Quality Measure of Color Scanning Filters

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Abstract

Quality of color filters is estimated by the differnce between the two fundamentals projected onto human visual subspace directly from input color space and indirectly via scanning filter subspace using projection operator P. The colorimetric error is graphically visualized in 3D color space assuming the input colors with sinusoidal spectral power distribution.

Introduction

Colorimetric **q** factor by Neugebauer[1] is a useful quality measure of color scanning filters. However, **q** is defined not for a set of 3 colors but for each single color, which is not always perfect in the total evaluation. Generalized goodness[2] of color filters is measured by the error between two fundamentals projected onto human visual subspace(**HVS**) directly from input color space(**ICS**) and indirectly via scanning filter subspace(**SFS**). There, the colorimetric error should be evaluated using the actual color targets and depends on the selected data sets. Here, we introduce a mathematical estimation method not using color targets but using Sinusoidal Spectral Power Distribution(**Sine SPD**). The phase-shifted Sine SPD is available to visualize the color errors by tracing its loci in 3D color space.

Projection to HVS

An input color spectrum C(λ) is described by n-dimentional vector **C** as follows.

$$\mathbf{C}=[c_1\ c_2\ .....\ c_n]^t\ ;\ c_i=C(\lambda_i),\ t=\text{transpose} \tag{1}$$

The projection of input color vector onto HVS from **ICS** via **SFS** is illustrated in Fig. 1.

In **HVS**, **C** is decomposed into the fundamental **C*** and the residue **B**. Fig. 2 shows the shape of $\mathbf{P_v}$ and how it decomposes a spectrum **C** of skin color into the two parts.

$$\mathbf{C=C^*+B,\ C^*=P_vC,\ B=(I-P_v)C} \tag{2}$$

Here, **I** means unit matrix and $\mathbf{P_v}$ represents the projection operator[3] onto **HVS**.

$$\mathbf{P_v=A(A^tA)^{-1}A^t} \tag{3}$$

where, **A** denotes color matching matrix given by CIE x(λ),y(λ),z(λ) as

$$A=\begin{bmatrix} x(\lambda_1) & x(\lambda_2) & & x(\lambda_n) \\ y(\lambda_1) & y(\lambda_2) & \dots & y(\lambda_n) \\ z(\lambda_1) & z(\lambda_2) & & z(\lambda_n)\end{bmatrix}^t \tag{4}$$

The fundamental **C*** carries a true tristimulus vector **T** percepted by human vision and a residue **B** with zero stimulus is not perceived.

$$\mathbf{T=A^t\,C=A^t\,C^*} \tag{5}$$

Next, the projection operator $\mathbf{P_f}$ from **ICS** onto **SFS** is given by the color filter matrix **F** as follows.

$$\mathbf{P_f=F(F^tF)^{-1}F^t} \tag{6}$$

$$F=\begin{bmatrix} r(\lambda_1) & r(\lambda_2) & & r(\lambda_n) \\ g(\lambda_1) & g(\lambda_2) & \dots & g(\lambda_n) \\ b(\lambda_1) & b(\lambda_2) & & b(\lambda_n)\end{bmatrix}^t \tag{7}$$

F is defined by its red,green,and blue color separation filter functions r(λ),g(λ),b(λ). Thus, the fundamental $\mathbf{C_f}$* from color scanning filter is obtained by cascading $\mathbf{P_f}$ and $\mathbf{P_v}$ as

$$\mathbf{C_f^*=P_vP_fC=A(A^tA)^{-1}A^t\,F(F^tF)^{-1}F^t\,C} \tag{8}$$

The tristimulus value of $\mathbf{C_f}$* is

$$\mathbf{T_f=A^t\,C_f^*} \tag{9}$$

Quality Measure of Filters

Neugebauer's q Factor

The ideal filters should be the linear transform of color matching matrix **A** as given by

$$\mathbf{F^t=MA^t}\ ;\ \mathbf{M}=\text{3x3 coefficient matrix} \tag{10}$$

In such case, $\mathbf{P_f=P_v}$ holds good in equ.(6) and **SFS** is equal to **HVS** to bring the correct tristimulus value. However, in practice, the actual color filters don't satisfy equ.(11) strictly. The filter functions r(λ),g(λ),b(λ) are expanded by orthonormal bases $u_1(\lambda),u_2(\lambda),...,u_N(\lambda)$ like as

$$\mathbf{F^t=K^t\,U} \tag{11}$$

$$K=\begin{bmatrix} r_1 & r_2 & \dots & r_N \\ g_1 & g_2 & \dots & g_N \\ b_1 & b_2 & \dots & b_N\end{bmatrix}^t \tag{12}$$

$$U=\begin{bmatrix} u_1(\lambda_1) & u_2(\lambda_1) & \dots & u_N(\lambda_1) \\ u_1(\lambda_2) & u_2(\lambda_2) & \dots & u_N(\lambda_1) \\ u_1(\lambda_3) & u_2(\lambda_3) & \dots & u_N(\lambda_3) \end{bmatrix} \quad (13)$$

Taking notice of $\mathbf{U}^t\mathbf{U}=\mathbf{I}$, $\mathbf{K}$ is given by the inner product of **F** and **U** as

$$\mathbf{K}=\mathbf{U}^t\mathbf{F} \quad (14)$$

Here, Neugebauer introduced MacAdam's orthonormal color matching functions[4] to the first 3 terms of **U**.

