# When to Use Linear Models for Color Calibration

Doron Sherman and Joyce E. Farrell Hewlett-Packard Laboratories, Palo Alto, California

# Abstract

With the advent of vector-space approaches, linear estimation techniques can be used in various ways for different imaging scenarios. We compare two methods for constructing linear models for surface reflectance spectra, the Principal-Components Analysis (PCA) and the One-Mode Analysis (OMA)<sup>1</sup> applied in simulating image capture under a number of realistic lighting conditions. We demonstrate that successfully using such methods depends on the exact problem at hand.

## Introduction

Color calibration methods involve prediction of device and human sensor responses as well as establishing transformations among them. For 3-sensor input devices, there are two cases in which the tristimulus values of the scene colors can be accurately determined; In the first case, spectral reflectances occurring in the recorded scene can be represented accurately by linear combinations of three basis vectors. Unfortunately, the actual number of basis vectors needed for natural scenes lies in the range of five to seven<sup>2</sup>. In the second case, spectral sampling functions of the recording device relate to the XYZ color matching functions by a linear transformation (i.e., a  $3 \times 3$  matrix)<sup>3</sup>. However, efforts attempting to achieve the latter condition have failed, mainly due to inability to manufacture color filters with the required sensitivities at affordable cost.

As a result, current image recording devices are subject to discrepancies between recorded RGB signals to XYZ tristimulus values of the scene colors, a phenomenon known as eye-versus-camera (or scanner) metamerism. Metamerism is one of the most basic and startling phenomena in color science, and stands for the lack of isomorphism (i.e., one-to-one correspondence) between a physical color stimulus to the evoked sensation (in case of humans) or response (in case of devices)<sup>4</sup>. Since images captured with current input devices are subject to a loss of information present in the original, some sort of estimation procedure is always necessary.

Generally, the illuminant under which the scene (or original) is recorded is different from the illuminant used for viewing the reproduced image. For a scanner, the device illuminant is fixed and can be measured and combined directly with the spectral sampling functions of the sensors. For a camera, the scene illumination can vary and need to be estimated either directly using spectrophotometric measurement or indirectly from the recorded image statistics. However, the spectral power distribution of scene illumination may not be fixed across the image in real situations, either due to interreflections from scene objects or because several types of illuminants may be present. In this paper, we will assume a single illumination that is known and fixed across the recorded image.

Linear models for representing surfaces, illuminants and sensors have become popular in solving color science problems. The popularity stems from the fact that human visual sensors as well as CCD sensors, commonly used in scanners and video and digital cameras, respond linearly to incident light spectra. Also, mathematical methods for dealing with linear models enable a relatively simple formulation of color estimation algorithms. In this paper, we apply such methods to predicting sensor responses from a commercially-available digital camera simulated under a number of different lights. We show that employing more knowledge about the imaging scenario can result in improved accuracy. However, we also demonstrate that applying the same criteria in a different context (i.e., RGB to XYZ color conversion) can result in degraded accuracy and, in fact, straightforward empirical methods perform best.

## Background

A substantial portion of the effort in using linear models is focused on constructing efficient vector-space representations for surface reflectances<sup>5</sup>. This efficiency is ordinarily quantified by computing the deviation of sample spectra from its approximation by a set of basis vectors. Deriving the basis vectors is generally done by performing a principal-component analysis (PCA) on sample spectra, generally through usage of the singularvalue decomposition (SVD)<sup>6</sup>.

Basis vectors derived using PCA indeed provide an efficient representation of the spectral content of surfaces. However, the derivation is based solely on sample spectra of surfaces and does not take into account other factors involved in the image formation process. These factors include the effect of different scene illuminants on the distribution of incident color spectra observed by the capture device, as well as the mass reduction of information performed by the device sensors converting the incident color spectra into three-dimensional RGB signals.

