

# Fast Colour Vesselness

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## Abstract

*Vesselness in an image is a map that conveys the extent to which certain image structures resemble blood vessels. The standard approach to this problem uses greyscale images. An earlier algorithm [1] derives a vesselness map for a colour image from the Hessian of a pure quaternion whose components are the colour channels. As an alternative to that method, we here divide the vesselness task into two parts: Convert the colour image to a grey image using the Fast Color2Grey algorithm [2, 3], and then use the traditional Frangi Hessian method [4] on that grey image to produce the vesselness map. Compared with the quaternion-based algorithm, the method proposed here is more accurate in identifying retinal blood vessels and also operates  $10^4$  times as fast.*

## 1. Introduction

In [2, 3] a fast method was developed for generating a greyscale equivalent to an input colour image such that contrast was approximately preserved. This took the place of a more complex method utilizing the structure tensor at each image pixel and, in its simplest form, used a very fast approach generating gradient maxima for  $x$  and  $y$ , taken over the three colour channels R,G,B. This replaced associating contrast with the main eigenvector of the structure tensor, which arguably provides the optimum map from colour contrast to grey contrast. In this max-gradient “Fast Colour2Grey” method, contrast-preserving grey gradients are the output, with a corresponding grey image developed using projection onto integrable gradient components and subsequent re-integration using a least-squares solution of the Poisson equation. The latter step is also fast because it can be carried out in the Fourier domain.

In [1] a method was proposed for incorporating colour information into the “vesselness” problem, a method in medical imaging for generating a measure indicating long tubes, and thus useful for problems such as identifying blood vessels in retinal images or blood flow within an artery from angiograms. The standard approach to this problem was enunciated in [4], and uses eigenvalues of a Hessian matrix (the  $2 \times 2$  array of second-derivatives of the image with respect to  $x$  and  $y$ ) to indicate the direction of least curvature. The method [1] for incorporating colour into this method was based on an elaborate construction utilizing colour as a pure quaternion, and a quaternion-based singular value decomposition to calculate the needed eigenvalues.

Here instead we use the Fast Colour2Grey method, as a mechanism for including colour contrast, and then apply

the standard vesselness algorithm. As well as verifying that indeed colour does improve accuracy of finding vessels, we obtain substantially better results than the previous colour method, along with an increase in speed of 4 orders of magnitude.

The Frangi vesselness measure [4] is a standard approach to obtaining an indicator of structures in medical imaging that have much higher curvature across a direction than along that direction.

The Frangi filter identifies vessels using the information in the eigenvalues of the Hessian matrix  $H_\sigma$  computed at scale  $\sigma$ . Since the Hessian involves second-derivatives, it is curvature that is being used as the feature. The steps of the algorithm are as follows:

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### Algorithm 1:

For each scale  $\sigma$ :

The image is convolved with a Gaussian kernel of size  $\sigma$ .

At this scale we calculate the Hessian matrix  $H_\sigma$ .

Derivatives are “ $\gamma$ -scaled” with a power, to maintain similarity of scales

(default  $\gamma = 2 \Rightarrow \partial^2/\partial_{x^2}$  is scaled by  $\sigma^2$ ).

For each Hessian matrix we calculate the eigenvalues  $\lambda_1$  and  $\lambda_2$  with  $|\lambda_1| < |\lambda_2|$

The main idea behind the Frangi filter is that for an ideal vessel we have  $|\lambda_1| \approx 0$  and  $|\lambda_1| \ll |\lambda_2|$

Frangi et al. used the ratio  $R_B = \lambda_1/\lambda_2$  as a vesselness measure since it attains its maximum value for a blob-like structure and is zero whenever  $\lambda_1 \approx 0$  or  $\lambda_1$  and  $\lambda_2$  tend to vanish (still remains bounded).

To dampen the effect of background pixels, the magnitude of the derivatives  $\mathcal{S} = \sqrt{\lambda_1^2 + \lambda_2^2}$  is also considered, where a low value indicates low vesselness.