If **F** is ideal so that (11) is equal to (10), then it is to be described completely by only first 3 terms of **U** and the remainings signify the errors. Thus, **q** factor is defined by the following ratio of the first 3 terms vs. total energy.

$$\begin{aligned} q_R&=r_1^2+r_2^2+r_3^2/(\mathbf{r}^t\mathbf{r}) \\ q_G&=g_1^2+g_2^2+g_3^2/(\mathbf{g}^t\mathbf{g}) \\ q_B&=b_1^2+b_2^2+b_3^2/(\mathbf{b}^t\mathbf{b}) \end{aligned} \quad (15)$$

The denominator means the norm of each color vector of $\mathbf{K}=[\mathbf{r},\mathbf{g},\mathbf{b}]$.

The **q** factor is useful for estimating each single filter by only its spectral sensitivity. However, the color reproducibility should be connected with a set of 3 color filters and the color error is to be measured in relation to that.

Error in Fundamentals

Human vision perceives the fundamental **C*** extracted from an input color **C**. **C*** carries the true tristimulus value as given in (5). Therefore, the goodness of color filters is measured by the mean square error between the true fundamental **C*** and the filtered fundamental $\mathbf{C_f}$* as given in (16).

$$\mathbf{e}=E[\|\mathbf{C}^*-\mathbf{C_f}^*\|^2] \;\; ; E=\text{averaging operator} \quad (16)$$

Similarly, the color difference between the original tristimulus vector **T** and the filtered $\mathbf{T_f}$ is

$$\Delta\mathbf{E}=\{E[\|\mathbf{T}-\mathbf{T_f}\|^2]\}^{1/2} \quad (17)$$

$\Delta\mathbf{E}$ is to be measured in CIELAB color space, where **T** and $\mathbf{T_f}$ are transformed into (L^*,a^*,b^*) and (L_f^*,a_f^*,b_f^*).

$$\Delta\mathbf{E_{LAB}}=\{E[(L^*-L_f^*)^2+(a^*-a_f^*)^2+(b^*-b_f^*)^2]\}1/2 \quad (18)$$

Estimation Using Sine SPD

Neumerical estimations of **e** and $\Delta\mathbf{E_{LAB}}$ need the actual $C(\lambda)$ data but it's troublesome to use the color chips with measured spectra. In stead of color chips, we applied **Sine SPD** in Fig. 3. **Sine SPD** is described by

$$C(\lambda)=C_0[1+m\sin\{2\pi f(\lambda_i-400)+\phi_j\}] \quad (19)$$

where, $400\text{nm}\le\lambda_i\le 700\text{nm}$, $i=1\sim n$, $0\le\phi_j\le 2\pi$, $j=0\sim N$, C_0 denotes the amplitude and m the modulation factor.

The hue of $C(\lambda)$ changes with the phase ϕ_j. Shifting $C(\lambda)$ according to $\phi_j=2\pi j/N$, the color coordinate draws a closed loop. The loci of **Sine SPD** can cover almost the gamut of **HVS** for its frequency range of $0.001 \le f \le 0.005$ cycle/nm.