Basis vectors derived using OMA deal better with the above factors by utilizing the filtering effects of the sensors' sensitivities and the illuminants' spectral content across the wavelength scale. Hence, predicting sensor responses using linear transformations based on OMA is less subject to loss of information. It should be noted, however, that linear models derived by OMA will not necessarily be optimal in representing the surface reflectance spectra, although OMA can be used effectively to predict sensors responses<sup>1</sup>.

#### Development

Representation of surface reflectance spectra using a finite-dimensional linear model is ordinarily expressed using the following equation:

$$S(\lambda) = \sum_{i=1}^{i=d} w_i B_i(\lambda)$$
(1)

where color spectra  $S(\lambda)$  is represented as a weighted average of **d** basis vectors  $B_i(\lambda)$  with weights  $w_i$ , or alternatively using matrix algebra as: S = BW where the basis vectors  $B_i(\lambda)$  are stored as columns of matrix **B** and  $w_i$  are stored as columns of matrix **W**. For convenience, the basis vectors can be orthonormalized using a Modified Gram-Schmidt<sup>6</sup>, since any linear non-singular transformation of the vectors spans the same subspace. The main advantage is that the basis vectors become the transpose of the sampling vectors instead of their pseudoinverse.

Mapping the original spectra onto the linear subspace is done using a projection matrix. This is basically a filtering operation of the original spectra using the sampling vectors followed by a span operation using the basis vectors:

$$S' = BW = B (B^{t}S) = (BB^{t}) S = PS$$
(2)

where  $\mathbf{P}$  is the projection matrix and  $\mathbf{S}$ ' is the approximated spectra. Conventional methods for deriving a linear model for sample spectra, such as PCA, perform minimization of the error in the spectral domain, usually via the SVD:

$$\mathbf{E} = \| \mathbf{S} - \mathbf{PS} \| \tag{3}$$

$$S = UDV^{t}$$
(4)

where **S** contains the sample reflectance spectra in column vectors and **D** is a diagonal matrix with monotonically decreasing singular values. The first **d** columns of **U** are taken as orthonormal basis vectors for the linear model,  $\mathbf{B}_{pca}$ . However, for calibration purposes, we are more interested in minimizing the error in the sensor responses domain, using the OMA method, again done via the SVD:

$$\mathbf{E} = \| \mathbf{TS} - \mathbf{TPS} \| \tag{5}$$

$$R = TS = UDV^{t}$$
(6)

where **T** contains all responsivity functions (sensors with illuminant) of interest in the imaging scenario, includ-

ing a human observer with desired viewing illuminants. We obtain the best least-squares approximation by factoring  $\mathbf{R}$  into:

$$(TB) (BtS) = (UdDd)(Vdt)$$
(7)

where  $U_d$ ,  $D_d$  and  $V_d$  are U, D and V with dimensionality reduced to d. The linear model, **B**<sub>oma</sub>, is obtained by solving the following expression for the sampling vectors B<sup>t</sup>:

$$\mathbf{B}^{\mathsf{t}} = \mathbf{V}_{\mathsf{d}}^{\mathsf{t}} \mathbf{S}^{\mathsf{t}} (\mathbf{S}\mathbf{S}^{\mathsf{t}})^{-1} \tag{8}$$

Note that  $\mathbf{B}_{pca} \neq \mathbf{B}_{oma}$  since  $\mathbf{B}_{pca}$  is based on factorization of spectra and  $\mathbf{B}_{oma}$  is based on factorization of sensor responses.

#### Experiment

The two methods, PCA and OMA, are used to derive linear models for predicting sensor responses. Also, we use the derived linear models to construct RGB to XYZ transformations. In both experiments, errors are computed between model-predicted (or transformed) values to the known data. We used a standard test target, the Macbeth Color Checker<sup>7</sup>, with simulation of a commercially-available Kodak DCS200 digital camera. The sensors sensitivities for an actual camera were taken from estimations<sup>8</sup> shown in Figure 1.



Figure 1. Spectral sensitivities of Kodak camera sensors

Recording images was simulated under three different illuminants; fluorescent, tungsten and D6500 daylight (Figure 2). For computing human sensation errors, we selected a 2-degree standard observer and a D50 viewing illuminant.

