CAT02 and HPE Triangles

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Abstract

Here, we investigate CAT02 and HPE triangles in both x,y and u',v' spaces. The findings of this study are key for solving the Yellow-Blue and Purple problem simultaneously. At the same time, some inaccurate descriptions concerning the HPE triangle in the literature are discussed.

Introduction

Ever since the recommendation of the CIECAM02 colour-appearance model [1,2] by CIE TC 8-01 Colour-appearance modeling for colour-management systems, it has been used to predict colour appearance under a wide range of viewing conditions, to specify colour appearance in terms of perceptual attributes, to quantify colour differences, to propose a uniform colour space and to provide a profile connection space for colour management. However, it was found that the domain of the CIECAM02 is smaller than the ICC connection space [3], causing unexpected computational failure during the transformations using the CIECAM02. Brill and Süsstrunk [4-7] pointed out that one of the problems comes from the CAT02, which is embedded in the CIECAM02. These researchers observed that the CAT02 triangle (on the red-yellow side of the CIE x,y chromaticity locus) falls slightly outside the HPE triangle. Because of this, the adaptation to a bluish illuminant will shift to a yellowish colour from inside to outside the HPE triangle, resulting in a negative achromatic response. They called this phenomenon the "Yellow-Blue" problem. They also observed that the CAT02 triangle lies partially inside the CIE spectrum locus (from the blue corner and purple side). Because of this, adaptation to a purple illuminant will make a purple colour outside the CAT02 triangle but inside the CIE chromaticity locus 'more purple' and located outside the HPE triangle, resulting in a negative achromatic response. They called this phenomenon the "Purple" problem. They proposed a solution to the Yellow-Blue problem, but failed to solve the Purple problem. In fact they pointed out that the CIECAM02 computational problem can be solved if both problems can be solved at the same time, i.e. the HPE triangle, the new CAT02 matrix triangle, and the domain enclosed by CIE chromaticity should be properly nested.

In the present paper, the triangles of CAT02 and HPE matrices are further investigated. Firstly, in the next section, the primaries of the CAT02 matrix and its associated region of non-negative responses are clearly defined. The region is found to be a triangle in x,y or u',v' spaces with the vertices being the three primary points, called the triangle of the CAT02 matrix [4-6]. In Section 3, the primaries and their associated region of non-negative responses for the HPE matrix are examined, revealing that the region in x,y space has a polygonal shape with three sides. One side is a segment linking the red and blue primary points, while each of the two other sides constitute a line with one definite end at either the red or blue primary points. In the u',v' space, the region of non-negative response is a four-sided polygon, which differs from the results reported by Brill and Süsstrunk [4-6], who found this to be a triangle with the three vertices at the three primary points. In fact, this triangle includes two regions with one corresponding to the non-negative responses and the other to the non-positive responses. In Section 4, the regions of the CAT02 and HPE non-negative responses are compared. The final section presents the conclusions.

Red, Green and Blue Primaries of a Chromatic Adaptation Matrix

In 2006, Brill [4] found that the nonlinear post-adaptation function in CIECAM02 has an infinite rate of change at the origin, which causes the inverse of the CIECAM02 to be computationally unstable or ill conditioned. Let M be a matrix of a chromatic adaptation transform (CAT) with the general form:

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{13} & m_{32} & m_{33} \end{pmatrix} \quad (1)$$

Then R, G, B are defined by

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (2)$$

and can be considered as cone/sharper sensor responses. If the matrix M equals to the CAT02 matrix M_{02} [1,2,8], it is defined as:

$$M_{02} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{bmatrix} \quad (3)$$

and R,G and B are the sharper sensor responses [9]. If the matrix M equals the HPE matrix M_{HPE} [1,2] defined as:

$$M_{HPE} = \begin{bmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.18340 & 0.04641 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix} \quad (4)$$

R, G and B are the cone responses [9].