The vesselness measure at scale  $\sigma$  is then defined to be:

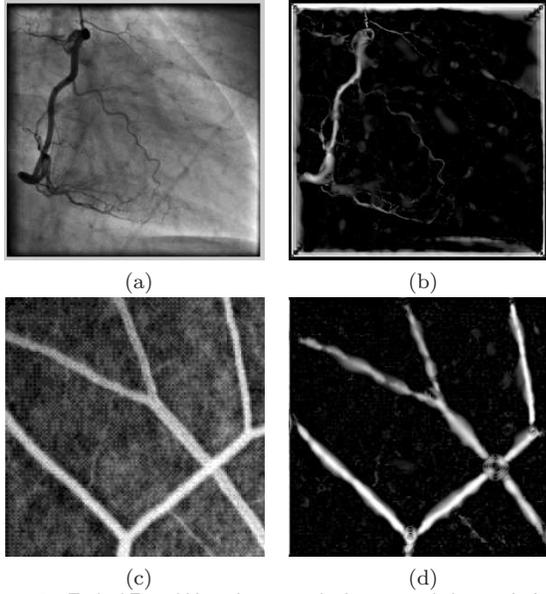
$\mathcal{V}_\sigma = 0$  if  $\lambda_2 < 0$   
else  $\exp[-\mathcal{R}_B^2/(2\beta^2)] (1 - \exp[-\mathcal{S}^2/(2c^2)])$ ,

where  $\beta$  and  $c$  are parameters which control the sensitivity of the filter to the measures  $\mathcal{R}_B$  and  $\mathcal{S}$ .

Defaults values are  $\beta=0.5$  and  $c$ =half of the max-value of the Hessian Frobenius norm.

A set of  $\sigma$  values is used (default = {1, 3, 5, 7, 9}) and the feature is taken to be the max-feature across scales.

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**Figure 1.** Typical Frangi-Vesselness results for greyscale inputs. Left: Input; Right: Output.

Fig. 1 shows typical results for medical image greyscale input images. We obtained these results by running a standard reference implementation [5] provided in (non-optimized) Matlab, on input greyscale coronary artery and angiogram images.

Other approaches for devising vesselness descriptors led up to and have evolved since the seminal paper [4]: Using the Hessian to capture second-order derivative information was first proposed by Koller et al. [6] and enhanced by Lorenz et al. [7], Sato et al. [8] and Frangi et al. [4], all analyzing eigenvectors of the Hessian matrix to estimate vessel orientation. Later methods building upon the Frangi vesselness algorithm are also typically built upon curvature and the Hessian (see, e.g., [9]).

As an alternative to the Hessian, Armande et al. [10] and Prinnet et al. [11] used the Weingarten matrix in differential geometry. In another effort, Westin et al. [12] used eigenvalue analysis of a tensor built from a set of 3D quadrature filters. In a recent work, Bauer and Bischof [13] applied the Frangi vesselness criteria to the local Jacobian of the vector space obtained from gradient vector flow diffusion [14] and claimed this alternative approach overcomes most of the shortcomings of multi-scale Hessian based filters.

Nevertheless the original method is that which is cited most frequently, notwithstanding its known tendency to perform poorly for crossings (when 3D is imaged in 2D) or bifurcations, and also for high-curvature points. The original method is typically retained as a preprocessing step for more complex vessel segmentation methods [15].

Lesage et al. (2009) state that “the method from Frangi et al. (1998) has been extensively used in practice, owing to its intuitive geometric formulation” and therefore, in order to make a comparison with the most standard technique, the Frangi filter [4] is that which we use here. The interested reader may peruse the extensive overview

of vessel detection techniques in [16, 9, 15].

## 2. Colour Contrast into Grey Contrast

Here we propose including colour information into the vesselness calculation by simply using the greyscale output of the Fast Colour2Grey method [2, 3] as input to the standardized Frangi-Vesselness method, rather than generating an elaborate (and slow) mathematical framework to incorporate colour as in §3 below. Moreover, in §4 we show that this simpler approach is considerably more accurate in terms of agreement with ground-truth vessel segmentation.

