Reconstruction of Surface Spectral Reflectances Using Characteristic Vectors of Munsell Colors

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Abstract

To represent a wide variety of spectral reflectance accurately with possibly small numbers of characteristic vectors, the vectors should be computed from as many as color samples. From the reflectance spectra of 1,565 Munsell color chips (glossy collection) color characteristic vectors are computed by using the Karhunen-Loeve transform (KLT). With the obtained color characteristic vectors, spectral reflectances of Macbeth ColorChecker's test colors are reconstructed and their spectra are shown graphically to see the fitness of our characteristic vectors. For each color, the chromaticity differences between the measured and reconstructed spectra were calculated and shown in the CIE xy chromaticity diagram. As a result, we can reconstruct the measured color reflectance spectra very closely by the first four characteristic vectors. With the first three characteristic vectors, however, color differences between the measured and reconstructed spectra were not negligible.

Introduction

Colors reproduced by color image processing system are affected by various physical factors such as surface reflectances, illuminant spectral power distributions, camera sensor spectral responsivities, monitor phosphor spectral distributions, and human photoreceptor spectral responsivities and all these factors are represented by functions of wavelength. Therefore it is difficult to reproduce the original object color accurately. Thus color calibrations of the input and output data of color image processing system based on spectral distributions are required. For the analysis and synthesis of a color image, color characteristic vectors or basis functions which can represent surface reflectances with a small number of characteristic vectors and their weighting values are essential.

The first study for characteristic vectors of color signal was done by J. Cohen in 1964[1]. Here he used arbitrary selected 150 surface reflectance spectra out of 433 Munsell color chips to evaluate autocorrelation matrix of the spectra. These reflectance spectra were sampled from 380nm to 770nm at 10nm intervals. Using the autocorrelation matrix, the characteristic vectors were computed and the first four vectors are published. The first four characteristic vectors computed by Cohen after normalization are shown in Figure 1(a). With the his first four characteristic vectors, he extracted 99.68% of the cumulative variance.

J.P.S.Pakkinen et al. also computed the autocorrelation matrix from the set of the 1,257 Munsell color chips (matte finish), and published the first eight characteristic vectors in [2]. These reflectance spectra were sampled from 400nm to 700nm at 5nm intervals. Figure 1(b) shows Pakkinen's the first four characteristic vectors out of eight characteristic vectors. He also concluded that the first four characteristic vectors were enough to reconstruct the measured spectra. The mean errors in xy color coordinate system were measured to be 0.0040 and 0.0054 in x and y, respectively. B. A. Wandell[3] has tried to represent the surface reflectances using Fourier basis functions.

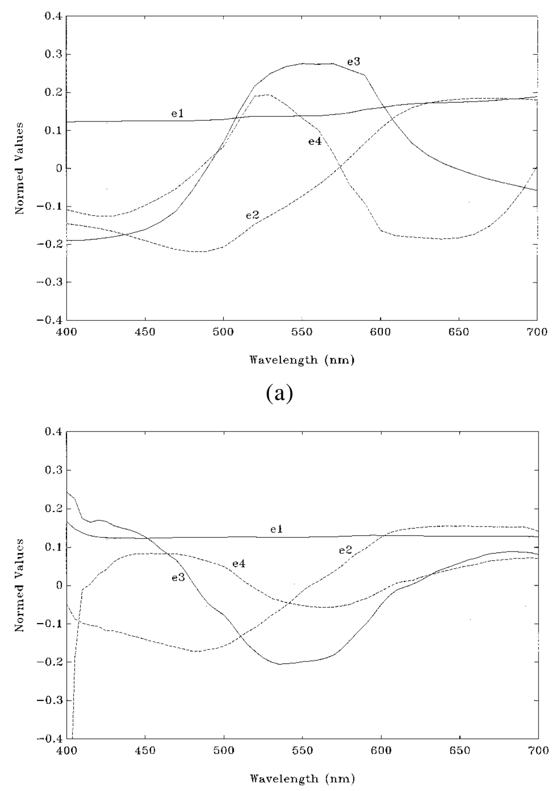


Figure 1. (a) J. Cohen's the first four characteristic vectors (b) Parkkinen's the first four characteristic vectors

Calculation of Color Characteristic Vectors

To compute our characteristic vectors we measured the whole spectra of 1,565 Munsell color chips(glossy collection) with the d/0 geometry and specular excluded method using cary 17 photospectrometer. These reflectance spectra were uniformly sampled from 380nm to 780nm at 10nm intervals.