If X, Y, and Z in Eq. (2) are the tristimulus values of the corresponding colour predicted using CAT02 and the matrix defined by Eq. (1) is the HPE matrix, then R, G, and B defined by Eq. (2) are the R', G', and B' in CIECAM02. Since the nonlinear post-adaptation function has infinity changing rate when one of R', G', and B' is zero, Brill [4] investigated what kind of tristimulus values can cause the cone responses to be zero. Since our interest concerns the situation in which R, G, or B is zero, a positive scaling factor to the tristimulus values in Eq. (2) will not affect the results. Hence, the tristimulus values X, Y, Z are limited to the chromaticity coordinates x, y, z, i.e. we have the constraint:

$$X+Y+Z=1 \qquad (X=x,\ Y=y,\ \text{and}\ Z=z) \tag{5}$$

Thus,

$$\begin{aligned} R &= m_{11}X + m_{12}Y + m_{13}Z = (m_{11}-m_{13})x + (m_{12}-m_{13})y + m_{13} \\ G &= m_{21}X + m_{22}Y + m_{23}Z = (m_{21}-m_{23})x + (m_{22}-m_{23})y + m_{23} \\ B &= m_{31}X + m_{32}Y + m_{33}Z = (m_{31}-m_{33})x + (m_{32}-m_{33})y + m_{33} \end{aligned} \tag{6}$$

Hence,

$$\begin{aligned} R=0 &\quad\Leftrightarrow\quad (m_{11}-m_{13})x + (m_{12}-m_{13})y = -m_{13} \\ G=0 &\quad\Leftrightarrow\quad (m_{21}-m_{23})x + (m_{22}-m_{23})y = -m_{23} \\ B=0 &\quad\Leftrightarrow\quad (m_{31}-m_{33})x + (m_{32}-m_{33})y = -m_{33} \end{aligned} \tag{7}$$

Here the symbol $\Leftrightarrow$ means equivalent. Thus, each response can be zero at a line in CIE x,y chromaticity space defined by Eqs. (7). Figure 1 shows the lines for R=0 (red line), G=0 (green line) and B=0 (blue line) in the CIE x,y chromaticity space for the CAT02 matrix defined by Eq. (3). The point P_{12} is the intersection point for lines R=0 and G=0, the coordinates of which can be found by solving the first and second equations in (7). P_{13} is the intersection point for lines R=0 and B=0, the coordinates of which can be found by solving the first and third equations in (7). P_{23} is the intersection point for lines G=0 and B=0, the coordinates of which can be found by solving the second and third equations in (7). All the chromaticity coordinates for the intersection points and corresponding R, G, B response values are listed in Table 1.

Table 1: Chromaticity coordinates of the blue (P_{12}), green (P_{13}) and red (P_{23}) primaries in the CIE x,y diagram, and R, G and B responses at these primary points for the CAT02 matrix.

	X	y	R	G	B
P_{23}	0.7114	0.2949	0.6490	0	0
P_{13}	−1.4758	2.5059	0	5.2920	0
P_{12}	0.1439	0.0568	0	0	0.7873

It should be noted that Eq. (2) defines the transform from X, Y, Z space to the R, G, B space. If we consider solving next Eq. (8) for X, Y, Z, the solution is the tristimulus values of the blue primaries [4] in X, Y, Z tristimulus space:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \tag{8}$$

Thus, the third column of the inverse of the matrix M is the tristimulus values for the blue primary. Let

$$S=X+Y+Z,\quad x=X/S,\ y=Y/S\ \text{and}\ z=Z/S \tag{9}$$

Where upon Eq. (8) becomes

$$\begin{pmatrix} 0 \\ 0 \\ 1/S \end{pmatrix} = M \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{10}$$

The first two equations of the above linear system of equations are exactly the R=0 and G=0 lines defined by Eqs. (7). Hence, the intersection point P_{12} of the R=0 and G=0 lines gives the chromaticity coordinates of the blue primary. Similarly, the intersection point P_{13} of the R=0 and B=0 lines gives the chromaticity coordinates of the green primary, while the intersection point P_{23} of the G=0 and B=0 lines gives the chromaticity coordinates of the red primary, leading to the formal definition below.

Definition: The chromaticity coordinates of the primaries of the matrix M are the intersection points for two lines from the three defined by Eqs. (7). Thus, the chromaticity coordinates of the blue primary, denoted as P_{12}, is the intersection point of the first and second lines in Eqs. (7); the green primary, denoted as P_{13}, is the intersection point of the first and third lines in Eq. (7); and the red primary, denoted as P_{23}, is the intersection point of the second and third lines in Eqs. (7). Linking the three primary points in chromaticity space forms a triangular region which is called the triangle associated with the matrix M.