The Fast Colour2Grey method forms a fast approximation for a standard and disciplined method for mapping colour contrast into greyscale contrast. Let a colour image be denoted  $\rho$ , with components  $k = 1..3$ . Then the gradient image for each channel  $k = 1..3$  has two components, denoted  $\rho_{,x}^k, \rho_{,y}^k$ : respectively the partial derivatives  $\partial\rho^k/\partial x$  and  $\partial\rho^k/\partial y$ . Di Zenzo’s *structure tensor*  $\mathbf{Z}$  is formed as the symmetric  $2 \times 2$  matrix

$$\mathbf{Z} = \begin{pmatrix} \sum_k \rho_{,x}^k \rho_{,x}^k & \sum_k \rho_{,x}^k \rho_{,y}^k \\ \sum_k \rho_{,x}^k \rho_{,y}^k & \sum_k \rho_{,y}^k \rho_{,y}^k \end{pmatrix} \quad (1)$$

Since  $\mathbf{Z}$  is symmetric, its eigenvectors are the 2-vector columns  $\{\mathbf{v}_1, \mathbf{v}_2\}$  of a real, orthogonal matrix  $\mathbf{V}$ . The eigenvector associated with the largest eigenvalue points in the (unsigned) direction of maximum contrast [17]. So a standard way to make a grey gradient from a colour set of gradients is to adopt as the output grey gradient the direction of maximum change, which is the maximum-eigenvalue direction  $\mathbf{v} = \mathbf{v}_1$ . Vector sense (sign) is taken to be that of increasing luminance value.

Now an excellent approximation of this approach, that is much faster than finding eigenvectors (and also dispenses with the need to watch for sign flips) was given in [2]. That method uses the straightforward scheme of using the maximum change over all the colour channels, R,G,B, as a simple but effective approximation to the eigenvector approach.

The method went on to distinguish between forward-facing or backward-facing derivatives, and generating a best-maximum in each case. Here, in the interests of speed we adopt the simplest formulation for the problem, as given in [2]: find the maximum, over R,G,B, for the right-facing  $x$ -derivative and the downward-facing  $y$ -derivative. Then the approximation of a gradient formed for a greyscale image,  $g$ , is  $\nabla g$  given by

$$\left\{ \begin{array}{l} \text{using colour gradients } \nabla\rho = \{\rho_{,x}^k, \rho_{,y}^k\}, k = 1..3, \\ \text{find scalar-field } g \text{ gradients:} \\ \nabla g_{,x}^E = \max_k^{abs} (\rho_{,x}^{k,E}) \\ \text{where } E \text{ is the East-facing } x\text{-derivative;} \\ \text{and similarly for the South-facing } y\text{-derivative.} \end{array} \right. \quad (2)$$

That is, we have chosen the maximum change in any colour channel as our tentative grey-gradient  $\nabla g$ .

However, it need not be the case that  $\nabla g$  actually forms the gradient of an image — i.e., the gradient pair  $\nabla g$

need not be “integrable” (and in fact it would be surprising if it were so). Therefore we must form a best least-squares solution to recover a grey image  $g$ . This is best carried out in the Fourier domain and here we make use of the method set out in [18], where the least-squares solution is directly found in Fourier space. The output of that method is a greyscale field, unique up to an unknown constant of integration, which we set here by mapping the mean to the mean of the luminance. The method [18] is extremely fast because it simply involves an analytic step combining the Fourier transform of the two gradient components into the Fourier transform of a greyscale whose Fourier gradient is closest in a least-squares sense to the given input gradients. Altogether, we thus combine a very fast max-finding routine with this also fast Fourier image generator, making for a fast greyscale generator.

The output of Fast Colour2Grey is then input to Frangi et al.’s vesselness procedure, which is also fast (assuming a fast built-in eigenanalysis). To indicate how this approach behaves consider an input colour image, shown in Fig.2(a) (reproduced from [1]<sup>1</sup>). If we simply use Luminance, defined here as  $L = R + G + B$ , then the Vesselness feature is as in Fig.2(b). On the other hand, if we first incorporate colour contrast by using the Fast Colour2Grey algorithm then output is as in Fig.2(c) — an examination reveals more features are discernible using colour information as part of the input.

