On the Information Content along Edges in Trichromatic Images

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Abstract

We introduce a theoretical framework for measuring the information content in the edges extracted from a color image. The main difficulty in estimating the amount of information (differential entropy [1, 2]) in an image signal is to fit an appropriate probability mass function to the trichromatic image data. To estimate the amount of information in the edges extracted from a color image, we first convolve the image with a derivative filter. By fitting a Kotz-Type probability distribution to the convolved image, we then estimate the differential entropy of the edge coefficients as a measure of the uncertainty involved in the edge content of a postreceptoral chromatic image. The proposed estimation of differential entropy provides an efficient means of processing the edge content information under a variety of natural illuminations, which might be further used as a quantitative measure for evaluating color constant image retrieval.

Introduction

Human visual system provides an efficient means of processing the color information of reflecting surfaces by discounting the contribution of illumination to the chromatic content of the image. This ability is called color constancy. In 1983, Gilchrist *et al.* [3] proposed that the edge content of a scene is critical for the visual system to decompose a scene into the components of reflectance and illumination. Computational models of lightness constancy are typically based on the assumption that illumination changes involve smooth variations in luminance level [4-6]. Then it is believed that the effects of illumination occur at higher level of spatial frequencies in a scene.

In the context of color constancy, the mechanism of visual system and the capability of an observer in representing constant colors are of particular interest [7]. Implementation of edge detection algorithms and image gradient [4, 8] were proposed as a computational approach to illumination detection based on the general scheme of Lands' Retinex model [6]. On the basis of von Kries diagonal transformation hypothesis, edge-based color constancy methods were proposed based on the assumption that the average of gradient coefficients in a scene under neutral illumination corresponds to an achromatic patch [9, 10]. Edge-based algorithms in color constancy have performed decently in detecting the color vector of illumination particularly when the signal-to-noise ratio of a scene is above medium [11]. Concerning the importance of edges in detecting illumination, it is noteworthy to mention that convolving an image with a high-pass derivative filter can extract a higher level of special frequencies in a scene. Then spatial frequencies in a scene appear to be crucial for the visual system in illumination detection.

The postreceptoral processing of color information in an opponent color system optimally removes the redundant information caused by correlations among receptoral cone responses due to overlap in the cones' spectral sensitivities [12]. Nieves *et al.* [13] found that the magnitude of the three post-receptoral Luminance, Red-Green and Blue-Yellow edge contrast changes was almost constant across daylights. However, they noted that the normalized edge contrasts in the Lum, RG, and BY opponent channels decreased by illumination changes up to 9000 K, before becoming almost constant beyond 10,000 K (Fig.4 in Ref. 13).

Considering a high threshold of spatial frequencies for the illuminant effects in a scene, in the present research, we are particularly interested in finding out how much information is captured by the gradient edges under different natural illuminations. The amount of postreceptoral information of the edges in a natural scene is estimated by transformations of LMS cones responses to Luminance (Lum) and the two opponent chromatic responses, RG and BY.

Image Formation

A computational simulation of image formation was carried out using different hyperspectral images. The LMS cone responses at a pixel (x,y) in an image with spectral reflectance $\boldsymbol{r}(x,y;\lambda)$ can be calculated under illumination $\boldsymbol{e}(\lambda)$ by,

$$\begin{aligned} L(x,y) &= \int l(\lambda)\, r(x,y;\lambda)\, e(\lambda)\,\mathrm{d}\lambda \\ M(x,y) &= \int m(\lambda)\, r(x,y;\lambda)\, e(\lambda)\,\mathrm{d}\lambda \\ S(x,y) &= \int s(\lambda)\, r(x,y;\lambda)\, e(\lambda)\,\mathrm{d}\lambda \end{aligned} \tag{1}$$

in which $l(\lambda)$, $m(\lambda)$, and $s(\lambda)$ are the Smith and Pokorny cone sensitivities [14] at wavelength λ, and the illumination is assumed to be constant over the scene. The LMS receptoral image was then transformed to a cone-opponent image representation using a simple approach as follows [12, 15]:

$$\begin{aligned} Lum(x,y) &= L(x,y) + M(x,y) \\ RG(x,y) &= L(x,y) - M(x,y) \\ BY(x,y) &= 2S(x,y) - Lum(x,y) \end{aligned} \tag{2}$$

where, Lum is luminance value, and RG and BY are the two perceptual chromatic axes of opponent colors at pixel (x,y).

The postreceptoral image, $\boldsymbol{I}(lum, rg, by)$, can be convolved with a gradient filter, f, for extracting the edges,

$$[I^{\sigma}_{lum} * f, I^{\sigma}_{rg} * f, I^{\sigma}_{by} * f] \tag{3}$$

where, σ is the standard deviation of a Gaussian filter, $\boldsymbol{G}^{\sigma}$, with which the image, $\boldsymbol{I}$, is convolved, as $\boldsymbol{I}^{\sigma} = \boldsymbol{I} * \boldsymbol{G}^{\sigma}$, for local smoothing.

