Spectrum-Locus Convexity: A Metric for Cameras?

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Abstract

Spectrum-locus convexity confers to human vision the property that optimal colors are 1-0 with at most two transition wavelengths. It also confers illuminant-invariance of the CW/CCW chromaticity ordering of certain reflectance triads. The same holds for cameras, and provides a less stringent criterion for camera quality than that of Maxwell and Ives. Unlike in applications that design reflectance spectra, the camera convexity criterion has the goal of ensuring that cameras and humans share the same non-reversing reflectance triads, not of ensuring illuminant-invariance of the triads themselves. Convexity may be a useful metric, but is undefined when the sensors are non-overlapping. This paper will qualitatively explore these issues.

Introduction

How good are the spectral responses of a given trichromatic color camera? Ideally, the camera should make the same color matches we do, in which case the camera-to-human metamerism would be zero. This ideal (called the Maxwell-Ives or Luther criterion) requires the camera-sensitivity functions (CSFs) to be different (nonsingular) linear combinations of human color-matching functions (CMFs). Departures from the ideal are commonly quantified by RMS departures of each CSF $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ (where λ is wavelength) from the space of CMFs $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$. Such a metric of departure from the perfect state does not completely represent the distortion of camera-rendered colors relative to those seen by humans.

Toward a different sort of metric, the present paper proposes the hypothesis that color distortion becomes qualitatively worse when the local hue ordering of three colors can reverse from camera to human. Here is a model of local hue ordering: If one looks at three colored patches meeting at a vertex in the visual field, and adapts to the average of the three, then the chromaticities of the three patches can be said to have a clockwise (CW) or counterclockwise (CCW) ordering about that adapted average, and this is roughly an analogue of hue ordering. As far as the mathematics is concerned, one can ignore the adapted average, and talk about CW or CCW *cyclic ordering* of reflectances. It will be the transfer of such ordering to new illuminants and from camera to human, whose importance will be the hypothesis of the current paper.

I have been writing about this hypothesis in several guises for 30 years. With H. Hemmendinger, I applied it to quantifying the vulnerability of color atlases of reflectances to color distortion when the light spectrum is changed (see [1] and earlier references therein). Later I described conditions under which the hypothesis allows avoidance of certain pathologies of the mapping from colorant concentrations to tristimulus values [2]. Then, based on the same formalism, J. Larimer and I described conditions that would allow avoidance of on-screen metamerism of displays with more than three primaries [3]. Now, I am extending the notion to cameras. But in the case of cameras, one is less concerned with the effect of illuminant change than with the effect of the camera-to-human transformation.

Definition

What is it that human spectral sensitivities have that is rare in cameras? A casual glance at the CIE's horseshoe diagram gives a hint. The spectrum locus is manifestly convex, in a way that is intuitively clear but requires some further words for mathematical discussion. Cameras don't usually have convex spectrum loci.

Informally, we can define spectrum-locus convexity (of either a camera or a human) to mean that the chromaticity region enclosed by the spectrum locus and a line between the spectrum-locus endpoints (e.g., line of purples) is convex. But we'll need a more precise formal definition:

Let $\mathbf{p}(\lambda)$ be a column 2-vector of chromaticity such as $(x,y)'$ from a monochromatic light of wavelength λ. The locus of $\mathbf{p}(\lambda)$ over visible wavelengths comprises the spectrum locus. Let λ_1, λ_2, λ_3 be three wavelengths such that $\lambda_3 > \lambda_2 > \lambda_1$. I call a spectrum locus convex if the quantity $\det[\mathbf{p}(\lambda_2) - \mathbf{p}(\lambda_1), \mathbf{p}(\lambda_3) - \mathbf{p}(\lambda_1)]$ is either always non-negative or always non-positive independent of the choice of λ_1, λ_2, λ_3.

An additional condition must be satisfied if part of the spectrum locus is a straight line: If $\mathbf{p}(\lambda_1)$, $\mathbf{p}(\lambda_2)$, and $\mathbf{p}(\lambda_3)$ lie on such a line, then $\det[\mathbf{p}(\lambda_2) - \mathbf{p}(\lambda_1), \mathbf{p}(\lambda_3) - \mathbf{p}(\lambda_1)] = 0$. When this condition occurs, the wavelength will be considered well ordered if $\mathbf{p}(\lambda_2)$ lies between $\mathbf{p}(\lambda_1)$ and $\mathbf{p}(\lambda_3)$.