Below, in §4 we go on to quantify this improvement stemming from colour information, utilizing a database of colour digital retinal images.

### 3. Complex Colour Hessian

Along with complex numbers, quaternions form another variant of generalized numbers. In this case there is still one real value, but now 3 pure quaternion values, making up a 4-component quantity. The rules for composition of quaternions were set out by Hamilton in 1843. Unit vectors for the three pure-quaternion values are denoted  $i, j, k$ .

In [1] it is suggested to move colour into the quaternion realm by forming a pure-quaternion Hessian matrix,

$$H_q = i \begin{pmatrix} R_{,xx} & R_{,xy} \\ R_{,yx} & R_{,yy} \end{pmatrix} + j \begin{pmatrix} G_{,xx} & G_{,xy} \\ G_{,yx} & G_{,yy} \end{pmatrix} + k \begin{pmatrix} B_{,xx} & B_{,xy} \\ B_{,yx} & B_{,yy} \end{pmatrix}$$

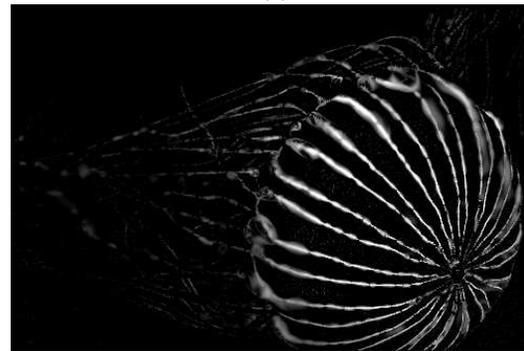
Then to develop the eigenvalues required in Algorithm 1 above, Ref. [1] suggests utilizing the quaternion version of Singular Value Decomposition. Here, we simply made use of the standard Matlab package [19] for this quaternion-SVD calculation. This quaternion SVD calculation is exceedingly slow, and this is compounded by the fact that derivatives are taken over five scales  $\sigma$ , with the maximum forming the feature. Calculation of vesselness then proceeds by substitution of these eigenvalues into the standard Frangi filter [5].

The two colour-based methods are summarized as fol-

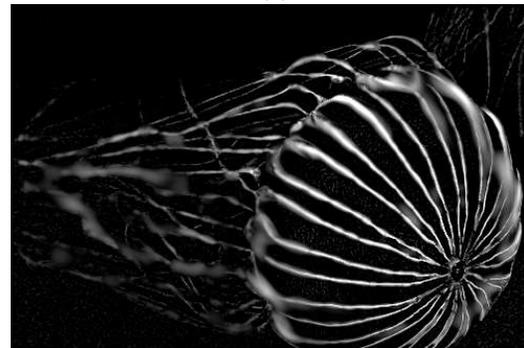
<sup>1</sup><http://www.funny-potato.com/jellyfish.html> (access date: 2011/3/15)



(a)



(b)



(c)

**Figure 2.** (a): Colour input image; (b): Vesselness result using greyscale Luminance from RGB image; (c): Vesselness using Fast Colour2Grey greyscale.

lows:

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**Algorithm 2:** Fast Colour2Grey

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Apply the Fast Colour2Grey algorithm to the input colour image.

Proceed with Algorithm 1.

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**Algorithm 3:** Quaternion Colour

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Assign the 3 channels of the input colour image to a pure quaternion.

Apply the quaternion-SVD routine.

Proceed with Algorithm 1.

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## 4. Application to Digital Retinal Images

Images such as the colour input in Fig.2(a) are fairly noise-free. However, many images required as input to vesselness determination are quite noisy. The colour retinal images in the DRIVE database (Digital Retinal Images for Vessel Extraction) [20] constitute a standard set of such difficult noisy images: as well as having low contrast, some background intensity values are larger than vessel intensities, and as well there is interreflected light present in the form of reflection of light on vascular walls. The influence of strong edges may typically result in two vessels being identified rather than one, for each vessel profile.