Edge Information Content

The difficulties in estimating the information content in a scene pertains to the way in which the entropy is estimated for a discretized random variable of pixel intensities in a scene. Field & Chandler [16] proposed a computational method for measuring the amount of information corresponding to the phase and power spectra of a natural scene. Due to computational complexity, however, they implemented the method for a relatively small patch size. The problem of estimating the information content of an image is even more pressing when dealing with a three dimensional chromatic image. Aiming at identifying the amount of information captured by colors in a scene, Franch & Foster [17] studied different estimation techniques for measuring the discreet entropy of a scene by partitioning the space of continuous RGB variable.

Edge Statistics

The edge coefficients of an image are distributed in a more kurtotic manner than a Gaussian distribution. Figure 1 represents typical contours of a log-histogram of the distribution of the $(rg\text{ - }by)$ postreceptoral edges extracted from the image, $\boldsymbol{I}(lum, rg, by)$, under neutral daylight illumination with CCTs ~ 6500 K. It can be seen that a multivariate heavy-tailed elliptically symmetric probability distribution can be fitted well into the data of the edge coefficients. Similar representations of the distribution can be found for other two sets of postreceptoral variables, *i.e.* $(lum\text{ - }by)$, and $(lum\text{ - }rg)$. Chakrabarti *et al.* [18] fitted a radial exponential distribution to the convolved image with a derivative filer. We propose a similar Kotz-Type probability density function [19],

$$p(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = c\left|\boldsymbol{\Sigma}\right|^{-1/2} \exp\left\{-k\,[(\boldsymbol{x}-\boldsymbol{\mu})^{T}\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})]^{\theta}\right\} \tag{4}$$

of n-dimensional multivariate random variable, $\boldsymbol{x} = [x_1, \ldots, x_n]$, in which, $c = [\theta k^{n/2\theta}\,\Gamma(n/2)]/[\Gamma(n/2\theta)\pi^{n/2}]$, and $\Gamma(\cdot)$ is Gamma function. The maximum likelihood estimation of the parameters in Kotz-type distribution was discussed by Naik and Plungpongpun [20].

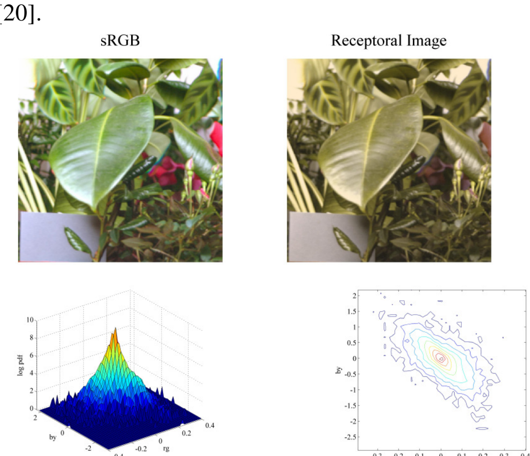

Figure 1. A typical log-histogram of the distribution of the postreceptoral edges extracted from a natural scene under neutral illumination.

Differential Entropy

The differential entropy of a probability distribution, $p(\boldsymbol{x})$, of a multivariate random variable, $\boldsymbol{x} = [x_1, \ldots, x_n]$, is defined as,

$$H(\boldsymbol{x}) = -\int p(\boldsymbol{x}) \log\, p(\boldsymbol{x})\, d\boldsymbol{x}\ , \tag{5}$$

in which $p(\boldsymbol{x})$ is the probability mass function [21]. It can be proved that the differential entropy, $H(\boldsymbol{x})$, of the Kotz-type probability distribution, $p(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$, in Eq. (4) is,

$$H(\boldsymbol{x}) = n/2\theta - \log c \tag{6}$$

A special case of Kotz-type distributions is multivariate Gaussian distribution, where $k = 0.5$ and $\theta = 1$, for which,

$$H(\boldsymbol{x}) = (1/2)(n + n\log 2\pi + \log|\boldsymbol{\Sigma}|) = (1/2)\log(2\pi e)^{n}|\boldsymbol{\Sigma}|. \tag{7}$$

The univariate Laplace distribution,

$$p(x; \mu, \sigma) = \frac{1}{2\sigma}\exp\left\{-\left|\frac{x-\mu}{\sigma}\right|\right\} \tag{8}$$

is another special case of probability distribution of Eq. (4), where $k = 1$, $\theta = 0.5$, and $n = 1$, and the entropy of which is,

$$H(x) = 1 + \log 2\sigma. \tag{9}$$

The mutual information, *MI*, can then be measured by the following equation,

$$MI(\boldsymbol{x}) = \sum_{i} H(x_i) - H(\boldsymbol{x})\ , \tag{10}$$

in which $H(x_i)$ is the entropy of the univariate random variable x_i, measured by Eq. (9).