The first method, PCA, based on the sample reflectance spectra only, derives the spectral functions (equations 3, 4) shown in Figure 3.



Figure 2. Spectral power distribution of scene illuminants



Figure 3. PCA-based spectral sampling functions



Figure 4. OMA-based spectral sampling functions (all lights)

The second method, OMA, is based on the sensor responses which are computed by multiplying the sample reflectance spectra by the sensors' responsivities. These sensors include the Kodak camera under each of the three light sources and the  $XYZ_{D50}$  color matching functions. The spectral functions derived (equations 5, 6, 7, 8) for all the four sensors combined are shown in Figure 4.

#### Results

To compare the performance of the two methods, we calculated the surfaces weights for all five linear models; PCA, OMA for each light (FLU, TUN and D65), and OMA for the three lights combined (ALL). Then, we computed the RMS errors for the best linear-regression fit between surface weights and sensor responses (Table 1). There are four sets of responses; device RGB under each of the three lights (RFLU, RTUN and RD65), and XYZ under the D50 light (XD50). The reference RGB and XYZ sensor responses were calculated directly from the known surface reflectances of the test target and the respective sensors responsivities.

Table 1. RMSE of linear fit for predicting sensor responses

	RFLU	RTUN	RD65	XD50
PCA	3.04	3.78	2.03	3.65
ALL	1.08	0.86	1.01	1.39
FLU	0.67	2.88	1.70	0.73
TUN	1.11	0.70	1.14	1.44
D65	1.32	2.10	1.02	0.92

In addition, we calculated 3×3 transformations between device RGB under each light and XYZ under D50 light for all five linear models (as above) using the following expression:

$$\Gamma^{t}_{xvz} \mathbf{B} \ (\mathbf{T}^{t}_{rgb} \ \mathbf{B})^{-1} \tag{9}$$

where  $T_{xyz}^t$  is the sensors responsivities for XYZ<sub>D50</sub> and  $T_{rgb}^t$  is the sensors responsivities for RGB under each light. Also, we calculated the empirical least-squares 3×3 transform between the 24 RGB values under each light to the 24 XYZ values (EMP), and also the least-squares 3×3 transform between the RGB sensors responsivities under each light to the XYZ<sub>D50</sub> color matching functions (CMF). Then, we computed the RMS errors for the RGB sensor responses under each of the three lights (RFLU, RTUN and RD65) between the reference XYZ values to the transformed XYZ values (Table 2).

#### Discussion

Table 1 demonstrates that using knowledge of the sensors in addition to knowledge of surface spectra is useful at predicting responses from surface weights; all OMA models perform better than the PCA model for all four sensors. As expected, the best fit for the RGB responses under each of the three lights, is always obtained with the linear model based on the same light. Also, the second-best fit for the RGB responses under each of the three lights, is obtained for the linear model based on all three lights combined.

Table 2, however, demonstrates that using the same knowledge for transforming device RGB into XYZ values actually does worse than without it; the PCA model performs better than all OMA models for all three RGB sensors. In fact, the PCA model is very close to the optimal empirical (EMP) transform. Also, it is not even true that the OMA models are always better than the CMF method based on the sensors' responsivities only.

	RFLU	RTUN	RD65
PCA	1.93	3.57	2.32
ALL	4.81	12.20	6.07
FLU	6.81	21.61	10.81
TUN	4.78	10.70	6.51
D65	5.49	16.24	8.16
CMF	17.36	19.42	4.00
EMP	1.90	3.49	2.30

Table 2. RMSE of reference XYZ versus transformed XYZ

In summary, linear models for surface reflectance spectra are useful for color calibration of input devices, but only when used in the appropriate context. Improved accuracy can be achieved in these cases when spectral characteristics of sensors and illuminants present in the captured scene is taken into account. Applications likely to benefit from such approach include those using multiple scanners (different illuminants, different sensors) as well as video and digital cameras (different illuminants, same sensor). Similar benefits can be realized for rendering of spectral waveforms on color monitors in computer graphics applications.

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