Fig. 4(a) shows an input colour image, and figures (b,c,d) show results for the standard Luminance-based vesselness measure, for the Fast Colour2Grey based vesselness, and for the method [1]. As well, Fig. 4(e) shows the manually segmented boolean result for ground-truth vessels and (f) shows the boolean result for the grey from Fast Colour2Grey. For each method, identical program parameters were used, in the reference implementation [5]. For Method [1], eigenvalues were calculated by that method instead, but all else remained the same. (Note that using the parameters as stated in [1] instead wrongly produced double edges: bi-modal distributions across vessels.)

To compare methods, we generated vesselness greyscale images, and identified pixels above the 95-percentile as vessels. Comparison with ground truth is via the sum of absolute differences between the boolean manually-selected vessels and those identified by each method, divided by the image size to form a percentage error. Results are shown in Table 1, over the test database consisting of 40 input images. The Fast Colour2Grey method had a typical runtime of 1.3 sec. for an image resolution of  $584 \times 565$  pixels; whereas the method [1] had an average runtime of  $2 \times 10^4$  sec. for the same sized image.<sup>2</sup>

## 5. Conclusions

As can be seen from Table 1, colour has an important effect on generating more accurate vesselness segmentation. The simple Fast Colour2Grey method provides sufficient

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<sup>2</sup>Mean errors in Table 1 are for all 40 database images for the first two rows; for the third row only 10 images were processed, on a colony of PCs with a great many cores and large distributed memory, before 50-hour walltime out-of-time job crash.

Method	Min%	Median%	Mean%	Max%
Luminance	5.62	7.26	7.36	9.16
FastCol2Grey	<b>5.23</b>	<b>6.89</b>	<b>6.93</b>	<b>8.91</b>
Quatern.	8.46	9.27	9.53	11.10

**Table 1: Mean sum of absolute errors for boolean vessel pixels (divided by image size, as percentage).**

colour contrast information so as to improve Frangi's vesselness procedure. Moreover, it improves on a previous colour method's accuracy, whilst speeding up run time by 4 orders of magnitude.

In regard to future work, the approach we have used here to combine colour and vesselness has been to simply use the Fast Colour2Grey procedure to make a grey, and then make a Hessian. Other approaches to be explored would also include (i) Finding the Hessian in R and in G and in B, and use each max matrix element overall; and (ii) Using the max-gradient over R and G and B and go on to develop a Hessian matrix from a further derivative of that gradient, as opposed to re-integrating to a greyscale image as an intermediate step.

As an improvement to output generated from the Frangi et al. algorithm, we have begun to consider how to provide an accented colour output, utilizing information in the input RGB image along with the output vesselness. Fig. 3 shows an input retinal image along with the same image, but with the achromatic channel replaced by the vesselness value. Here, we have used Wandell's colour opponent space [21] to split brightness from chroma: the original chroma is put back into the accented image, with luminance replaced. Further work could extend such images and explore how colour may be best put to use in vesselness visualizations.

So far, we have used the most-utilized embodiment of the vesselness idea [4]. For future work, we shall consider the alternative descriptors for vesselness as discussed in §1 and determine what effect colour contrast has on these competitor methods.