By "visible wavelength" I mean any wavelength for which at least one of the CMFs (or CSFs) is nonzero.

It is readily shown that, except for a possible overall change of sign for the whole spectrum locus, the algebraic sign of $\det[\mathbf{p}(\lambda_2) - \mathbf{p}(\lambda_1), \mathbf{p}(\lambda_3) - \mathbf{p}(\lambda_1)]$ is the same as that of $\det[\mathbf{q}(\lambda_1), \mathbf{q}(\lambda_2), \mathbf{q}(\lambda_3)]$, where column 3-vectors $\mathbf{q}(\lambda)$ comprise the CMFs (or CSFs). This equivalence allows us to discuss the chromaticity and tristimulus domains interchangeably.

Impact of Spectrum-Locus Convexity

It is interesting that the human-vision spectrum locus is substantially a simple horseshoe-shaped curve, and one might ask what survival pressure might have encouraged such an evolution. As Schrödinger [4] and others noticed, the convex spectrum locus is such that the optimal-color reflectances are 1 or 0 at each wavelength with at most two transitions. A maximum of n transitions occurs when a line through the spectrum locus intersects it n times [5]. When n = 2, the spectrum locus (plus line of purples) is a convex curve. Another, less obvious impact is that, if the spectrum locus is convex, then certain triads of reflectances preserve their CW/CCW cyclic ordering in chromaticity when the illuminant is changed. This is a spectrally general, but weak and empirically untested, form of color constancy.

What mathematical property distinguishes these "certain triads" of reflectances? Given one such triad $r_1(\lambda)$, $r_2(\lambda)$, $r_3(\lambda)$, the property is as follows: If one plots a "chromaticity" diagram with coordinates $[r_1(\lambda), r_2(\lambda)]/[r_1(\lambda) + r_2(\lambda) + r_3(\lambda)]$, the spectrum locus of this diagram is convex in the sense noted above (a property we call *reflectance-triad convexity*). To understand why this is so, it suffices to apply the Binet-Cauchy theorem, described below.

Binet-Cauchy Theorem

The Binet-Cauchy theorem [6] concerns the determinant of the product of two non-square matrices **A** and **B**. The existence the product **AB** requires the number of columns N in **A** to be the same as the number of rows of **B**, but otherwise the dimensions of **A** and **B** are unconstrained. Rather than state the Binet-Cauchy theorem for the general case, it is sufficient here to discuss the case in which **A** has three rows and **B** has three columns. In that case, the theorem has the following simple form:

$$\det[\mathbf{AB}] = \sum_{k,m,n} \det[\mathbf{A}(k,m,n)]\ \det[\mathbf{B}(k,m,n)], \qquad (1)$$

where (for example) **A**(k,m,n) is the 3x3 block of **A** that comprises columns k, m, n, and $1 \leq k < m < n \leq N$. This is a sum is over all corresponding 3-by-3 blocks of **A** and **B**. Clearly N is going to be the number of wavelengths in a Riemann-sum approximation to a spectrum integral, and the number 3 will refer to the dimensions of color space, or to the number of compared objects within that color space.

Now we interpret the quantities in Eq. 1: **A** has components $A_{jk'} = S(k')\, q_j(k')\, d\lambda$ (where k' is the wavelength index, S(k') is the illuminant spectral power distribution, and q_j is the j'th CMF (or CSF). Also, **B** has components $B_{k'i} = r_i(k')$, where r_i is the i'th reflectance in a particular reflectance triad. Then, **Q** = **AB** will be the 3-by-3 matrix whose column vectors are the tristimulus vectors (or camera values) of each of the reflectances r_i.

