A Simplified Overprint Model and Why the Masking Equations "Sort Of" Work for Color Halftone Hardcopy

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Abstract

The linear masking equations, inspired by continuous tone photographic color, were applied by Murray, et al., to halftone printing. This model is considerably simpler than models customarily used for halftone color hardcopy, such as those based on Neugebauer. Despite unrealistic assumptions regarding the image microstructure, the masking equations often perform reasonably well.

In this paper, the differences between the linear masking model and the model of Yule and Colt (Neugebauer with wideband Yule-Nielsen correction applied) are parsed. It is shown that, under conditions for which the Yule-Nielsen parameter n grows without bound (or its reciprocal, u, approaches zero), the differences can be attributed to sub-additivity of densities. An approximation is offered to model the sub-additivities.

The largest difference between the two models can be expected to occur under normal circumstances when all colorants are printed solid.

Symbols and Notation

Reflectance vector

We shall adopt the notation $\mathbf{R}$ for a vector of wideband reflectances. Normally, this vector will have three components, which shall be denoted R, G, and B. We further assume that the tristimulus values X, Y, and Z, may be approximated (if not computed exactly) by a linear combination of the components of the vector $\mathbf{R}$. That is,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \approx \mathbf{A} \cdot \mathbf{R} \tag{1}$$

The transformation may be exact, relative to an illuminant with spectral power distribution S, if the spectral products are independent linear combinations of $\bar{x}(\lambda)S(\lambda)$, $\bar{y}(\lambda)S(\lambda)$, and $\bar{z}(\lambda)S(\lambda)$; the matrix A will then be inverse of the matrix that defines these linear combinations.

Density vector

We shall adopt the notation $\mathbf{D}$ for a vector of densities. Absolute densities may be computed as the negative of the common logarithm of the corresponding reflectance. Relative densities are computed by subtracting the absolute density vector of the printing substrate from the absolute density vector of the printed area of interest.

Unless otherwise noted, relative density vectors will be used in this paper.

Metric and Parametric Colorant Amounts

Binary inking levels are assumed in this paper. Within a halftoned patch of visually uniform fill, the fraction of area for which a cyan ink is printed will be denoted c'. This shall be referred to as the *metric colorant amount* for cyan. A corresponding *parametric colorant amount*, c, may be estimated using photometric measurements and an appropriate formula. For example, when using the masking equations, as we shall in the next section, the parametric colorant amount for cyan may be computed as the red-filter density of a cyan tint relative to the paper, divided by the relative red-filter density of a solid cyan print.

Two apparently very different models

The Yule-Colt model [1] is a multi-ink generalization of the Yule-Nielsen model, and is analogous to the Neugebauer model. For three colorants using conventionally-angled halftones:

$$\mathbf{R} = \begin{bmatrix} (1-c)(1-m)(1-y)\mathbf{R}_p^u + cmy\mathbf{R}_{cmy}^u \\ +c(1-m)(1-y)\mathbf{R}_c^u + my(1-c)\mathbf{R}_{my}^u \\ +m(1-c)(1-y)\mathbf{R}_m^u + cy(1-m)\mathbf{R}_{cy}^u \\ +y(1-c)(1-m)\mathbf{R}_y^u + cm(1-y)\mathbf{R}_{cm}^u \end{bmatrix}^{1/u} \tag{2}$$

Considerably less complicated is the linear masking model for three inks. Using densities relative to the substrate, these are:[2, 3]

$$\mathbf{D} = c\mathbf{D_c} + m\mathbf{D_m} + y\mathbf{D_y} \tag{3}$$

Even though a different set of linearization functions may be necessary, it seems remarkable that both equations (2) and (3) can both provide approximations to the same set of actual print conditions accurate enough for sensitivity analysis, black generation, and other applications. This paper asks, and aims to answer, how can two equations that have such different forms and complexity both model the same set of printing conditions?

Reconciliation of Eqs 2 and 3

Pollack's Limit

Pollack [4] pointed out that the limit of R^u, as $u \to 0$, is essentially the corresponding density. In addition to the continuous tone scenario mentioned by Pollack, conditions under which u becomes close to zero, and even becomes negative, have been discussed by Lewandowski, Ludl, Byrne, and Dorffner [5], as well as this author. [6, 7] Using relative densities as in Eq (3), and noting that the first term then cancels, Eq (2) becomes:

$$\mathbf{D} = \begin{array}{l} cmy\mathbf{D}_{cmy} \\ +c(1-m)(1-y)\mathbf{D}_c + my(1-c)\mathbf{D}_{my} \\ +m(1-c)(1-y)\mathbf{D}_m + cy(1-m)\mathbf{D}_{cy} \\ +y(1-c)(1-m)\mathbf{D}_y + cm(1-y)\mathbf{D}_{cm} \end{array} \quad (4)$$

While a power function of the reflectances has been replaced by the corresponding densities, this is still rather different from the linear masking equation.