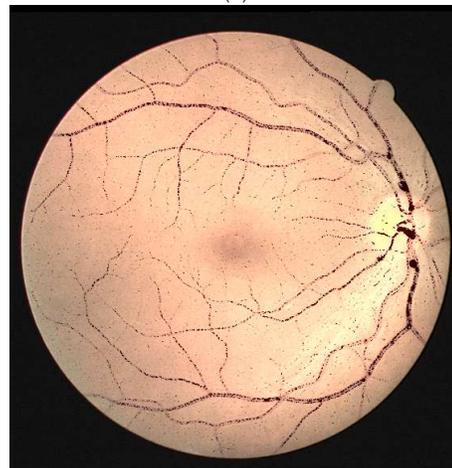
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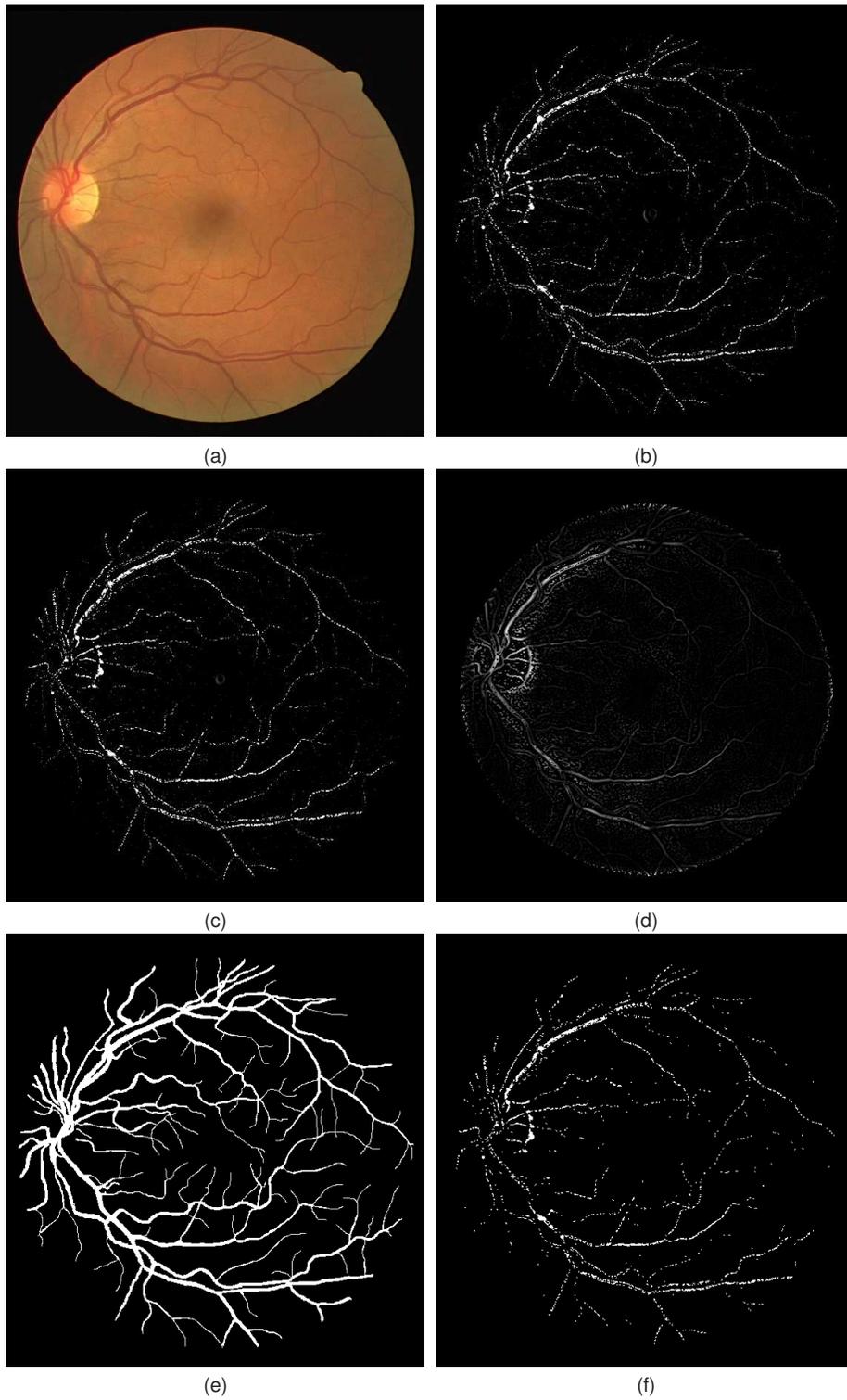


(a)



(b)

**Figure 3.** (a): Input colour retinal image; (b): Output replacing Luminance channel by Vesselness measure.



**Figure 4.** (a): Input image; (b): Vesselness value, using Luminance in the standard algorithm [4]; (c): Vesselness using the output from Fast Colour2Grey [2]; (d): Quaternion-based method [1]; (e): Manually segmented boolean vessel pixels. (f): Fast Colour2Grey boolean vessel identification.