Now we need to calculate the autocorrelation matrix from measured spectra to compute the color characteristic vectors. Let us denote the measured jth spectrum by the column vector S_j

$$S_j = [s_j(\lambda_1), s_j(\lambda_2), \ldots, s_j(\lambda_N)]^T \quad (1)$$

where $s_j(l_n)$ is the measured surface reflectance coefficient of *j*th color chip at the sampled wavelength l_n and T denotes the transpose. To evaluate the characteristic vectors, autocorrelation matrix R which represents the statistical characteristic of the reflectance spectral distributions is computed using the equation,

$$R = \frac{1}{P}\sum_{j=1}^{P} S_j S_j^T \quad (2)$$

where *P* is total number of color chips used in measurements. The number of color chips used in measurements are shown in ten different color groups in Table 1.

Table 1. The number of Munsell color chips in ten different color groups used in calculation of the autocorrelation matrix *R*.

Used Colors	B	BG	G	GY	P	PB	R	RP	Y	YR
Number of Colors	133	127	143	143	144	155	193	175	181	171

From the calculated autocorrelation matrix *R*, the eigenvectors and the corresponding eigenvalues were computed by the Jacobi method using Karhunen-Loeve transform. Then the eigenvectors of the autocorrelation matrix *R* constructed from the surface spectral reflectances are the very color characteristic vectors. The first eight vectors among the calculated 31 vectors with their corresponding eigenvalues are listed in Table 2. Figure 2 shows the first four eigenvector graphically in the range from 400nm to 700nm to compare with the characteristic vectors obtained by Pakkinen's and Cohen's. Figure 2(b) shows the second four characteristic vectors. Since the first characteristic vector represents the mean of spectra used and it increases as the wavelength goes larger, it means that we have used red-side's color spectra slightly more than the blue-side's in our evaluation.

Reconstruction of Surface Spectral Reflectances

Let us represent *B* characteristic vectors in matrix form in the order of eigenvalue's magnitude among the obtained characteristic vectors, then the matrix is

Table 2. The first eight charateristic vectors computed from 1,565 Munsell color chips.

