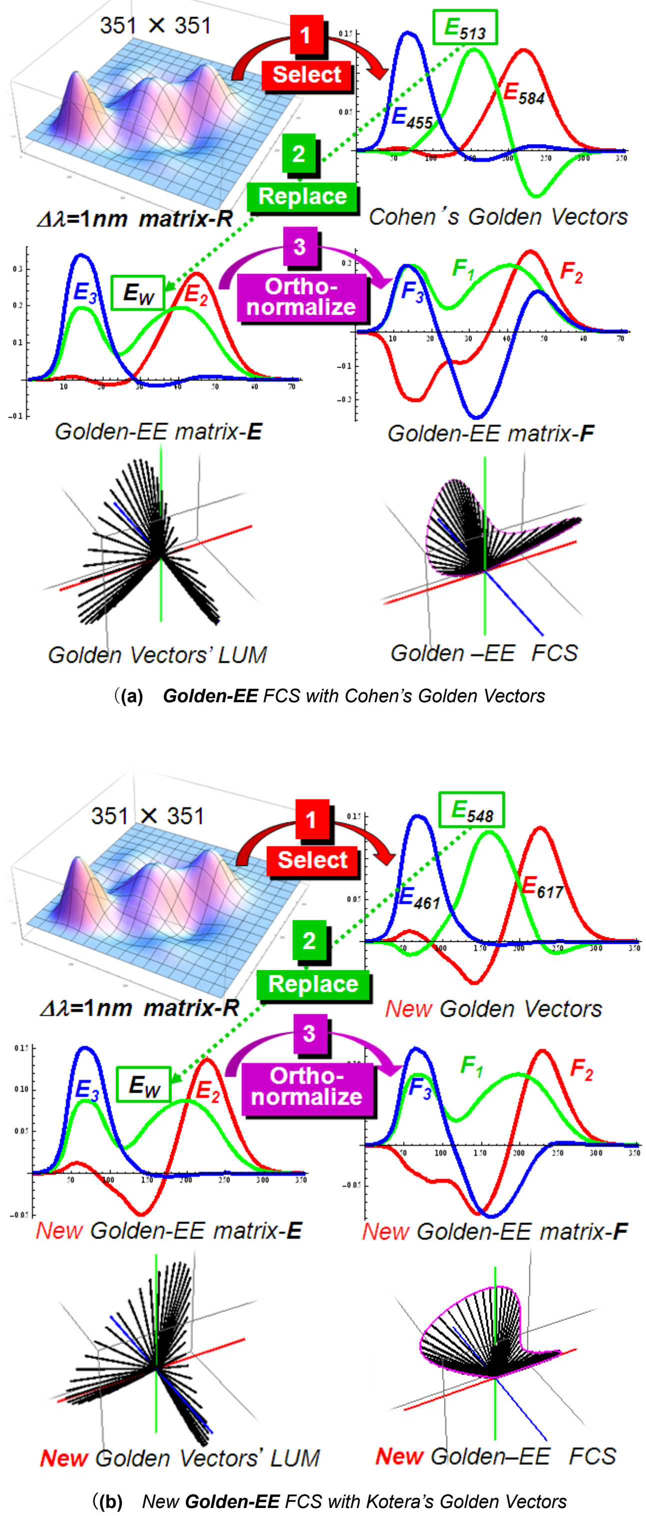
According to the *matrix-**R*** theory, the projection operator ***R*** itself is decomposed to the luminance-chrominance components of $\{R_Y, R_R, R_B\}$. Also, the *fundamental* ***C****(λ) for an input spectrum ***C***(λ) is decomposed into $\{Y, C_R, C_B\}$ components as given by the following Eq. (23).

$$R = FF^t = R_Y + R_C,\ R_Y = F_1F_1^t$$
$$R_C = R_R + R_B,\ \ R_R = F_2F_2^t,\ R_B = F_3F_3^t$$
$$C^*(\lambda) = RC(\lambda) = Y(\lambda) + C_R(\lambda) + C_B(\lambda)$$
$$Y(\lambda) = R_Y C^*(\lambda),\ C_R(\lambda) = R_R C^*(\lambda),\ C_B(\lambda) = R_B C^*(\lambda) \tag{23}$$

Where, Y, C_R and C_B denote *luminance, Red-Green* and *Yellow-Blue opponent-color* signals respectively.