Experiment

We used a total of 50 hyperspectral images, consisting of 29 natural scenes collected at the Bristol University [22], 12 images gathered by Ruderman *et al.* [12], and 9 spectral images of nonurban scene collected by Foster *et al.* [23, 24]. The LMS response of each image at a pixel was computed under 140 outdoor illuminations with CCTs from 2,900 K to 36,660 K (Fig. 2). The images were also normalized by the maximum value in a scene to keep the range within 0 to 1. The postreceptoral image, $\boldsymbol{I}(lum, rg, by)$, was then computed using Eq. (2), and afterwards the postreceptoral edges were extracted as explained in Eq. (3).

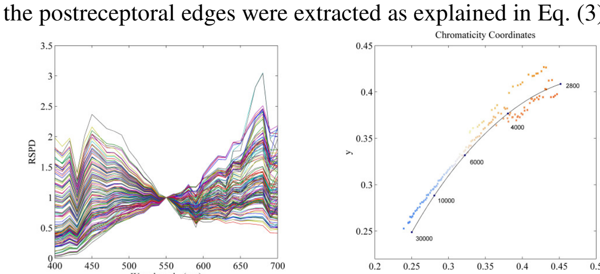

Figure 2. The spectral power distributions together with the chromaticity coordinates of 140 outdoor illuminations under which the images were rendered.

The entropy, $H(\boldsymbol{x})$, and mutual information, $MI(\boldsymbol{x})$, of postreceptoral edges, where $\boldsymbol{x}$ represents the (lum, rg, by) variable, were measured for each natural scene rendered under 140 outdoor illuminations. Since we are interested in the information content along edges in natural scenes, the average of the measured entropies and mutual information across the all 50 images can be calculated for each illumination separately. In the present work, we computed the average of the 50 measured entropies and mutual information under each outdoor illumination. Figure 3 shows the results of the average entropy and mutual information of the postreceptoral edges as a function of the CCT of the illumination under which the scene was rendered. It can be observed that mutual information decreases rapidly as CCT increase up around 10,000 K. Besides, the information content, $H(\boldsymbol{x})$, increased up to around 10,000 K and then slightly decreases. The results are consistent with Nieves *et al.* findings [13] which state that edge contrasts were almost constant for the postreceptoral mechanism in daylight with CCT above 10,000 K.

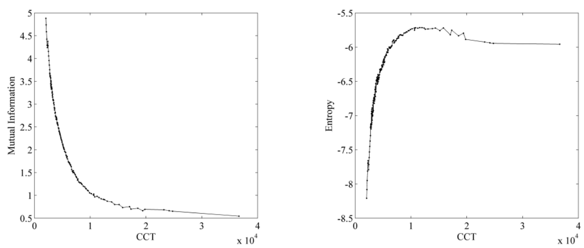

***Figure 3**. The average of mutual information and entropies (in nats) of postreceptoral edges across the all 50 natural scenes versus the correlated color temperature (CCT) of illuminations.*

In order to investigate how changes in natural illumination influence the entropy and mutual information of edges in the postreceptoral domain after adaptation, the modified postreceptoral responses were computed as [12, 13],

$$\begin{aligned} \hat{l} &= (\hat{\mathrm{L}} + \hat{\mathrm{M}}) \\ \hat{\alpha} &= (\hat{\mathrm{L}} + \hat{\mathrm{M}} - 2\hat{\mathrm{S}}) \\ \hat{\beta} &= (\hat{\mathrm{L}} - \hat{\mathrm{M}}) \end{aligned} \tag{11}$$

where, $\hat{\mathrm{L}}$, $\hat{\mathrm{M}}$ and $\hat{\mathrm{S}}$ are,

$$\begin{aligned} \hat{\mathrm{L}} &= \log(L) - \langle \log(L) \rangle \\ \hat{\mathrm{M}} &= \log(M) - \langle \log(M) \rangle \\ \hat{\mathrm{S}} &= \log(S) - \langle \log(S) \rangle \end{aligned} \tag{12}$$

and $\langle \cdot \rangle$ is the mean operator. Figure 4 shows the distribution of the modified postreceptoral edges extracted from a natural image under the neutral illumination (the scene in Fig. 1). As shown in Fig. 4, the redundancy between edges in the postreceptoral mechanisms measured by Eq. (11) decreased comparing with the responses calculated by Eq. (2).