Wave-length (nm)	The First Eight Characteristic Vectors							
	1	2	3	4	5	6	7	8
380	0.0276	0.0243	-0.0270	0.0391	-0.0217	0.2828	-0.3544	-0.1673
390	0.0451	0.0472	-0.0565	0.0561	-0.0060	0.3252	-0.4441	-0.2330
400	0.0793	0.1024	-0.1248	0.1035	0.0599	0.3252	-0.3859	-0.2234
410	0.1091	0.1633	-0.1911	0.1485	0.1512	0.1904	-0.0226	-0.0354
420	0.1177	0.1867	-0.2105	0.1500	0.1835	0.1033	0.1674	0.0634
430	0.1191	0.1962	-0.2116	0.1342	0.1778	0.0528	0.1807	0.0416
440	0.1204	0.2049	-0.2079	0.1212	0.1543	-0.0080	0.1569	-0.0084
450	0.1212	0.2142	-0.1997	0.1071	0.1184	-0.0741	0.1259	-0.0615
460	0.1218	0.2231	-0.1834	0.0906	0.0645	-0.1243	0.0836	-0.0711
470	0.1233	0.2347	-0.1580	0.0621	-0.0219	-0.1647	0.0003	-0.0327
480	0.1249	0.2460	-0.1187	0.0143	-0.1269	-0.1706	-0.0890	0.0586
490	0.1258	0.2536	-0.0645	-0.0548	-0.2420	-0.1427	-0.1396	0.1622
500	0.1278	0.2519	-0.0008	-0.1181	-0.3231	-0.0799	-0.1423	0.2230
510	0.1329	0.2383	0.0754	-0.1603	-0.3217	0.0012	-0.1070	0.1876
520	0.1377	0.2146	0.1634	-0.1932	-0.2137	0.0749	-0.0196	0.0307
530	0.1393	0.1915	0.2326	-0.2182	-0.0751	0.1350	0.1211	-0.1486
540	0.1401	0.1705	0.2682	-0.2032	0.0346	0.1804	0.2259	-0.2297
550	0.1433	0.1431	0.2865	-0.1296	0.1279	0.1828	0.2322	-0.1842
560	0.1493	0.1030	0.3057	-0.0087	0.2199	0.0888	0.0806	-0.0175
570	0.1538	0.0635	0.3141	0.0982	0.2774	-0.0539	-0.0943	0.1561
580	0.1586	0.0290	0.2917	0.1838	0.2520	-0.1450	-0.1817	0.1971
590	0.1651	-0.0127	0.2367	0.2515	0.1334	-0.1932	-0.2011	0.0979
600	0.1705	-0.0554	0.1703	0.2773	-0.0056	-0.1860	-0.1474	-0.0243
610	0.1740	-0.0917	0.1031	0.2609	-0.1447	-0.1225	-0.0330	-0.1091
620	0.1763	-0.1147	0.0585	0.2287	-0.2156	-0.0525	0.0535	-0.1534
630	0.1780	-0.1277	0.0274	0.1934	-0.2423	0.0001	0.1105	-0.1708
640	0.1796	-0.1340	0.0021	0.1599	-0.2261	0.0319	0.1392	-0.1540
650	0.1810	-0.1368	-0.0223	0.1263	-0.1827	0.0495	0.1470	-0.1142
660	0.1823	-0.1373	-0.0425	0.0879	-0.1279	0.0606	0.1318	-0.0585
670	0.1835	-0.1373	-0.0555	0.0456	-0.0803	0.0822	0.0982	0.0096
680	0.1844	-0.1384	-0.0592	0.0062	-0.0443	0.1075	0.0640	0.0927
690	0.1852	-0.1399	-0.0592	-0.0296	-0.0161	0.1315	0.0315	0.1669
700	0.1860	-0.1408	-0.0590	-0.0606	0.0087	0.1499	0.0048	0.2136
710	0.1868	-0.1415	-0.0610	-0.0842	0.0324	0.1565	-0.0096	0.2350
720	0.1872	-0.1422	-0.0679	-0.0973	0.0541	0.1460	-0.0159	0.2343
730	0.1876	-0.1414	-0.0765	-0.1097	0.0711	0.1219	-0.0239	0.2143
740	0.1893	-0.1356	-0.0841	-0.1343	0.0833	0.0807	-0.0453	0.1517
750	0.1915	-0.1252	-0.0906	-0.1966	0.1085	-0.1004	-0.0273	0.0130
760	0.1942	-0.1124	-0.0974	-0.2405	0.1144	-0.1921	-0.0749	-0.1301
770	0.1959	-0.1032	-0.1047	-0.2717	0.1185	-0.2629	-0.1127	-0.2440
780	0.1972	-0.0977	-0.1078	-0.2891	0.1287	-0.3082	-0.1369	-0.3318
Eigen Values	5.1775	0.3695	0.1193	0.0204	0.0091	0.0050	0.0031	0.0022

$$\phi = [\phi_1, \phi_2, \ldots, \phi_b, \ldots, \phi_B] \quad (3)$$

where the bth characteristic vector ϕ_b is given by

$$\phi_b = [\phi_b(\lambda_1), \phi_b(\lambda_2), \ldots, \phi_b(\lambda_N)]^T \quad b = 1, 2, \ldots, B \quad (4)$$