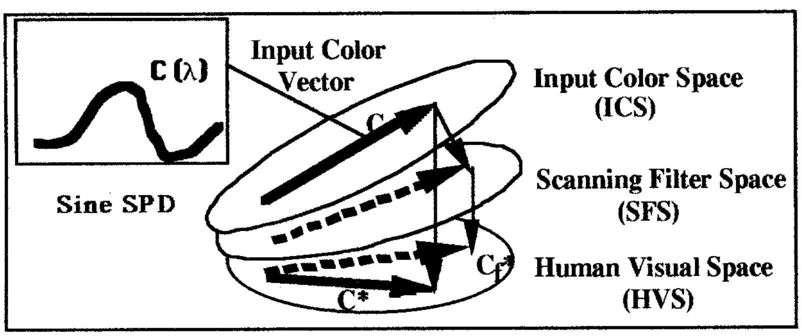

Figure 1. Relation of Projection in Vector Color Space

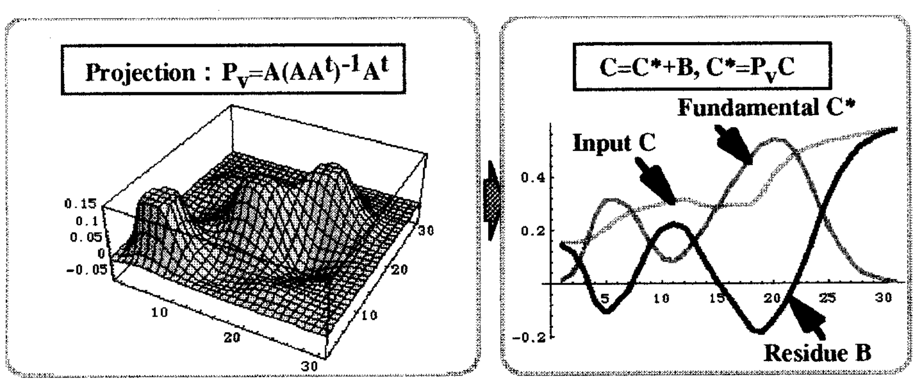

Figure 2. Projection Operator $\mathbf{P_v}$ and its Spectral Decomposition Function

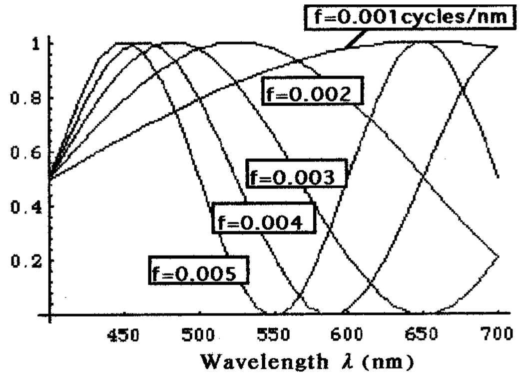

Figure 3. Sine SPD as Input Colors

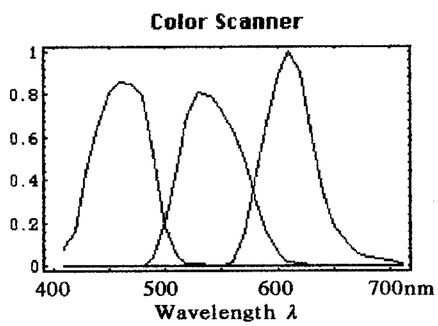

Figure 4. Color Filter Response

The following three typical color scanning filters were tested.

1. Wratten color filter(KODAK #22,#58,#47B)
2. CCD TV camera(Broadcasting)
3. Color Scanner(Model SC2000 made by Matsushita)

Figure 4 shows the filter response of scanner. The total estimation diagram for color scanning filters is given in Figure 5.

Table 1 summarizes the mean squre errors and color differences together with Neugebauer's **q** factors. The estimated projection operator $\mathbf{P_f}$ for color scanner are shown in Fig.6(a). **Sine SPD** loci are traced in XYZ space and visualized in Fig.7(a). The thin lines show the true tristimulus vector **T** and thick lines the filtered $\mathbf{T_f}$. These two line pairs tell how the errors change in 3D color space. It is clear that Wratten is rather worse than the other two. But, CCD camera and color scanner are not always enough and need electronic filter correction for the higher precision color reproduction.

Filter Correction

The color signal from scanning filter can be approximated to the true tristimulus value by electronic color correction. The most simple correction is done by linear matrix operation. A tri-color signal **X** from an input color **C** scanned by filter **F** is given by

$$\mathbf{X}=[R,G,B]^t=\mathbf{F}^t\mathbf{C} \tag{20}$$

Operating a matrix $\mathbf{M_c}$ on **X**, the corrected signal $\mathbf{X_c}$ is obtained by

$$\mathbf{X_c}=\mathbf{M_c X}=\mathbf{M_c F^t C} \tag{21}$$

$\mathbf{X_c}$ is hoped to match with the true tristimulus vector $\mathbf{T}=\mathbf{A}^t\mathbf{C}$ in (5).