The completeness in OCS is also verified by checking the orthogonality and achromatic gyayness between these components.

Figure 5 illustrates the spectral decomposition for the proposed ***Golden-EE FCS*** with Kotera's ***Golden Vectors***, where the almost perfect completeness is verified.



(a) ***Golden-EE*** *FCS with Cohen's Golden Vectors*

(b) *New* ***Golden-EE*** *FCS with Kotera's Golden Vectors*

Figure 4 *Construction of Complete OCS with Golden Vectors*

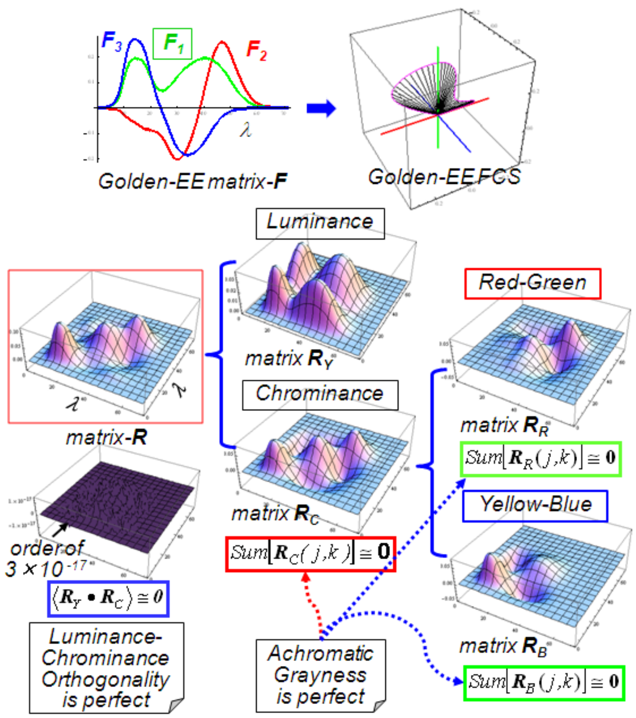


Figure 5 *Completeness by spectral decomposition of Golden-EE FCS*

XYZ to/from Luminance-Chrominance Transform

The proposed complete OCS with *Golden-EE FCS* is simply described by a linear transformation from *CIE XYZ* space as

$$F_{Opt}^{t} = MA^{t};\ M = 3\times3\ matrix \tag{24}$$

The transform matrix M is given by

$$M = \left(A^{t}A\right)^{-1} F_{Opt}{}^{t} A \tag{25}$$

The concrete linear transformations are given as follows.

[Complete OCS with *Cohen*'s *Golden-EE FCS*]

$$F_{Opt}^{t} = \begin{bmatrix} 0.26534 & 0.44199 & 0.29267 \\ 0.91865 & -0.37688 & -0.54177 \\ 0.97503 & -1.13998 & 0.16495 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \tag{26}$$

[Complete OCS with *Kotera*'s *Golden-EE FCS*]

$$F_{Opt}^{t} = \begin{bmatrix} 0.26534 & 0.44199 & 0.29267 \\ 1.29078 & -0.91271 & -0.37807 \\ 0.35844 & -0.78009 & 0.42165 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \tag{27}$$