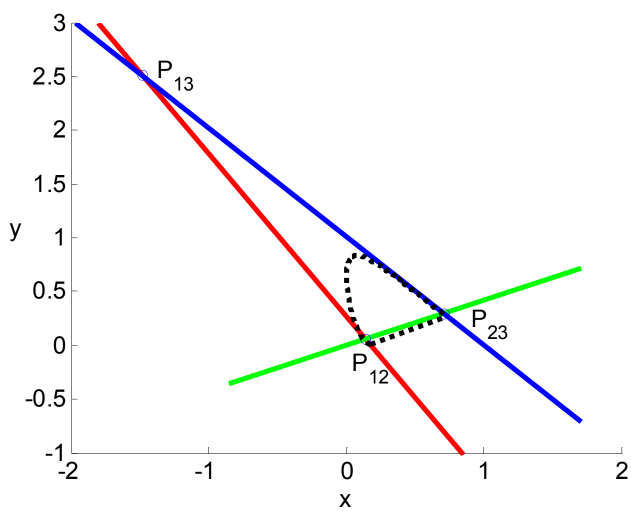

***Figure 1**: Response lines R=0 (red), G=0 (green) and B=0 (blue), and red (P_{23}), green (P_{13}) and blue (P_{12}) primaries in the x,y chromaticity diagram for the CAT02 matrix. The dotted curve is the CIE spectral locus and purple line.*

From Figure 1, the red line divides the whole plane into two parts, one with a strictly positive R response and the other with a strictly negative R response. In particular, from Table 1, the point P_{23} above the red line has a positive response R, and therefore all the points located above the red line have a strictly positive R response. Following similar arguments, it can be concluded that all points located above the green (G=0) line have a strictly positive G response and all points located below the blue (B=0) line have a strictly positive B response. Thus the triangle of the CAT02 matrix in Figure 1 indicates the non-negative R, G, and B responses, which was observed first by Brill and Süsstrunk [4-7].

Note that normally we are interested only in the first quadrant of the x,y space, since the CIE spectral locus and all real colours appear in this region. However, the green primary point is far away from this region, as shown in Figure 1. If they are transformed to the u',v' space, the situation becomes much better, as shown in Figure 2.

Note that Figure 2 shows the side of P_{12} and P_{23} of the CAT02 triangle is mainly inside the region enclosed by CIE spectral locus and purple line. Also, if the side of P_{13} and P_{23} of the triangle is enlarged (see Figure 7 later), this side of the triangle falls outside the region enclosed by the CIE spectral locus and purple line. These facts were observed first by Brill and Süsstrunk [4-7]. The first one is the source of the Purple problem and the second one is the source of the Yellow-Blue problem.

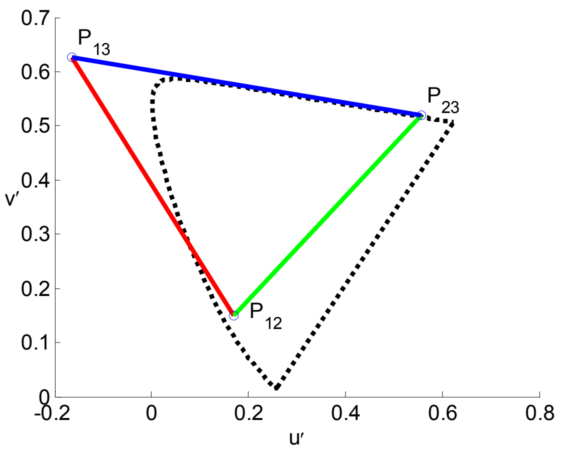

***Figure 2**: Response lines R=0 (red), G=0 (green) and B=0 (blue), and red (P_{23}), green (P_{13}), and blue (P_{12}) primaries in the u',v' space for the CAT02 matrix. The dotted curve is the CIE spectral locus and purple line.*

Table 2: Chromaticity coordinates of the blue (P_{12}), green (P_{13}) and red (P_{23}) primaries in the CIE x,y diagram, and R, G and B responses at these primary points for the HPE matrix.

	x	y	R	G	B
P_{23}	0.8374	0.1626	0.4384	0	0
P_{13}	2.3022	−1.3022	0	−2.0701	0
P_{12}	0.1680	0	0	0	0.8320

Primaries and Region of Non-Negative R, G, B Responses for the HPE Matrix

If the matrix M in the previous section is replaced by the HPE matrix defined in Eq. (4), the primaries of the HPE matrix can be determined analogously, and the resulting chromaticity coordinates of these primaries together with the R, G, B responses at these three primary points are listed in Table 2. The R=0 (red), G=0 (green), and B=0 (blue) lines and their corresponding intersection points P_{23}, P_{13}, and P_{12} together with the CIE spectral locus and purple line (dotted curve) are shown in Figure 3. Comparing the primary points and R, G, B responses with the ones from CAT02 matrix, the following differences can be found:

1) While the green primary point P_{13} was located above the G=0 line for the CAT02 matrix, for the HPE matrix it is located below the G=0 line.
2) At each of the red, green, and blue primary points, the associated response (R G, or B) is positive for the CAT02 matrix, but for the HPE matrix, the response G behaves differently because at the green primary point the G response has a negative value.
3) The CAT02 triangle has non-negative R, G, and B responses. However, the triangle of the HPE matrix has non-negative R and B responses, but non-positive G response. Thus, the triangle of the HPE is not in the region enclosed by the spectral locus and purple line.
4) If the three (R=0, G=0 and B=0) lines divide the whole x,y plane into 7 regions, the region with non-negative R, G, and B responses for the CAT02 matrix is the CAT02 triangle (see Figure 1), but it is the region with the CIE spectral locus for the HPE matrix (see Figure 3). For the CAT02 matrix, there is no region with simultaneous non-positive R, G, and B responses, but, for the HPE matrix, there is one region (labeled N in Figure 3) where all the R, G, and B responses are non-positive.

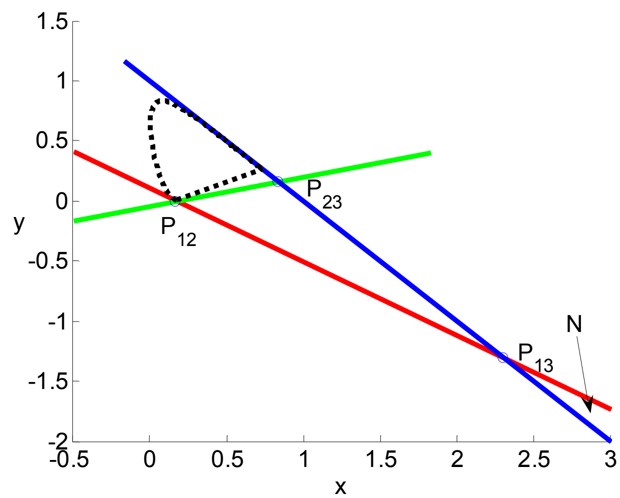

***Figure 3**: Response lines R=0 (red), G=0 (green) and B=0 (blue), and red (P_{23}), green (P_{13}), and blue (P_{12}) primaries in the x,y chromaticity diagram for the HPE matrix. The dotted curve is the CIE spectral locus and purple line. The letter N indicates the region where all the R, G, and B responses are non-positive.*

Note that the HPE triangle does not delimit the region of non-negative R, G, and B responses for the HPE matrix, as was first observed by Brill [4]. However, Brill noted (see Figure 2 in reference [4]) that the region of non-negative R, G, and B responses for the HPE matrix has two parts —the part with the CIE spectral locus and the part labeled with N in our Figure 3— which is not completely correct according to our observations (see item 4 above). The part containing the CIE spectral locus is the region of non-negative R, G, and B responses, but the part labeled with N is the region of non-positive R, G, and B responses.

As noted for the CAT02 triangle and primary points, Figure 3 shows a wide range of values in the CIE x,y diagram, although normally, we are interested only in the part in the first quadrant. This difficulty is overcome once again if the primaries and R=0, G=0, and B=0 lines are represented in the u',v' space. Figure 4 shows all the information for the HPE matrix in the u',v' space. Now it is clear that the HPE triangle in u',v' space (like the CAT02 triangle) clearly encompasses the region enclosed by CIE spectral locus and purple line, which was observed first by Brill. This researcher also observed that the part with non-negative R, G, and B responses and the part with non-positive R, G, and B responses shown in Figure 3 are transformed into the HPE triangle shown in Figure 4; unfortunately, however, he thought that the two parts, hence the HPE triangle in u',v' space, had non-negative R, G, and B responses. As noted above, it is now clear that this is not the case, and we need to consider where the separation occurs between the two parts inside the HPE triangle. To understand this problem fully, we need to investigate the transformation from x,y to u',v' [10]:

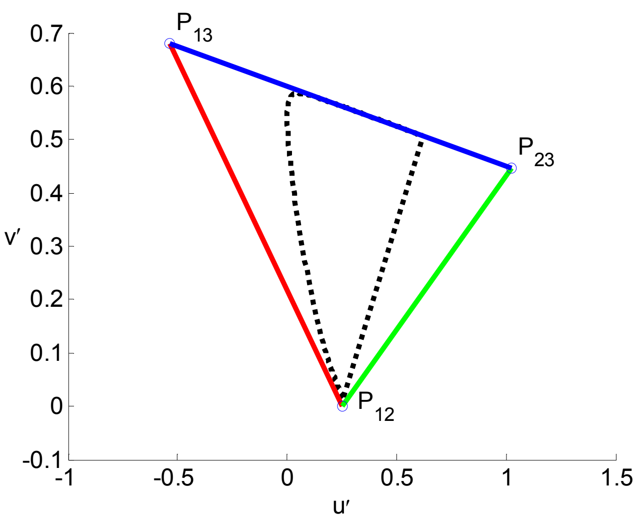

***Figure 4**: Response lines R=0 (red), G=0 (green) and B=0 (blue), and red (P_{23}), green (P_{13}) and blue (P_{12}) primaries in the u',v' space for the HPE matrix. The dotted curve is the CIE spectral locus and purple line.*

$$u' = \frac{4x}{-2x+12y+3} = \frac{4}{-2+12y/x+3/x}, \quad v' = \frac{9y}{-2x+12y+3} = \frac{9}{-2x/y+12+3/y} \quad (11)$$

Since the third row of the HPE matrix defined by Eq. (4) is (0, 0, 1), then, from Eq. (7), the equation in terms of x,y coordinates for the B=0 line is:

$$x+y=1\text{, or } 1+y/x=1/x\text{, or } x/y+1=1/y \quad (12)$$

Thus, when x,y along the B=0 line approach infinity in both directions, u' approaches the same limit. This is also true for v'. If we denote the u' limit by u_{binf}, and the v' limit by v_{binf}, we can show that

$$u_{binf}=-4/14 \text{ and } v_{binf}=9/14. \quad (13)$$

Similarly, when x,y along the R=0 line approach infinity in both directions, u' and v' also have a limit. If we denote the u' limit by u_{rinf}, and the v' limit by v_{rinf}, we can show that

$$u_{rinf}=-0.4291 \text{ and } v_{rinf}=0.5891. \quad (14)$$

Let there be the point B_{inf} with coordinates (u_{binf} , v_{binf}) and R_{inf} with coordinates (u_{rinf} , v_{rinf}). The line linking these two points B_{inf} and R_{inf} is shown in Figure 5 and divides the HPE triangle in the u',v' space into two parts. The lower part has non-negative R, G, and B responses while the upper part has non-positive R, G and B responses.

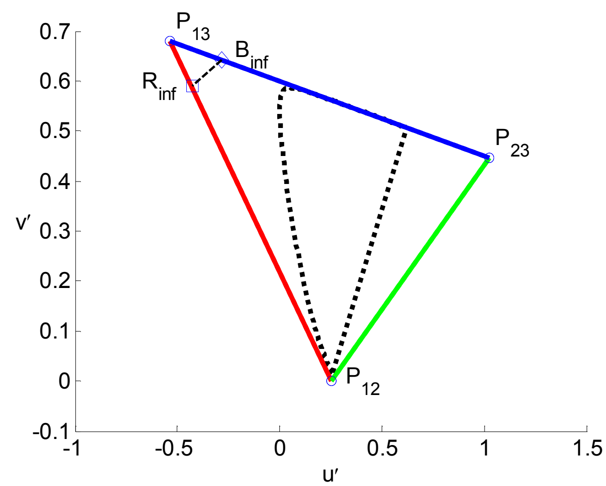

***Figure 5**: Response lines R=0 (red), G=0 (green) and B=0 (blue), and red (P_{23}), green (P_{13}) and blue (P_{12}) primaries in u',v' space for the HPE matrix. The dotted curve is the CIE spectral locus and purple line. The points R_{inf} and B_{inf} are the limits in u',v' space when x,y values along the R=0 and B=0 lines, respectively, approach infinity in both directions.*

Comparison of the CAT02 and HPE triangles in the u',v' Space

It is clearly not easy to compare the primaries and triangles of the CAT02 and HPE matrices in the x,y space, as shown in Figures 1 and 3. However, it is fairly easy to compare them in the u',v' space by plotting them together, as shown in Figure 6. The dotted red, green, and blue lines form the HPE triangle, while the solid red, green, and blue lines correspond to the CAT02 triangle. The black dotted curve is the CIE spectral locus while the purple line links the two ends of the CIE spectral locus. The blue dotted and solid lines and the part of the spectral locus on the side of blue lines appear to overlap, but do not, as observed first by Brill and Susstrünk [4-7]. In fact, this can be seen clearly by enlarging this part, as shown in Figure 7. The B=0 side of the CAT02 triangle is outside the HPE triangle and the CIE spectral locus is nested inside the HPE triangle, which is the source of the Yellow-Blue problem, as noted by Brill & Süsstrunk [4-7]. In addition, the blue primary or the intersection point of the R=0 and G=0 sides of the CAT02 triangle is located inside the domain enclosed by the CIE spectral locus and the purple line, and most of the G=0 side of the CAT02 triangle is located inside the domain enclosed by the CIE spectral locus and the purple line, which is the source of the Purple problem, as noted by Brill & Süsstrunk [4-6].

