A Review of Sampling Effects in the Processing of Color Signals

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Abstract

This paper reviews the sampling of color spectra and its effect on the accuracy of derived properties such as CIE tristimulus values and color rendering indices. The details of numerical computation are considered; the errors and their sources are discussed.

Introduction

With the introduction of digital devices to measure and reproduce color signals, it is commonly assumed that the radiant or reflective spectrum of the color signal can be accurately represented by discrete samples, $r = [r_1 r_2 \ldots r_N]^T$. While color scientists are aware of Shannon's sampling theorem, there is little published work on the determination of the practical sampling rate and its effect on the accuracy of the derived color parameters. Let us consider the simple task of computing the tristimulus values of a reflective surface. The value is defined by

$$t_i = \int_{-\infty}^{\infty} r(\lambda) a_i(\lambda) d\lambda \qquad (1)$$

where $r(\lambda)$ is the power of the radiant spectrum as a function of wavelength and $a_i(\lambda)$ is the i^{th} CIE color matching function[1,9]. It is assumed that the color matching functions are within a linear transformation of the cones sensitivities of the eye and represent continuous analog functions.

The color matching functions were derived from experimental data which represented the functions at discrete samples of wavelength. The experimental data was interpolated, transformed, and tabulated at 1nm increments. The computation of the tristimulus values of the reflective sample, **r**, is represented by

$$\mathbf{t} = \mathbf{A}^T \mathbf{L} \mathbf{r} \qquad (2)$$

where $\mathbf{A} = [a_j(\lambda_i)]$ is an Nx3 matrix of the CIE color matching functions sampled at N wavelengths, **L** is an NxN diagonal matrix representing the spectrum of the illuminant under which the sample is viewed (called the viewing illuminant) and **t** is a 3x1 vector of the tristimulus values.

Color measurements of reflective or transmissive materials using actual hardware can be modelled by a two step process. First the data is obtained from a scanner which uses an illuminant, a set of filters and a detector. The filters are designed so that when combined with the detector and illuminant, the result is approximately a linear transformation of the matrix $\mathbf{A}_L = \mathbf{LA}$. The second step is to transform the data by a linear transformation to the CIE tristimulus values. The entire process is represented algebraically by

$$t = A_L^T r \approx BM^T ODL_0 r \qquad (3)$$

where **M** is the filter set, $\mathbf{L}_0$ is the diagonal matrix whose elements define the instrument illumination, **D** is the diagonal matrix whose elements define the detector sensitivity, **O** is the diagonal matrix whose elements represent the transmission of the optical path and **B** is the 3x3 transformation to obtain the CIE tristimulus values under illuminant **L**.

Bandwidths of Color Signals

From eq. (3), it is seen that there are three basic types of color signals to consider: reflectances (or transmissivities), illuminants and sensors. Reflectances usually characterize common everyday objects but occasionally man-made items with special properties such as filters and gratings are of interest. Illuminants vary a great deal and include natural daylight, office lighting and special lamps used in imaging equipment. The sensors include the cones of the human eye, CCD's and photomultiplier tubes. Let us consider the bandwidths of these classes of signals and the implied sampling rates.

A note on bandwidth is appropriate here. The color matching functions represent continuous functions with finite support. Because of the finite support constraint, they cannot be bandlimited. However, they are clearly smooth and have very low power outside of a very small frequency band.

The most important sensor characteristics are the cone sensitivities of the eye and their transformations defined by the CIE. These are smooth functions and have limited bandwidths. Using 2nm representations of the functions, the power spectra of these signals have shown that 10 nm sampling is adequate. This is based on the observation that the power above 0.05 cycles/nm is down more than 40 dB from the peak. Most sensors have smooth sensitivity curves which have bandwidths comparable to those of the color matching functions.

Several ensembles were used for the study of reflective objects in an attempt to include the gamut of spectra encountered in practice. Reflective spectra were obtained with a spectroradiometer using a sampling interval of 2nm. The collection included the spectra from 170 natural objects (skin, hair, cloth, plastics, etc.), printed material (gravure, inkjet printers, color copiers, etc.), and automotive paint samples. The average spectra of the natural objects shows a narrower bandwidth than that of the color matching functions. However, the wavelength spectra of the few objects with the largest bandwidths (e.g. dark red leaf) require finer sampling.

There are many types of illuminants used for viewing, scanning and measurement. The properties of three standard illuminants, daylight, incandescent and fluorescent, can be used as a guideline for sampling and signal processing which involves other types. It can be shown that the illuminant is the determining factor for the choice of sampling interval in the wavelength domain.