Application to Color Image Clustering

Separation of Opponent-Color Image Components

Recently, Bratkova et al [10] reported that their model ***oRGB*** has an excellent property to separate the opponent-color components for natural images. Here the performance of proposed complete OCS is examined in comparison with ***oRGB*** using the same test image as Bratkova et al. Fig. 6 shows the resultant images separated to (Y, C_R, C_B) components. Since the opponent-color components (C_R, C_B) have negative values, they are visualized by subtracting the reproduction by (Y, C_R) or by (Y, C_B) from that by full (Y, C_R, C_B). The ***oRGB*** model is based on a nonlinear hue rotation for its OCS axes matching the orthogonal opponent-color axes of C_B in vertical and C_R in horizontal. Although ***oRGB*** is not orthonormalized but incomplete as OCS, it has the excellent opponent-color separability better than *CIELAB* or *YIQ* and the proposed OCS worked much the same as ***oRGB*** as shown in Fig.6 (a). In addition, Fig.6 (b) shows the achromatic grayness test result using monochrome image "Lena". As clearly seen, the proposed model keeps the achromatic grayness without coloring as well as *YIQ* even if adding the C_R or C_B to the *luminance* Y, because of taking C_R=C_B=0 for achromatic gray input.

Color Image Segmentation

CIELAB is a most popular OCS characterized by cube-root nonlinear scaling but scaled uniform to human perceptual color difference. This perceptual uniformity as metric space is expected to be useful for color clustering or image segmentation. While, since the proposed *Golden-EE FCS* or *YIQ* are linear scaled OCS, they are perceptually non-uniform but have excellent complete color-opponency. Now it's interesting whether these simple linear OCSs are useful or not for separating the clustered color objects due to the complete opponent-color characteristics.

Lastly the paper introduces an experiment on the application to color image segmentation. First, a ***bilateral filter*** is operated to the test images in order to reduce the background noise. Then ***k-means*** clustering method is applied to the segmentation. Fig. 7 shows a result. The rose "*Sakuragai*" is segmented to class number K=2, where, its petal area is correctly separated with shadowed area in CIELAB, but the shadowed pinkish area is confused with the background in *YIQ* and the proposed OCS. On the other hand, the image "*Baboon*" (K=4), looks to be naturally segmented in *YIQ* and the proposed OCS better than CIELAB. In this sample, only the proposed OCS succeeded in clustering the both eyes to the identical class. For the image "*Parrot*"(K=8), CIELAB worked best, the proposed OCS came next, and *YIQ* to the worst. In spite of simple linear transformation from *CIE XYZ*, the proposed OCS proved to be useful to color clustering or image segmentation.