Then an arbitrarily chosen *j*th reflectance spectrum can be represented approximately by the number of *B* characteristic vectors and corresponding eigenvalue vector ψ^j, i.e.,

$$\Psi^j = [\Psi_1^j, \Psi_2^j, \ldots, \Psi_B^j]^T \quad (5)$$

The *b*th eigenvalue ψ^j_b of the eigenvalue vector ψ^j in Eq. (5) is given by the sum of product of each characteristic vector and the reflectance spectrum as

$$\Psi_b^j = \sum_{i=1}^{N} \phi_b(\lambda_i)\, s_j(\lambda_i) \quad (6)$$

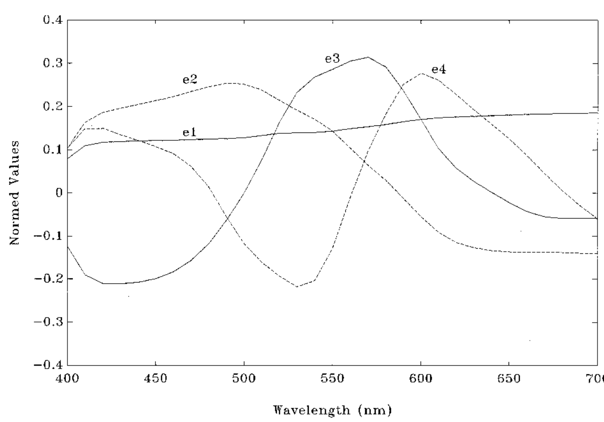
In vector equation form, ψ^j of Eq. (5) can be written as

$$\psi^j = \phi^T S_j \tag{7}$$

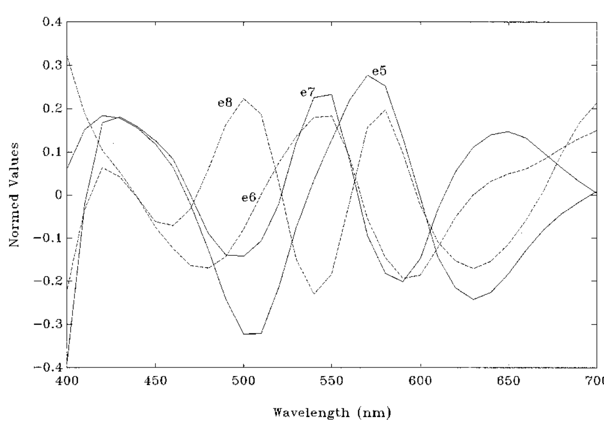
Using Eq. (7) and with B characteristic vectors, the approximated reflectance spectrum S_j of each spectrum is given by the reconstruction equation as follows.

$$\hat{S}_j = \phi \psi^j \tag{8}$$

By choosing Macbeth ColorChecker's twentyfour color spectra as a test color set, we tried to reconstruct each color's reflectance spectrum with the characteristic vectors obtained using Eq. (8). All the reflectance spectra were sampled at 10nm intervals. Figure 3 shows the experimental results in part. Solid curves in Figure 3 show the measured reflectance spectra using Cary2000 photospectrometer and dashed and dashdot curves in Figure 3 represent reconstructed spectra with the first three and four characteristic vectors, respectively. In the experiment, the twentyfour reflectance spectra were not included in the process of obtaining autocorrelation matrix R.



(a)



(b)

Figure 2. (a) our the first four characteristic vectors (b) our the second four characteristic vectors