The frequency power spectra for CIE illuminants A (incandescent) and D65 (daylight) indicate that 20 nm sampling is adequate. It is with the fluorescent lamps that even 2nm sampling becomes suspect. The fluorescent lamp can be modelled as the sum of a bandlimited signal, $l_b(\lambda)$, and a delta function series:

$$l(\lambda) = l_b(\lambda) + \sum_{k=1}^{q} \alpha_k \delta(\lambda - \lambda_k) \qquad (4)$$

where α_k represents the strength of the spectral line at wavelength λ_k.

It is clear that the fluorescent signals are not bandlimited. Estimation of the power in the peaks can be done by signal restoration methods which can use the information about this specific signal[4]. Typical frequency power spectra of fluorescent lamps indicate that the bandlimited part can be sampled at 10 nm but the total spectrum has power above -40dB everywhere.

Color Operations

Measurements of the CIE tristimulus values for a variety of illuminants can be obtained from an estimate of the reflectance spectrum of an object. Design of filters for scanners is done by compensating for the effects of illuminants and detectors. The illuminants of scanning devices are characterized digitally. The effect of illuminants on the appearance of colors (color rendering) can be estimated by using sampled spectra. The appropriateness of this sampling rate depends on the accuracy of the values to be computed.

The most common quantity computed is the tristimulus vector associated with a particular object as seen under a particular illuminant, eq. (2). The sampling theorem states that a continuous signal can be reconstructed from discrete samples taken at an interval $\Delta\lambda$ if the signal is bandlimited to $F_{max} < \frac{1}{2\Delta\lambda}$. If the continuous signal can be reconstructed then the continuous integral can be computed numerically with arbitrary accuracy. For this case, the product in the integral is the function which should be bandlimited. The relation of the bandwidth of the signals to the bandwidth of the product is discussed in detail in [6]. For this work, let us review those results.

Consider the simple computation of eq. (1) in the continuous domain. The product

$$p(\lambda) = a_i(\lambda) r(\lambda) \qquad (5)$$

is represented in the frequency domain by convolution

$$P(\omega) = \int_{-\infty}^{\infty} S_i(\alpha) R(\omega - \alpha) d\alpha \qquad (6)$$

It is easily shown that the worst case condition (both functions are rectangular) results in doubling the bandwidth. Fortunately, the CIE color matching functions and most reflectance spectra are low bandwidth signals and the increase in bandwidth is negligible. The exception is products with fluorescent illuminants.

Computation with Sampled Signals

Several applications have been described where proper sampling and computation was critical to computing accurate color values. Consider the common problem of determining the tristimulus values of a measure reflectance under various illuminations. The color matching functions and the standard illuminants, A and D65, are tabulated at 1nm intervals from 360nm to 830nm (471 samples) by the CIE[1]. The resolution of the measurements of the reflecting object is determined by the particular instrument. Assume N samples at $\Delta\lambda > 1nm$ resolution. The computation to approximate the integral in eq. (1) can be done in several ways.

1. subsample the tabulated values; sum the product over the N samples
2. interpolate the measured samples to 1nm resolution; sum the product over 471 samples
3. bandlimit the tabulated values to $\Delta\lambda$nm resolution; sum the product over N samples

It can be shown that if ideal bandlimiting filters and ideal interpolation are used that the results of 2 and 3 are identical. The CIE recommends a procedure similar to 2 which uses a Lagrangian interpolating function[1,8]. A comparison of methods 1 and 3 was done in [6]. It is of interest to compare 2 and 3 under realistic but suboptimal conditions.

From the previous section it is clear that the largest errors will be caused by aliasing (undersampling) of the signals when tristimulus values under fluorescent illumination are desired. These errors are discussed in detail in [7]. Related work is found in [5]. Let us review the general problem here.

If $r(\lambda)$ and $a(\lambda)$ are bandlimited, they may be represented by their samples:

$$r(\lambda) = \sum_{n=-\infty}^{\infty} r(\lambda_n) s(\lambda - \lambda_n) \text{ and } a(\lambda) = \sum_{n=-\infty}^{\infty} a(\lambda_n) s(\lambda - \lambda_n) \qquad (7)$$

where $s(\lambda)$ is an interpolating function, usually the *sinc* function. For this analysis, assume $a(\lambda)$ represents the product of a color matching function and an illuminant. The tristimulus value can then be written as

$$t = \int r(\lambda)a(\lambda)d\lambda = \sum_m \sum_n g(\lambda_m)b(\lambda_n)\int s(\lambda-\lambda_m)s(\lambda-\lambda_n)d\lambda \quad (8)$$