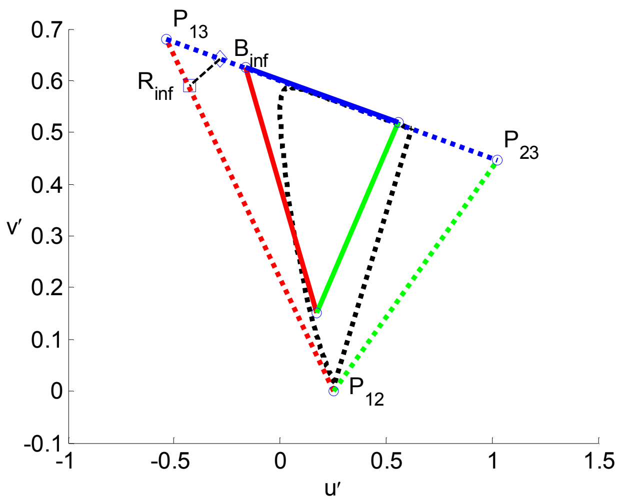

Figure 6: *Response lines R=0 (dotted red), G=0 (dotted green) and B=0 (dotted blue), and red (P_{23}), green (P_{13}), and blue (P_{12}) primaries in u',v' space for the HPE matrix. The black dotted curve is the CIE spectral locus and purple line. The points R_{inf} and B_{inf} are the limits in u',v' space when x,y along the R=0 and B=0 lines, respectively, approach infinity in both directions. The three solid lines (red for R=0, green for G=0 and blue for B=0) correspond to the CAT02 triangle.*

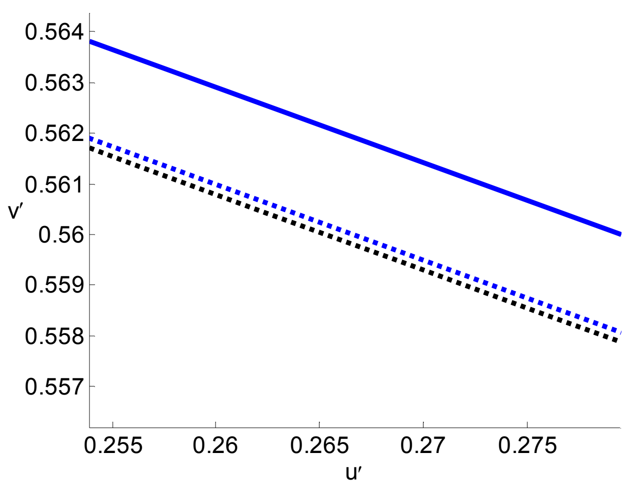

Figure 7: *Detail of Figure 6 along the lower part of the B=0 side of the CAT02 triangle. The solid blue line is part of the CAT02 triangle, the dotted blue line is part of the HPE triangle, and the dotted black curve is the CIE spectral locus.*

It should be mentioned that Brill & Süsstrunk [5-7] gave a solution for the Yellow-Blue problem by setting the last row of the CAT02 matrix to be (0, 0, 1). However, they [5,6] noted that the Purple problem is hard to solve and the CIECAM02 computational failure can be solved if the CAT02 matrix is modified in such a way that the associated triangle lies inside the HPE triangle and encompasses the domain enclosed by the CIE spectral locus and purple line.

Conclusions

In this paper, the primaries, triangles, and regions of the associated non-negative R, G, and B responses for the CAT02 and HPE matrices were investigated and compared. It was found that the HPE triangle in u',v' space encompasses two parts separated by a line linking the point B_{inf} with coordinates (u_{binf} , v_{binf}) defined by Eq. (13) and the point R_{inf} with coordinates (u_{rinf} , v_{rinf}) defined by Eq. (14), as shown in Figure 5, which differs from the findings of Brill & Süsstrunk [4-6]. The results of the present paper are key to overcome the problem of CIECAM02 computational failure and solve the Yellow-Blue and Purple problems simultaneously, which will be discussed in a separate paper.

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