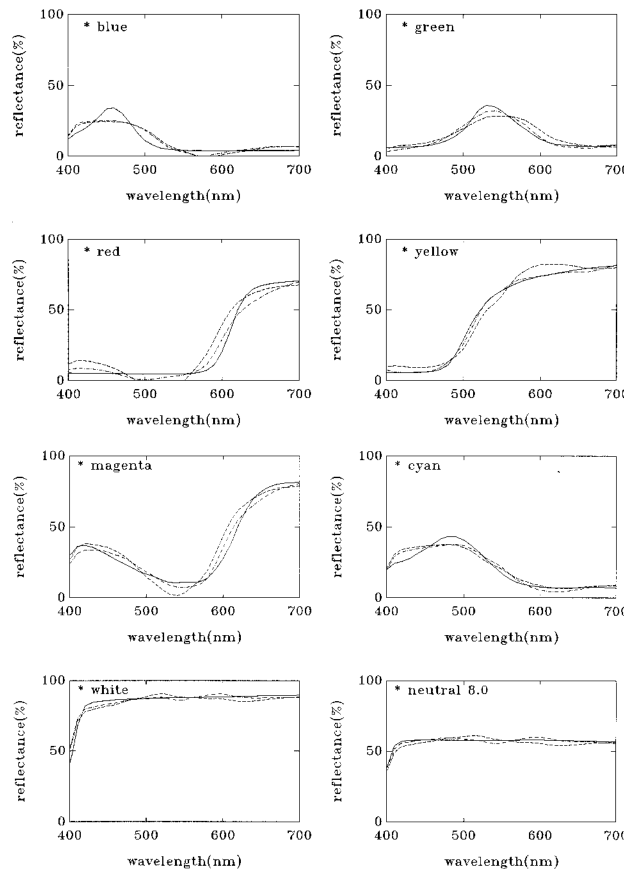


Figure 3. The reconstructed reflectance spectra (shown 8 colors out of 24 Macbeth test colors).

Now, in order to see the color differences of measured and reconstructed spectra on xy chromaticity diagram, let us evaluate the chromaticity coordinates x and y of the reconstructed color signal. The color signal $c(\lambda_i)$, i = 1, 2, ..., N, under *CIE* standard illuminant *C* whose spectrum is shown in Figure 4(a)[4], is given by

$$c(\lambda_i) = s(\lambda_i)\, e(\lambda_i), \quad i = 1, 2, \ldots, N \tag{9}$$

where $e(\lambda_i)$ is sampled spectrum of *CIE* standard illuminant C. Or in matrix form, the equation is

$$C_j^T = S_j^T E \tag{10}$$

where,

$$C_j = [c_j(\lambda_1), c_j(\lambda_2), \ldots, c_j(\lambda_N)]^T, j = 1, 2, \ldots, P \tag{11}$$

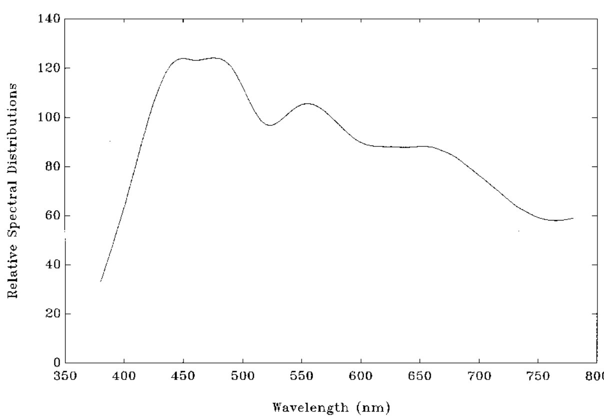
and E is $N \times N$ diagonal matrix whose diagonal entity is the sampled spectrum of CIE standard illuminant C which is $e(\lambda_1)$. Now, the tristimulus values X,Y,Z of the jth color signal C_j are given by

$$X = \sum_{i=1}^{N} C(\lambda_i)\bar{x}(\lambda_i)$$
$$Y = \sum_{i=1}^{N} C(\lambda_i)\bar{y}(\lambda_i) \tag{12}$$
$$Z = \sum_{i=1}^{N} C(\lambda_i)\bar{z}(\lambda_i)$$