Gathering terms in c, m, y, and their products

Further similarity may be gleaned by expanding the coefficients of the density vectors in the previous equation, and gathering the terms with the same coefficients. This yields:

$$\mathbf{D} = \begin{array}{l} c\mathbf{D}_c + m\mathbf{D}_m + y\mathbf{D}_y + cm\left(\mathbf{D}_{cm} - \mathbf{D}_c - \mathbf{D}_m\right) \\ +cy\left(\mathbf{D}_{cy} - \mathbf{D}_c - \mathbf{D}_y\right) + my\left(\mathbf{D}_{my} - \mathbf{D}_m - \mathbf{D}_y\right) \\ +cmy\left(\begin{array}{c} \mathbf{D}_{cmy} - \mathbf{D}_{cm} - \mathbf{D}_{cy} \\ -\mathbf{D}_{my} + \mathbf{D}_c + \mathbf{D}_m + \mathbf{D}_y \end{array}\right) \end{array} \quad (5)$$

The first three terms on the right hand side of Eq (5) correspond to the linear masking formula in Eq (3); the remaining terms are difference between the two formulas. The next sections will be devoted to further parsing of these components of the residual.

Parsing the cm residual term

Clearly, this term will be zero when either c, m, or both are zero. We now examine the more common (and interesting) case for which $cm \gg 0$. Under additivity of density, the first residual term, $cm\left(\mathbf{D}_{cm} - \mathbf{D}_c - \mathbf{D}_m\right)$, will be zero, because the density (relative to the paper) of the cyan-magenta overprint will equal the sum of the relative densities of the cyan and magenta solids. This term may be attributed to additivity failure alone.

However, it is possible to further parse this term, in particular, the multiplier of the cm factor. Such closer examination, using a less aggressive assumption than additivity of densities, will permit assessment of the magnitude of this term's contribution to error. The appendix contains a model for approximating the density vector of overprinted solid inks. Substituting this into the residual term yields:

$$cm\left[(\alpha_{cm} - 1)\mathbf{D}_c + (T_{v,cm} - 1)\mathbf{D}_m\right] \quad (6)$$

where the parameters α_{cm} and $T_{v,cm}$ are described in the appendix. A reasonable value for α is 0.75, and a reasonable value for T_v is 0.90. This implies that this term can be expected to contribute approximately $cm(0.25\mathbf{D}_c + 0.10\mathbf{D}_m)$ to the difference between the linear masking and Yule-Colt models.

Approximating the cy and my terms

In a similar vein, the term in (5) with the leading factor cy may be approximated as:

$$cy\left[(\alpha_{cy} - 1)\mathbf{D}_c + (T_{v,cy} - 1)\mathbf{D}_y\right] \quad (7)$$

In analog, the residual term with the leading factor my may be approximated using:

$$my\left[(\alpha_{my} - 1)\mathbf{D}_m + (T_{v,my} - 1)\mathbf{D}_y\right] \quad (8)$$

Approximating the cmy term

The final component of the residual may be arranged as:

$$cmy\left(\begin{array}{c} [\mathbf{D}_{cmy} - \mathbf{D}_{cm} - \mathbf{D}_y] - [\mathbf{D}_{cy} - \mathbf{D}_c - \mathbf{D}_y] \\ -[\mathbf{D}_{my} - \mathbf{D}_m - \mathbf{D}_y] \end{array}\right) \quad (9)$$

Each bracketed term may approximated using the methodology described in the appendix:

$$\begin{array}{l} \mathbf{D}_{cmy} - \mathbf{D}_{cm} - \mathbf{D}_y \approx (\alpha_{cmy} - 1)\mathbf{D}_{cm} + (T_{v,cmy} - 1)\mathbf{D}_y \\ \mathbf{D}_{cy} - \mathbf{D}_c - \mathbf{D}_y \approx (\alpha_{cy} - 1)\mathbf{D}_c + (T_{v,cy} - 1)\mathbf{D}_y \\ \mathbf{D}_{my} - \mathbf{D}_m - \mathbf{D}_y \approx (\alpha_{my} - 1)\mathbf{D}_m + (T_{v,my} - 1)\mathbf{D}_y \end{array} \quad (10)$$

While the first approximation in (10) could be parsed further by approximating the density vector of the cyan-magenta overprint, this is peripheral to the discussion in the present paper.

Discussion

Each of the residual terms is multiplied by the product of either two or all three of the parametric colorant amounts. Therefore, the difference between the two formulas is expected to be small when c, m, and y are all small. Indeed, both equations agree when all three colorant amounts are zero. On the other hand, a large difference can be expected when all colorants are solid, that is, $c = m = y = 1$.