Another geometric interpretation: the handedness of the chromaticity ordering of r_1, r_2, r_3 changes when the algebraic sign on the determinant det(**Q**) changes. Therefore, if we can show the sign on det(**Q**)= det(**AB**) does not change with S(k'), then we will have shown the illuminant-invariance of the ordering. But the sign on the left-hend side of Eq. 1 is equal to the sign on the right side, so if none of the terms on the right undergoes a sign reversal, the left-hand side has the requisite invariance of ordering.

With this interpretation, we go on: Each 3-by-3 determinant det[**A**(k,m,n)] evaluates to S(k)S(m)S(n) det[**q**(k,m,n)], whose sign is the same as that of det[**q**(k,m,n)]. {**Note:** Here, I use det[**q**(k,m,n)] as an abbreviation for det[**q**(k), **q**(m), **q**(n)].} Because the spectrum locus is convex in wavelength, the sign of det[**q**(k,m,n)] is independent of the choice of k,m,n. Therefore, the sign of det(**Q**) will be illuminant-invariant if the three reflectance spectra have the following convexity property: the signs of det[**B**(k,m,n)] = det[**r**(k,m,n)] do not depend on the choice of k, m, n, for k < m < n. That property is mathematically the same as the reflectance-triad convexity described at the end of the last section.

Camera Discussion

Cameras that have convex spectrum loci will enjoy the same properties I mentioned two sections above. But many cameras have disturbingly nonconvex spectrum loci. That means not only that the ordering of the "certain triads" above will be prone to illuminant change, but also that the ordering of triads may be different from camera to human. This may be a more serious problem than just violation of the Maxwell-Ives criterion that besets nearly all cameras. It needs empirical (perceptual) test. Meanwhile, I present here some examples of camera spectrum loci, to illustrate the possibilities.

Examples of spectrum loci

Real camera spectrum loci are shown in Fig. 1-3 below (digitized courtesy of Jim Worthey). In each of the figures, the spectrum locus (B, G)/(R + G + B) of a camera is plotted as a curve with wavelength being the parameter. The data are plotted for the entire wavelength range, with the additional constraint that R + G + B is greater than one percent of the maximum value (over all wavelengths) of R + G + B. The red part of the curve represents the wavelengths over which R+G+B is greater than 10 percent of $[R+G+B]_{max}$, and the black part of the curve is the domain over which R+G+B is between 1 and 10 percent of $[R+G+B]_{max}$.

In Fig. 1, note that the reported spectral sensitivities were curtailed so there is no black part of the curve. However, even without the low-amplitude part, there is a clear nonconvexity at the short-wavelength end that will likely interfere with camera-to-human mappings. Fig. 2, for a Foveon camera [8], shows a mostly convex locus, but with nonconvexities at the ends. These nonconvexities are not likely to disturb object colors very much, because they correspond to low amplitudes of the color-matching functions. The introduction of a filter [8] to optimize the correspondence to CIE CMFs does not change the shape of the spectrum locus, but shaves off the low-amplitude parts (with their nonconvexities) until they are hardly present at all (see Fig. 3).

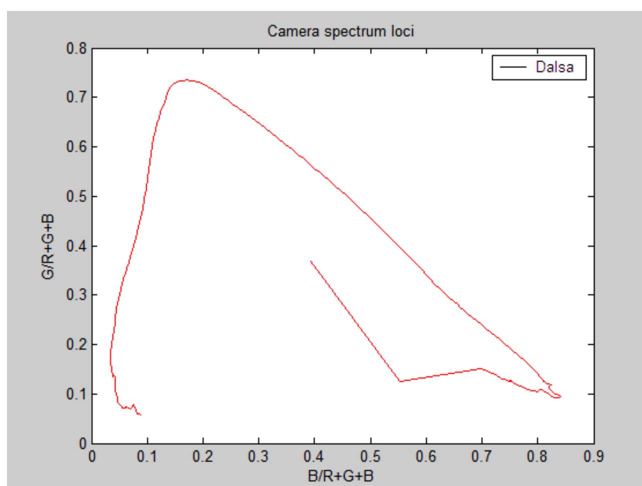

Figure 1. *Dalsa FTF3020C. Data were supplied as quantum efficiency. They have been converted to normal response per unit optical power. Ref: Product specification, 2004 Jan 15.*