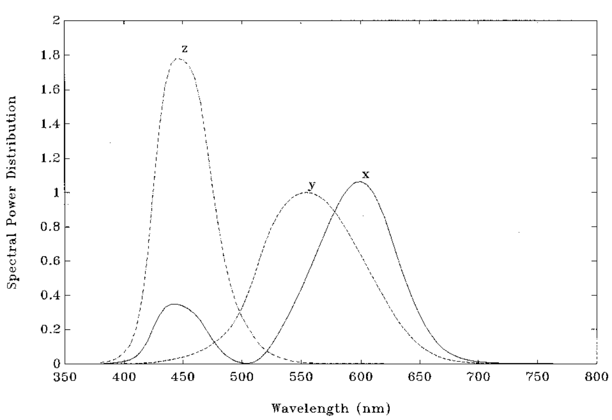
where $\bar{x}(\lambda_i), \bar{y}(\lambda_i), \bar{z}(\lambda_i)$ are sampled CIE xyz color matching functions (2° standard observer)[4, 5] (see Figure 4(b)). From Eq. (12), the chromaticity coordinate x,y is given by the equation

$$x = \frac{X}{X+Y+Z}, \; y = \frac{Y}{X+Y+Z} \tag{13}$$

For the reflectance spectra of Macbeth ColorChecker's twentyfour colors as a test color set, the calculated xy values are represented on xy chromaticity diagram of Figure 5(a) and (b). The symbol o represents x,y coordinates of reflectance spectra for the measured colors and symbol + represents x,y coordinates of reconstructed spectra. Figure 5(a) shows x,y chromaticity coordinates of reconstructed reflectance spectra with the first three characteristic vectors and Figure 5(b) with the first four characteristic vectors. As a result, we can reconstruct the measured reflectance spectra very closely by the first four characteristic vectors. With the first three characteristic vectors, however, color differences of reconstructed spectra were not negligible. Consequently, we can say that for the reasonably accurate reconstruction of the measured spectra we need at least the four characteristic vectors.

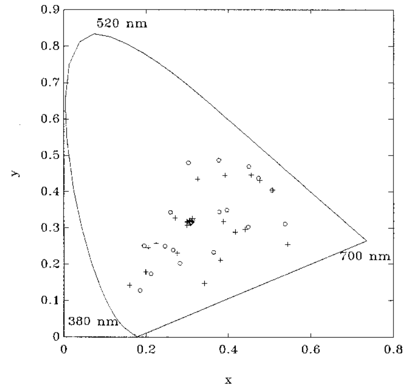


(a)

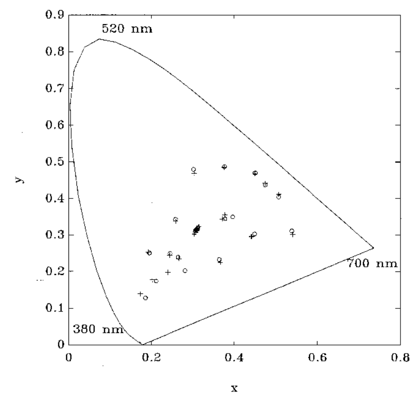


(b)

Figure 4. (a) CIE standard illuminant C spectrum (b) CIE xyz color matching functions



(a)



(b)

Figure 5. The xy chromaticities of the reconstructed spectra with (a) three and (b) four characteristic vectors.

Conclusion

The color characteristic vectors which can be applied to color reconstruction technique are computed from autocorrelation matrix of the entire measured 1,565 sample reflectance spectra of Munsell color chips (glossy collection) based on the Karhunen-Loeve transformation method. With the obtained characteristic vectors, we attempted to reconstruct the measured reflectance spectra of Macbeth ColorChecker's twentyfour color spectra as a test color set and investigated the fitness of reconstructed reflectance spectra with the first three and four characteristic vectors. The results are shown graphically for a number of test colors. Also we represented the reconstructed chromaticities on CIE xy chromaticity plane under CIE standard illuminant C along with the measured ones. Average distance between the measured and reconstructed spectra were computed for goodness of fit. Using the obtained characteristic vectors, we plan to develop an algorithm which can correct input/output color data of imaging system in terms of color image system's spectral distributions to achieve device independent color constancy.

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