Appendix

Predicting density vectors of overprinted solid inks

Models for predicting a two ink overprint have recently been investigated by this author in conjunction with other researchers. [8, 9] Among these is the following two-colorant overprint prediction formula, cast here in terms of relative spectral densities:

$$D_{\lambda,12} = D_{1,\lambda} + T_v D_{2,\lambda} - T_v \frac{D_{1,\lambda} D_{2,\lambda}}{D_{\infty 2,\lambda}} \quad (11)$$

where $D_{1,\lambda}$ is the spectral density above the substrate of the first (bottom) ink printed; $D_{2,\lambda}$ is the spectral density above the substrate of the second (top) ink printed; T_v is a so-called *trapping factor* that accounts for the incomplete transfer of the second-down ink on top of the first-down ink; $D_{\infty 2,\lambda}$ is the spectral relative *saturation density* (or relative masstone density) of the top ink; and $D_{12,\lambda}$ is the spectral density above the substrate of the two ink overprint. For inks printed in the correct (high to low) tack sequence, the trapping factor T_v is approximately 0.9.

In these citations, it is further suggested that the relative density masstone spectrum of the second-down ink may be approximated as a multiple of the second-down ink's relative density spectrum, i.e., $D_{\infty 2,\lambda} \approx k \cdot D_{2,\lambda}$. (The parameter k will be unity when the second-down ink is opaque, infinity when it is perfectly transparent; for a real ink, the value will of course be between these two extremes.) Substituting this into Eq (11) yields a two-parameter approximation to the spectrum of the overprint:

$$D_{\lambda,12} \approx D_{1,\lambda} + T_v D_{2,\lambda} - T_v \frac{D_{1,\lambda} D_{2,\lambda}}{k \cdot D_{2,\lambda}}$$
$$\approx D_{1,\lambda} + T_v D_{2,\lambda} - T_v \frac{D_{1,\lambda}}{k} \qquad (11a)$$
$$\approx \alpha D_{1,\lambda} + T_v D_{2,\lambda}$$

where $\alpha = \frac{k - T_v}{k}$. We complete the approximation for the wideband density vector of two overprinted inks by substituting the wideband relative density components for their spectral counterparts:

$$\mathbf{D}_{12} \approx \alpha \mathbf{D}_1 + T_v \mathbf{D}_2 \qquad (12)$$

More than two inks

When more than two inks are sequentially printed, there will of course be a first-down ink, a second-down ink, and a third-down ink. For lithographic printing, magenta is usually printed after (i.e., on top of) cyan, and yellow after magenta.

Clearly, Eq (12) may be used to approximate the magenta over cyan overprint, allowing the relative density vector for the cyan solid to play the role of D_1, and using the relative density vector of the magenta solid for D_2. The result will be an approximation for the relative density vector of the cyan-magenta overprint.

Similarly, the relative density vector for the green or cyan-yellow overprint may be approximated similarly, substituting the density vector of the yellow solid for D_2. Because the yellow ink will, in general, have different tack than the magenta, the value of T_v will be different. Because the magenta and yellow inks will exhibit different optical scatter, it is prudent to admit a different value of k. Together, these indicate the desirability for a different value of α. If values of T_v and α specific to this overprint are used, we may attach the subscript "$_{cy}$" to them, and denote their counterparts for the cyan-magenta overprint using instead the subscript "$_{cm}$."

The density vector of the yellow-over magenta overprint may be estimated analogously, permitting yet different values for T_v and α. Finally, the three-color overprint's relative density vector may be approximated by allowing the role of the first-down ink to be played by the cyan-magenta overprint, and the role of the second-down ink by the yellow. As always, values of T_v and α specific to this overprint may be used, attaching, as necessary, the appropriate subscript.

References

[1] J A C Yule and R Colt. Colorimetric investigations in multicolor printing. In *TAGA Proceedings*, pages 77–82. Technical Association of the Graphic Arts, 1951.

[2] Alexander Murray. The masking method of color correction applied to three- and four-color reproduction. *Photoengravers Bulletin*, 23:13–21, January 1934.

[3] John A C Yule. *Principles of Color Reproduction*. Wiley, New York, 1967.

[4] Felix Pollack. The relationship between the densities and dot sizes of multi-colour halftone images. *Journal of Photographic Science*, 3(4):308–313, 1955.

[5] Achim Lewandowski, Marcus Ludl, Gerald Byrne, and Georg Dorffner. Applying the Yule-Nielsen equation with negative *n*. *Journal of the Optical Society of America A*, 23(8):1827–1834, 2006.

[6] J A Stephen Viggiano. Physical significance of negative Yule-Nielsen n-value. In *ICIS '06: Proceedings of the International Congress of Imaging Science*, pages 607–610. IS&T: the Society for Imaging Science and Technology, 2006.

[7] J A Stephen Viggiano. Ink penetration, isomorphic colorant mixing, and negative values of Yule-Nielsen n. In *Eighteenth Color and Imaging Conference*, pages 285–290. IS&T, 2010.

[8] J A Stephen Viggiano and Sri Hemanth Prakhya. Prediction of overprint spectra using trapping models: A feasibility study. *RIT TAGA Student Chapter Journal*, pages 113–133, 2008.

[9] Franz Sigg, J A Stephen Viggiano, and Khalid Husain. Predicting the color of an overprint of two spot colors using a mathematical model. In *2011 TAGA Proceedings*, 2011.

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