Noting that $\int s(\lambda-\lambda_m)s(\lambda-\lambda_n)d\lambda = \delta(n-m)$, we get

$$t = \sum_n g(\lambda_n)b(\lambda_n) \quad (9)$$

The integral of eq. (8) cannot be obtained exactly from sampled values of nonbandlimited signals. For an arbitrary signal, let

$$f(\lambda) = f_b(\lambda) + f_u(\lambda) \quad (10)$$

where $f_b(\lambda)$ is the bandlimited part of $f(\lambda)$ and $f_u(\lambda)$ is the portion outside the sampling bandlimit. The samples desired are $f_b(\lambda_n)$. For convenience, let us assume that $r(\lambda)$ is bandlimited, i.e. $r_u(\lambda) = 0$, while $a(\lambda)$ is not. This eliminates two cross-terms and makes the equations less cumbersome. Now we can write

$$t = \sum_n r_b(\lambda_n)a_b(\lambda_n) + \int a_u(\lambda)r_b(\lambda)d\lambda \quad (11)$$

The integral term can be treated as aliasing noise which should be compared to the measurement noise and quantization noise.

Let us consider how to estimate the magnitude of the aliasing error. The sampled (aliased) signal, $s(\lambda_n)$, has a frequency spectrum represented by

$$S(\omega) = \sum_{m=-\infty}^{\infty} S_a(\omega - mF_s) \quad (12)$$

where F_s is the sampling rate and $S_a(\omega)$ represents the analog spectrum. The computation from the discrete samples gives

$$\hat{t} = \hat{P}(0) = \int_{-F_s/2}^{F_s/2} \sum_{m=-\infty}^{\infty} A_a(\omega - mF_s) \sum_{n=-\infty}^{\infty} R_a^*(\omega - nF_s)d\omega \quad (13)$$

If the reflectance is bandlimited, the aliasing error can be approximated by

$$\epsilon_a = \int_{-F_s/2}^{F_s/2} \sum_{m\neq 0} A_a(\omega - mF_s)R_a^*(\omega) \quad (14)$$

Interpolation with Non-Sinc Functions

Bandlimiting the signals is a natural way for a signal processor to approach the problems caused by aliasing. In practice, *sinc* functions are not used since they have infinite support. There is a vast literature on the design of approximate bandlimiting filters, e.g. [3]. The CIE has made recommendations on interpolation of sampled signals which were made independently of signal processing considerations. It is instructive to compare the two approaches.

The interpolation of eqs. (7) is a convolution where the interpolating function is the kernel. The CIE recommends using the Lagrange interpolating function which is based on the assumption that the function to be interpolated is a polynomial[2]. The most common practical interpolating functions for bandlimited signals are windowed *sinc* functions[3]. These are obtained by multiplying the ideal bandlimiting filter by a finite window.

A comparison of the performance of the Lagrange and windowed sinc interpolators depends of the signals used. If synthetic color spectra are used which are, in fact, polynomials, the Lagrange method will perform better since that is the assumption under which it was derived. If bandlimited functions are used, the windowed sinc method will perform better because of the more nearly constant transfer function in the passband of the filter. Experiments have verified these predictions. It is noted that the CIE color matching functions were obtained by interpolation. This was most likely a polynomial fit similar to the Lagrange method. In this case, the characteristics of this signal should be taken into account when evaluating the performance of the methods.

Truncation Effects

The CIE recommends computation over the range 360nm to 830nm. Unfortunately, most spectral measuring devices do not cover this entire range. In the case of truncated data, the CIE recommends summing the values of the CMF outside of the data range and multiplying the sum by the last data value. This is equivalent to extrapolating the spectra by repeating the last value and interpolating as before.

The comparison of performance over the extrapolated region is inconclusive. The Lagrange method will interpolate the constant value exactly. The sinc function will result in an ocsillating approximation which will give different results from the CIE recommended values. The experiment which would give interesting results is to use a database of spectra which are known over the entire CIE range and use both methods to extrapolate and compute the tristimulus values. At this time the author does not have such a data base and using a synthetically generated one is likely to bias the results.

Summary

The characteristics of color signals has been discussed. It was noted from previous work that most color signals are sufficiently bandlimited to allow sampling at 10nm. The exception is illuminants which have sharp spectral peaks. A discussion of methods for computation of tristimulus values indicated that the recommendation of the CIE are not optimal in terms of the sampling rate but usually give results that are close to optimal. The effects of truncation were noted and a method of testing extrapolation was proposed.

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