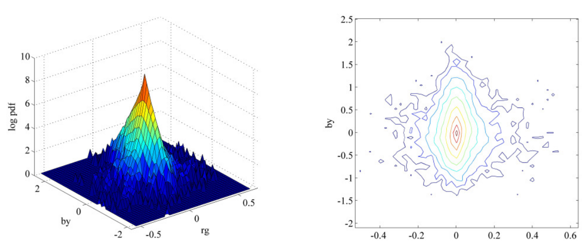

***Figure 4**. A typical log-histogram of the distribution of edges extracted from the modified postreceptoral responses for the scene shown in Figure 1, under the neutral illumination.*

Figure 5 represents the average entropy and mutual information of the modified version of the postreceptoral edges versus the CCT of the illumination under which the scene was rendered. According to the results of the information content in the edges of the modified postreceptoral responses, the entropy and mutual information changed as a function of CCT. In most of the cases, the mutual information represented a minimum value around 3600 K to 8000 K. The entropy of the modified postreceptoral edges decreased in general as the CCT of the illumination increased. As can be seen in Fig. 5, the mutual information of the modified postreceptoral edges represents a minimum value at CCT around 5000 K.

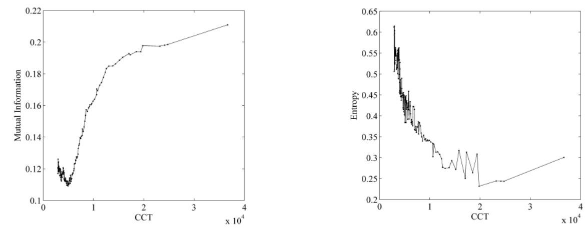

***Figure 5**. The average of mutual information and entropies (in nats) of edges extracted from the adapted postreceptoral responses across all 50 natural scenes versus the correlated color temperature (CCT) of illuminations.*

Discussion

According to previous results, mutual information decreases when the visual information ascends the visual pathway from receptoral to post-receptoral stages [15]. Besides, as daylight changes, chromatic and luminance edges also change but only by a few percent. Considering the vast range of different daylight CCTs the edge contrast change rate is probably not visually relevant [13]. However, the amount of information shared by the edges in the three cone-opponent mechanisms decreases rapidly up to 10,000 K, followed by an almost constant magnitude for illuminations with CCT above 10,000 K. The entropy in cone-opponent edges represents a maximum value around 10,000 K (Fig. 3). For the daylight illumination with CCT above 10,000 K, the magnitude of entropy in cone-opponent edges slightly decreases.

The results of the analysis of the information content in the edges from the modified postreceptoral mechanism based on Eq. (11) were different from that of obtained from Eq. (2). At a first glance it could be a stunning result because of translation and scaling invariance of the mutual information. Nevertheless the whole nature of the data was changed by the logarithm transformation on Eq. (12) and thus results for MI of adapted and unadapted are different. The information shared by adapted postreceptoral responses decreased for the edges extracted from scenes under daylight with CCTs around 3,600-8,000 K. It seems that the optimal daylight for constant color falls within the range of CCT in which the mutual information is moderately small. Furthermore, the results of postreceptoral edges obtained from Eq. (2) show that the unadapted postreceptoral systems tend to moderately increase the information content in a scene around neutral illumination. A relatively small value in the amount of information shared by the postreceptoral responses was also observed around neutral illumination with CCTs ~ 6,500 K. It has been argued that information estimates shed light on the amount of elements identified by human visual system in terms of their colors, independent of spatial position [23-24]. It should be further analyzed whether this statement holds when the visual signal is processed at a higher-level, *e.g.* scenes edges, under different illuminant changes.

Conclusions

A theoretical framework for measuring the entropy and mutual information of the edges extracted from a color image was introduced by fitting a Kotz-Type probability distribution to the edges of trichromatic image data. To estimate the amount of information in the edges extracted from a color image, we first convolve the image with a derivative filter. By fitting a Kotz-Type probability distribution to the convolved image, we then estimate the differential entropy of the edge coefficients as a measure of the uncertainty involved in the edge content of a postreceptoral chromatic image. The proposed estimations were analyzed by processing information content in the edge under a variety of outdoor illuminations. Considering the minimum value of CCT 5,000 K at which the modified postreceptoral edges represented an average minimum value of mutual information, finding the optimal daylight to account for constant color representation still remained unsolved. Thus, it cannot be concluded that adapted color mechanisms can discount illumination changes using the information content in the edges. However, the entropy in the edges of postreceptoral mechanisms represented a moderately higher magnitude around neutral illuminations with CCTs ~ 6,500 K. In order for visual system to represent constant color by discounting the contribution of illumination, further investigation is necessary to verify whether the information content in the edges of a scene plays an important role in possessing this fascinating ability.

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