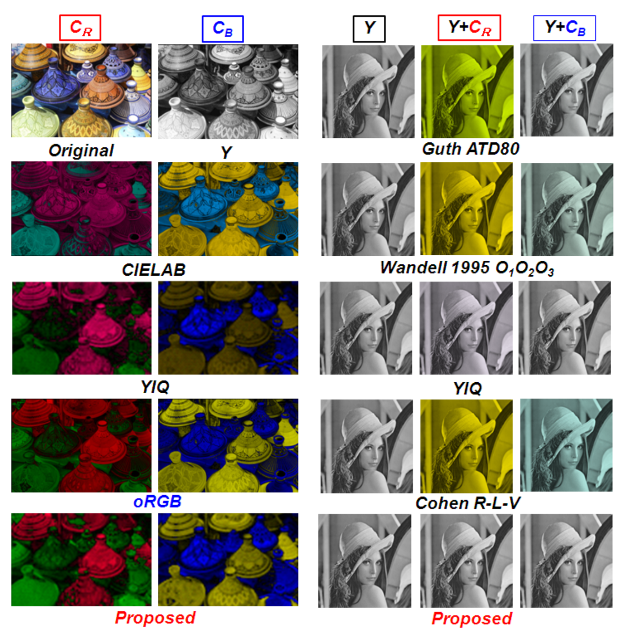


Figure 6 *(a) Opponent-color components (b) Achromatic grayness*

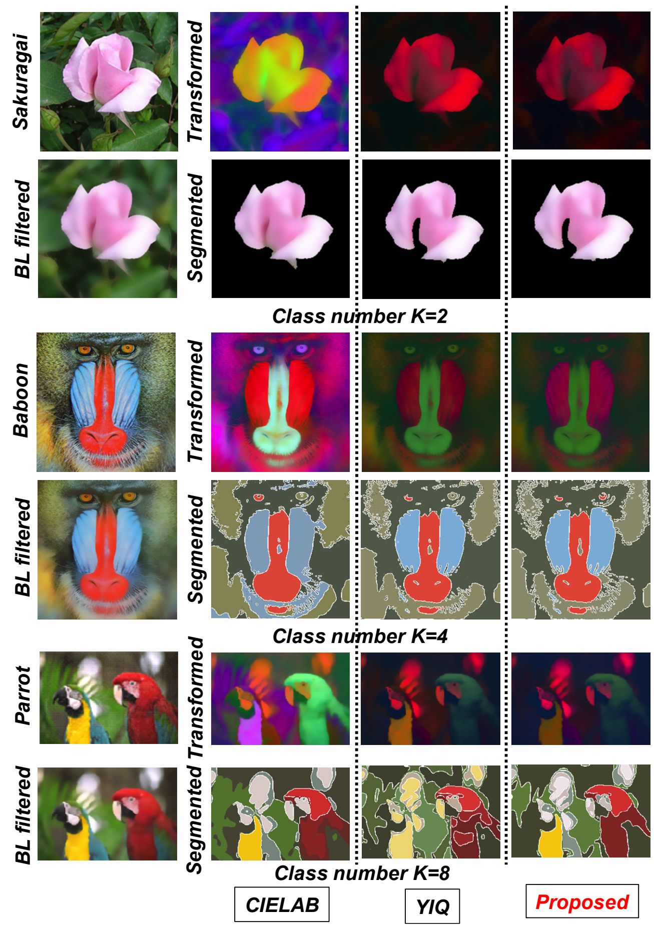


Figure 7 *Color image segmentation sample*

Conclusions

The paper proposed a complete *Opponent-Color Space* (*OCS*) based on the concept of *FCS* defined by *matrix-**R*** theory. The two conditions of [1] *orthonormality* and [2] *achromatic grayness* are to be satisfied at the same time to construct a complete *OCS*.

In general, it's hard to satisfy the condition [2] under the condition [1]. While, the condition [1] is not always necessary, but may be enough in practice if the condition of *orthogonality* is satisfied instead. Though *YIQ* system is not *orthonormal*, it's considered as a complete OCS copeing with *orthogonality* and perfect *achromatic grayness*. Nevertheless, the paper persuited the complete OCS with the both conditions of [1] and [2] from a point of mathematical beauty as a metric distance space.

The paper clarified that the primary factor lies in the seletion of ***luminance vector*** E_1 in the basis *matrix-**E*** and the problem is solved by using the ***fundamental*** E_W* of white spectrum W_{EE} as a vector E_1. The proposed model used ***Golden Vectors***, the best orthogonal *triplet*, extracted from the column vectors in *matrix-**R*** as an initial *matrix-**E***. Although the Cohen's ***Golden Vectors*** is already known, the author had a question whether it's truely the best or not. After re-searching for the best *triplet* with the high resolution of $\Delta\lambda$ =1 nm, a set of optimal ***Golden Vectors*** is newly found out, which is different from Cohen's and better in orthogonality.

The ***Golden-EE*** *FCS* as a complete OCS, is spanned by the *basis matrix-**F*** with the new ***Golden Vectors***. The proposed complete OCS is described by a linear transformation from *CIE XYZ* and worked better than *YIQ* in the application to the color image segmentation. Further improvement towards perceptually uniform metric space may be possible by introducing a nonlinear scaling but left behind as a future work.

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Author Biography

Hiroaki Kotera *joined Panasonic in 1963. He received PhD from Univ. of Tokyo. After worked at Matsushita Res. Inst. Tokyo during 1973-1996, he was a professor at Dept. Information and Image Sciences, Chiba University. He retired in 2006 and is working as TLO associate at Chiba University. He received 1993 IS&T honorable mention, 1995 SID Gutenberg prize, 2005 IEEE Chester Sall award, 2007 IS&T Raymond. C. Bowman award, 2009 SPSTJ and 2012 IIEEJ best paper awards. He is a Fellow of IS&T.*