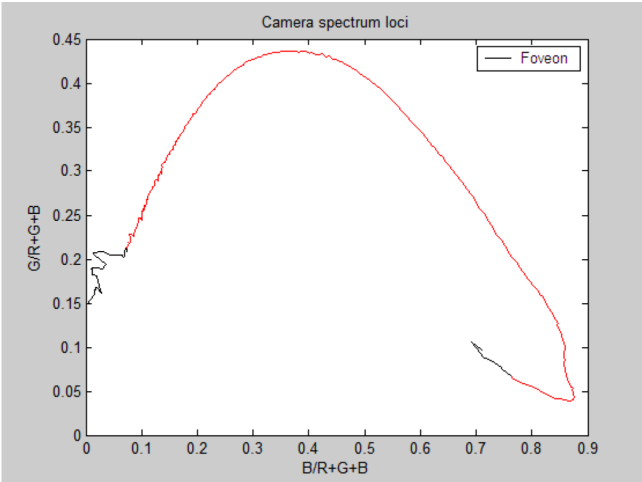

Figure 2. *FoveonX3. Ref: Lyon & Hubel [8]. Note that the high-amplitude (red) part of the curve is mostly convex.*

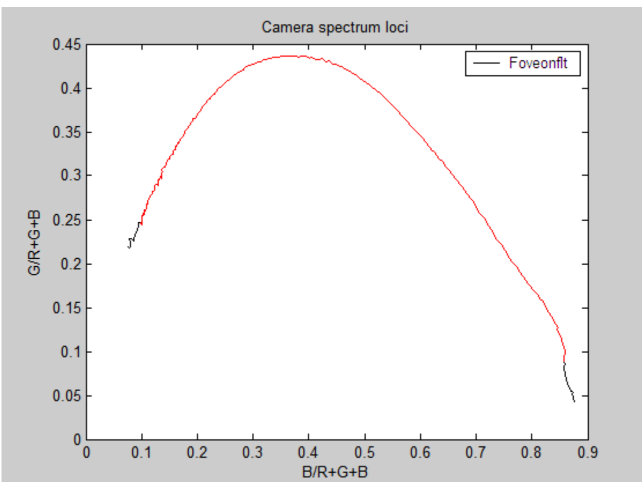

Figure 3. *FoveonX3 with optimum filter. Ref: Lyon & Hubel [8]. The filter was optimized for best match to the CIE color-matching functions, but that does not affect the spectrum locus except to attenuate below the plotting threshold certain wavelength contributions relative to max (R+G+B). Note that the red+black curve is mostly convex, but the red subset is even more predominantly convex.*

It is interesting to imagine empirical tests to determine the severity and idiosyncrasy of non-convexity artifacts. Do observers see the hues in rendered photographs reverse in a startling way? Does the hue reversal survive color-management transformation from camera values to CIE values?

A possible application

The convexity analysis finds an interesting interaction with the work of Andersen and Hardeberg [7]. This work proposed a camera-to-CIE mapping algorithm that finds the white point of a scene, divides the chromaticity space (camera or CIE) into pie wedges about this white point, and separately maps each camera-chromaticity pie wedge to a corresponding pie wedge in CIE space. When this is done, each ray in camera chromaticity space defined by the white specular reflection admixed (in various proportions) with a body reflection will map to a similar ray in CIE chromaticity. Hence the rays of constant dominant wavelength (or, informally, iso-hue lines) of chromaticity will map intact.

Define the angle about the white point in camera chromaticity space as θ, which is a function of the dominant wavelength λ. Andersen's pie-wedge-mapping algorithm depends on $\theta(\lambda)$ being single-valued, for otherwise there is a possibility for a ray to cross into another pie wedge in the mapping from camera to human.

When the camera spectrum locus is convex (as is the CIE locus), Andersen's pie-wedge mapping may have fewer line-crossings than otherwise. In that event the line-crossing rate will reduce to zero if any two patches that are adjacent to each other in θ, together with the white reflectance, have the reflectance-triad convexity property we have been talking about.

The utility of our convexity idealization for helping Andersen's algorithm has yet to be explored.

