A New Method for Calculating Textile Color-Recipe in Uniform Chromaticity Scale System

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Abstract

This article is based on the theory of chromaticity, using the brief mathematical transformation, presenting a method for calculating textile color-recipe in uniform chromaticity scale system (UCS) by linear programming. A lot of experimental result has shown that matching precision may be achieved in the practice.

Introduction

From 1931, CIE XYZ color system has solved the question of the chromaticity by numerical method, but it is nonuniform that we can't use the distance between two color points in color space to express the color-difference of sample and matching. A lot of works have been done to want to find a better Uniform Chromaticity Scale System (UCS) for many years, and now there are many UCS have been used in the color area. For computer color matching in theory, it is much better to use UCS than in XYZ color space. This article is presenting a method for calculating textile color-recipes in UCS (the example is CIE 1976 LAB system) by linear programming.

We are assuming that both the absorption coefficient and the scattering coefficient of the film are linear functions of the concentrations of the colorants, and simplified form of Kubelka-Munk (K-M) theory (opaque films) applies[1]:

$$K/S = (1-R)^2/2R \tag{1}$$

K and S being the K-M spectral absorption and scattering coefficients, and R is the reflectance at wavelength i.

For textile, there are less molecules of dye in the grey cloth, it is considered that the main scattering is only caused by the grey cloth, the scattering effect of dye can be ignored. For mixtures, the K/S is linear with the product of concentration C and K/S of the n individual colorants:

$$(K/S)_{mix} = \sum_{j=400}^{700nm} C_n (K/S)_{nj} = \sum_{j=400}^{700nm} C_n \cdot \varphi_{nj} \tag{2}$$

here $\varphi_{nj} = (K/S)_{nj}$ is called K-M function because of that delegate the optical characteristic of colorants at the dyeing cloth.

This content has been discussed by many papers, so we aren't detail to repeat in this paper.

The Theory of Color Matching in UCS

Making used of the CIE 1976 LAB system, whose transformation equations are[2]:

$$L^* = 116\,(Y/Y_o)^{1/3} - 16 \tag{3}$$

$$a^* = 500\,[(X/X_o)^{1/3} - (Y/Y_o)^{1/3}] \tag{4}$$

$$b^* = 200\,[(Y/Y_o)^{1/3} - (Z/Z_o)^{1/3}] \tag{5}$$

where X_o, Y_o , Z_o and X, Y, Z are the tristimulus values of the illuminant and the sample. In this case, the whole visible spectrum is represented by values at 30 equally spaced wavelengths, from 400 nm to 700 nm. Assuming that the aspectrophotometric match is possible, L^*, a^*, and b^* values of sample and match differ only slightly, and the difference between them can be approximated by a first-order expansion, according to E. Allen's assuming:

$$\Delta R \approx dR = \left(\frac{dR}{d\varphi}\right)\Delta\varphi = -\frac{2R}{1-R}(\varphi_{sam} - \varphi_{mat}) = \frac{2R}{1-R}(\varphi_{bas} - \varphi_{sam} + C_1\varphi_1 + C_2\varphi_2 + \ldots\ldots C_n\varphi_n) \tag{6}$$

R is the spectrum of the sample, φ_{bas} , φ_{sam} and φ_{mat} are the K-M functions of the basic textile, sample and the match, C_1, C_2 ....C_n are the concentrations of n individual colorants.

In UCS, the best condition is the L^*, a^*, b^* value of sample are as same as the matching, but we only can lead the different is the smallest between the sample and the matching in computer color matching practice, according to the differentiation:

$$\Delta L^* \approx dL^* = \left(\frac{\partial L^*}{\partial X}\right)dX + \left(\frac{\partial L^*}{\partial Y}\right)dY + \left(\frac{\partial L^*}{\partial Z}\right)dZ$$
$$= 116\left(\frac{1}{3Y_o}\right)\left(\frac{Y}{Y_o}\right)^{(-2/3)}\Delta Y \quad (7)$$

$$\Delta a^* \approx da^* = \left(\frac{\partial a^*}{\partial X}\right)dX + \left(\frac{\partial a^*}{\partial Y}\right)dY + \left(\frac{\partial a^*}{\partial Z}\right)dZ$$
$$= \left(\frac{500}{3}\right)\left[\left(\frac{X}{X_o}\right)^{(-2/3)}\left(\frac{1}{X_o}\right)\Delta X - \left(\frac{Y}{Y_o}\right)^{(-2/3)}\left(\frac{1}{Y_o}\right)\Delta Y\right] \quad (8)$$