The mean squre error $\mathbf{e_x}$ between $\mathbf{X_c}$ and **T** for the selected color chips $\mathbf{C}_j$(j=1~N) is represented by

$$\mathbf{e_x}=E[\|\mathbf{T}-\mathbf{X_c}\|^2]=E[\|\mathbf{A}^t\mathbf{C}_j-\mathbf{M_c F}^t\mathbf{C}_j\|^2] \tag{22}$$

$\mathbf{M_c}$ is determined to minimize $\mathbf{e_x}$ as

Table 1. Results of Error Estimation

	Wratten	CCD	Scanner
Neugebauer's q_R	0.597	0.921	0.929
q_G	0.914	0.956	0.926
q_B	0.719	0.945	0.979
Square error e	0.2051	0.0293	0.0299
Corrected e_c	0.2199	0.0045	0.0031
Color difference ΔELAB	19.53	9.87	9.48
Corrected ΔE_{LABC}	18.04	3.23	3.97

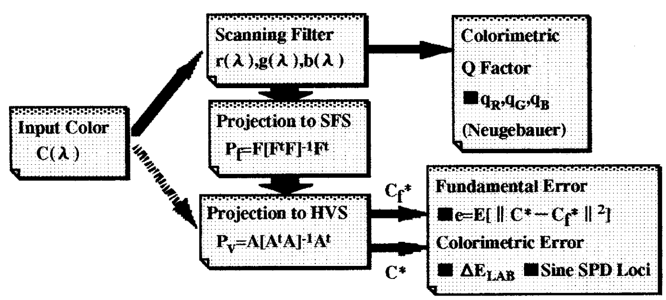

Figure 5. Flow Diagram of Quality Estimation of Color Filters

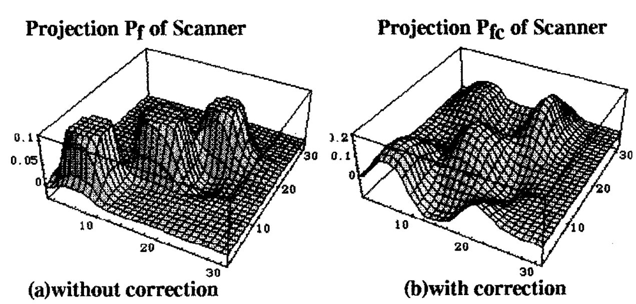

Figure 6. Projection Operator of Color Scanner

$$\mathbf{M_c}=(\mathbf{A}^t\mathbf{RF})(\mathbf{F}^t\mathbf{RF})^{-1}\ ;\ \mathbf{R}=E[\mathbf{C}_j\mathbf{C}_j^t] \tag{23}$$

where, **R** denotes correlation matrix for $\mathbf{C}_j$.

This means to correct the projection operator $\mathbf{P_f}$ like as

$$\mathbf{P}_{fc}=\mathbf{RF}(\mathbf{F}^t\mathbf{RF})^{-1}\mathbf{F}^t \tag{24}$$

Hence, the corrected tristimulus vector $\mathbf{T}_{fc}$ and fundamental $\mathbf{C}_{fc}$* are calculated by

$$\mathbf{T}_{fc}=\mathbf{A}^t\,\mathbf{P}_{fc}\mathbf{C}=\mathbf{A}^t\,\mathbf{RF}(\mathbf{F}^t\mathbf{RF})^{-1}\mathbf{F}^t\mathbf{C} \tag{25}$$

$$\mathbf{C}_{fc}^{*}=\mathbf{P}_v\mathbf{P}_{fc}\mathbf{C}=\mathbf{A}(\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t\,\mathbf{RF}(\mathbf{F}^t\mathbf{RF})^{-1}\mathbf{F}^t\mathbf{C} \tag{26}$$

The filter correction effects for color scanner are shown in **Figure 6(b)** and **Figure 7(b)** as compared with **(a)**. Similarly, $\mathbf{e}_c$ and $\Delta\mathbf{E}_{LABc}$ in **Table 1**, show how the correction reduces the errors. Although Wratten is still poor in spite of correction, CCD and Scanner are possible to be much improved by the correction enough to high quality color imaging.

Conclusions

The quality of color scanning filters is totally estimated by measuring the errors in fundamental spectra projected onto human visual subspace. **Sine SPD** is very useful to evaluate the numerical errors without using actual color targets and to visualize them in 3D color space. Further study should be continued on the generation of the better **Sine SPD**s to fit to the effective color gamuts necessary for the goal of system.

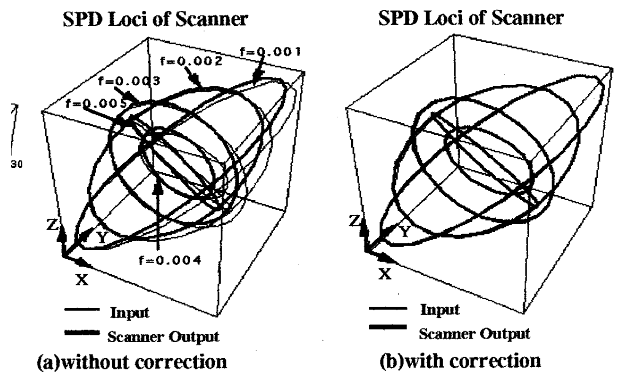

Figure 7. Estimated Sine SPD Loci of Color Scanner

References

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