An inconvenient truth

A camera with three non-overlapping spectral sensitivities (of which Po-Chieh Hung has shared an example with me) will have only three points for its spectrum locus. For such a three-peak camera, the above definition of spectrum-locus convexity is not satisfied. In that case, the object-color solid is not bounded by 1-0 reflectances, but by linear combinations of them. Nonetheless, Eq. (1) still applies to such a camera, and reflectance-triad convexity is enough to ensure that the CW/CCW chromaticity ordering of object-color triads is not affected by illuminant. Furthermore, when three reflectances do not satisfy reflectance-triad convexity, illuminant change can reverse their CW/CCW ordering. The three-peak camera shares this property with the human visual system (although the illuminant changes that reverse a CW/CCW ordering may not be the same for human and camera). Hence we have found a case in which the criterion of reflectance-triad convexity applies even when spectrum-locus convexity does not. [Note: See Appendix for an illustrative implementation of Eq. (1) using a two-peak camera and only four wavelengths, and invoking convexity of reflectance dyads rather than triads.]

Outlook

In other applications, I have tried to posit metrics for non-convexity in order to guide designers of color atlases and multi-primary displays. However, in the case of cameras I believe we are not ready for a metric. The basic utility of the construction has yet to be shown. There are problems with cameras that are not present in the other applications. For one thing, the transformation that needs an invariant order is one that incurs an observer metamerism, which is more severe than the metamerism of illuminant change. (Illuminant change cannot alter the spectrum locus, because all three CMFs are multiplied by the same function. However, observer metamerism affects the CMFs in different and arbitrary ways.) For another thing, the wavelength support on the camera spectrum locus may be different enough from the human visible-wavelength range that colors may be severely distorted by this mechanism alone. Finally, the possibility of non-compact wavelength support of certain camera-sensitivity functions (by design in some cases) renders the camera spectrum locus nonconvex by our definition. This last problem is mitigated by the fact that, even though such a camera does not have a convex spectrum locus, any three reflectances that have reflectance-triad convexity will have illuminant-invariant chromaticity CW/CCW ordering for both the camera and the human. Other issues of camera-to-human fidelity for such a camera have yet to be explored. Convexity analysis, for both camera spectrum loci and for reflectance triads, is an interesting problem and approach, worth a panel discussion and more thought.

Appendix

This appendix describes the application of a two-dimensional version of Eq. (1) to a two-peak camera that has four wavelengths, two per sensor band. The spectral sensitivities are [1, 1, 0, 0] and [0, 0, 1, 1]. The illuminant spectrum is [S_1, S_2, S_3, S_4]. We are talking about a reflectance *dyad* in this case, in which the two reflectances are [r_{11}, r_{12}, r_{13}, r_{14}] and [r_{21}, r_{22}, r_{23}, r_{24}]. The analogue of Eq. (1) is then

$$\det(\mathbf{Q}) = \det \begin{bmatrix} S_1 & S_2 & 0 & 0 \\ 0 & 0 & S_3 & S_4 \end{bmatrix} \begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \\ r_{13} & r_{23} \\ r_{14} & r_{24} \end{bmatrix}$$

$$= S_1 S_3 \det \begin{bmatrix} r_{11} & r_{21} \\ r_{13} & r_{23} \end{bmatrix} + S_1 S_4 \det \begin{bmatrix} r_{11} & r_{21} \\ r_{14} & r_{24} \end{bmatrix}$$

$$+ S_2 S_3 \det \begin{bmatrix} r_{12} & r_{22} \\ r_{13} & r_{23} \end{bmatrix} + S_2 S_4 \det \begin{bmatrix} r_{12} & r_{22} \\ r_{14} & r_{24} \end{bmatrix}.$$

It can be seen that the sign of det(**Q**) can change if the signs on the 2-by-2 reflectance determinants have different signs and the numbers S_1 to S_4 are changed in magnitude. Conversely, if the reflectance determinants all have the same sign, there is *reflectance dyad convexity*, and no changes in the positive numbers S_1, S_2, S_3, S_4 can influence the sign of det(**Q**). Hence if a reflectance dyad is convex in its 2D spectrum-locus analogue, then the ordering of the dyad in chromaticity is illuminant-invariant.

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Author Biography

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