$$\Delta b^* \approx db^* = \left(\frac{\partial b^*}{\partial X}\right)dX + \left(\frac{\partial b^*}{\partial Y}\right)dY + \left(\frac{\partial b^*}{\partial Z}\right)dZ$$
$$= \left(\frac{200}{3}\right)\left[\left(\frac{Y}{Y_o}\right)^{(-2/3)}\left(\frac{1}{Y_o}\right)\Delta Y - \left(\frac{Z}{Z_o}\right)^{(-2/3)}\left(\frac{1}{Z_o}\right)\Delta Z\right] \quad (9)$$

where $\Delta X, \Delta Y, \Delta Z$ are differences of tristimulus between sample and match, under an illuminant of spectral energy distribution $E(\lambda)$:

$$\Delta X = \Sigma E_\lambda \bar{x} \Delta R \Delta \lambda \quad (10)$$

$$\Delta Y = \Sigma E_\lambda \bar{y} \Delta R \Delta \lambda \quad (11)$$

$$\Delta Z = \Sigma E_\lambda \bar{z} \Delta R \Delta \lambda \quad (12)$$

here $\bar{x}$, $\bar{y}$, $\bar{z}$ are the CIE weighting functions for the standard observer, and ΔR is the difference of reflectance of sample and matching.

In theory it is the best when the $\Delta L^* = 0$, $\Delta a^* = 0$ and $\Delta b^* = 0$, but in practice we consider C_{n+1} is the difference of L*, a*, and b* between the sample and the matching, and when C_{n+1} is the utmost small, let Eq. (7) into (11~13), then into Eq. (8~10) we have gotten the forms of linear programming:

Objective function:

$$F_{min} = C_{n+1} \quad (13)$$

Bound equations:

$$D_{L1}C_1 + D_{L2}C_2 + ... D_{Ln}C_n \le C_{n+1} \quad (14)$$

$$D_{a1}C_1 + D_{a2}C_2 + ... D_{an}C_n \le C_{n+1} \quad (15)$$

$$D_{b1}C_1 + D_{b2}C_2 + ... D_{bn}C_n \le C_{n+1} \quad (16)$$

$$C_1, C_2, ... C_n \ge 0$$

D_L, D_a and D_b are constants that have to do with illuminant, spectral of sample and observer. We may get the best recipes concentrations (C_1, $C_2 \cdots C_n$) by dual simplex method. Because of the K-M theory is approximate theory, any one colorant's K-M function value φ_i isn't a constant under the different concentration. We must repair 6-8 grades from low concentration to high concentration to build the data-base of the every colorant. According to these data-base to calculate the recipe concentration by the interpolation. The computing program is shown in Figure 1.

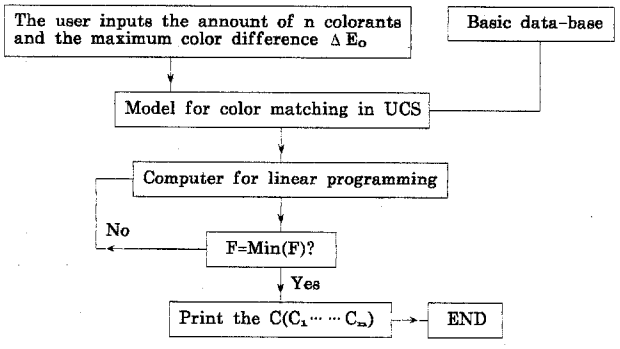

Figure 1. The computer color matching algorithm

Experimental Results and Conclusions

We performed experiments to test this method for many times under the D65 illuminant, CIE 1964 weighting function for the supplementary standard observer and CIE 1976 LAB uniform color space color-difference formula. For example, we use the acid colorants to dye the silk cloth, the results is shown in Table 1.

Table 1. Part results of color matching in XYZ system and UCS, dyeing at the silk by acid colorants.

No.	Tristimulus values of sample	Name of colorants	Matching in XYZ		Matching in UCS	
			C ($C_1 \ldots C_n$)	ΔE	C ($C_1 \ldots C_n$)	ΔE
1	X=22.306 Y=22.619 Z=46.382	Orange GSN Blue 6B Cyanic 4BNS	0.0604% 0.275% 0.099%	0.64%	0.0619% 0.254% 0.103%	0.37
2	X=14.091 Y=11.122 Z=15.983	Orange GSN Black BR Bright Red 10B	0.34% 0.17% 0.48%	0.64%	0.341% 0.171% 0.484%	0.64

Δ E is the color-difference of sample and matching.
$C(C_1 \ldots C_n)$ are the concentration recipes.

In conclusion one can see that this method is simple and effective in practice. There isn't contradictory between two color matching methods. In theory, the color-difference has been considered as a standard of matching quality in UCS is better than in XYZ system.

References

1 E. Allen, "Basic Equations Used in Computer Color Matching", *J. Opt. Soc. Am.*, **10**, 1966.

2. J. A. Cogno, "Linear and Quadratic Optimization Algorithms for Computer Color Match", *Color Research and Application* 1988, **2**, pp. 40-42.