

# A Study on Spectral Response for Dichromatic Vision

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## Abstract

The paper proposes a novel approach to analyze the dichromatic color vision defects from a point of spectral responses based on the projection theory of spectral space to/from 2-D dichromatic Human Visual Sub-Space. The visible spectra to the dichromats (protanopes, deutanopes, and tritanopes) are extracted from an  $n$ -dimensional spectral input with the 2-D version of Matrix- $\mathbf{R}$  notated as  $\mathbf{R}_{dichro}$ . Since the matrix  $\mathbf{R}_{dichro}$  is an identical and invariant mapping operator inherent in human vision that is independent of any linear transformation or any illuminant, the fundamental spectra  $\mathbf{C}^*_{dichro}$  sensed with matrix  $\mathbf{R}_{dichro}$  are also inherent in the dichromats. The lost spectra are easily obtained as a difference in the fundamentals between the normals and the dichromats. These lost spectral profiles tell us why the color appearances are similar to the protanopes and deutanopes, and dissimilar to the tritanopes. The perceived colors are simulated based on the two hypotheses of substitution and nulling processes.

## Introduction

Color vision deficiency is quite common, about 8% males have any of color blindness. So far, many approaches to simulate the color deficiencies have been reported.

Dichromatic color vision arises from missing one of the  $LMS$  cones,  $L$  type in protanopes,  $M$  type in deutanopes, and  $S$  type in tritanopes. In comparison with normal trichromatic vision, dichromats mistake a color discrimination in the reduced 2-D color gamut. Brettel et al [1] proposed a solution to this problem and Viénot et al [2] advanced the model to a palette-based graphical simulation system on sRGB display. Since different colors (Gamut I) can be perceived by a dichromat as equal to the test stimulus  $T$ , the simulation  $S$  is to get the intersection of Gamut I and to find the set of colored stimuli (Gamut II) whose appearance is the same for normals as for dichromat. However their algorithm has the drawbacks in the high computation costs and especially in the difficulty of determining the Gamut II.

Recently, P.Capilla et al [3], simplified the corresponding pair procedure through the Gamut I and Gamut II into a systematic color transform model of  $T$  to/from  $S$  based on the following two hypotheses as

[A] Substitution hypothesis: Dichromacy arises from the substitution of one photopigment by another ( $M$  by  $L$  for protanopes,  $L$  by  $M$  for deutanopes, and  $L$  or  $M$  by  $S$  for tritanopes), but the subsequent neural circuitry is normal.

[B] Nulling hypothesis: One of the opponent chromatic mechanisms (red-green for protanopes and deutanopes, blue-yellow for tritanopes) is nulled in some or all of the opponent stages of the model.

These models simulate a dichromatic vision on display screen just as a normal observer experiences the same sensation viewing  $S$  as a dichromat viewing  $T$  through  $RGB$  to  $LMS$  and further into the opponent-color space such as  $ATD$  and through their inverse transformations. However, since these models are described by

tri-color signals, we can know the lost colors in dichromatic vision only as the color differences in tri-color space, not in the spectral space.

This paper discusses the dichromatic color vision based on the spectral responses and clarifies what parts in the spectra visible to the normals are lost for the dichromats of protanopes, deutanopes and tritanopes. A dichromatic view on display is also simulated according to the method by P.Capilla not using  $ATD$  (Guth) but with  $IPT$  [4] (Ebner & Fairchild) as an opponent color space. A simulated image is compared with those by existing algorithms.

## Basic Concept

The paper introduces a novel approach to the dichromatic vision from a point of spectral analysis as follows.

### [1] Projection to Dichromatic Fundamentals

Based on the projection theory, a  $n$ -dimensional spectral input is projected onto 2-dimensional dichromatic HVSS (Human Visual Sub-Space), that is, the dichromatic FCS (Fundamental Color Space). In the dichromatic FCS, the *fundamentals*, are extracted as the spectral components visible to the dichromats based on the 2-D version of matrix- $\mathbf{R}$  theory.

### [2] Extraction of Lost Spectra for Dichromats

The spectral responses are analyzed using sinusoidal SPDs (Spectral Power Distributions) and typical spectral color targets such as Macbeth, IT8 or Munsell. The lost spectra are simply extracted as the differences in the *fundamentals* captured by the normals and the dichromats through the projection matrix  $\mathbf{R}_{dichro}$ .

### [3] Color Appearance Simulation

The substitution and the nulling hypotheses on the dichromats are applied to the  $LMS$  cone space and the  $IPT$  opponent-color space respectively. The simulated colors for dichromats are calculated by inserting the substitution and the nulling processes in between the forward and backward transformation path ways.  $IPT$  is adopted because it's an excellent opponent-color space with hue linearities and chromatic uniformities [5].

**Figure 1** overviews the proposed model. The key to the new approach lies in the introduction of modified dichromatic version of matrix- $\mathbf{R}$ , here notated as  $\mathbf{R}_{dichro}$ . The model is roughly divided into two stages. In the first half stage, the *fundamentals*, spectra visible to dichromats are extracted through  $\mathbf{R}_{dichro}$  and the lost spectra are easily analyzed by comparing with *fundamentals* for normals. The latter half stage simulates the colors to be viewed on display for normals just as experiencing the similar color appearance as the dichromats, where the *fundamental* spectra perceived by dichromats are converted to the corresponding  $LMS$  /sRGB signals through the inverse processes. In the path of latter stage, the conversion from  $LMS$  to  $IPT$  is included to reflect an opponent-color encoding function in human brain. The inverse signal processings in the latter stage are generally the same as Capilla, originating from Brettel and Viénot.

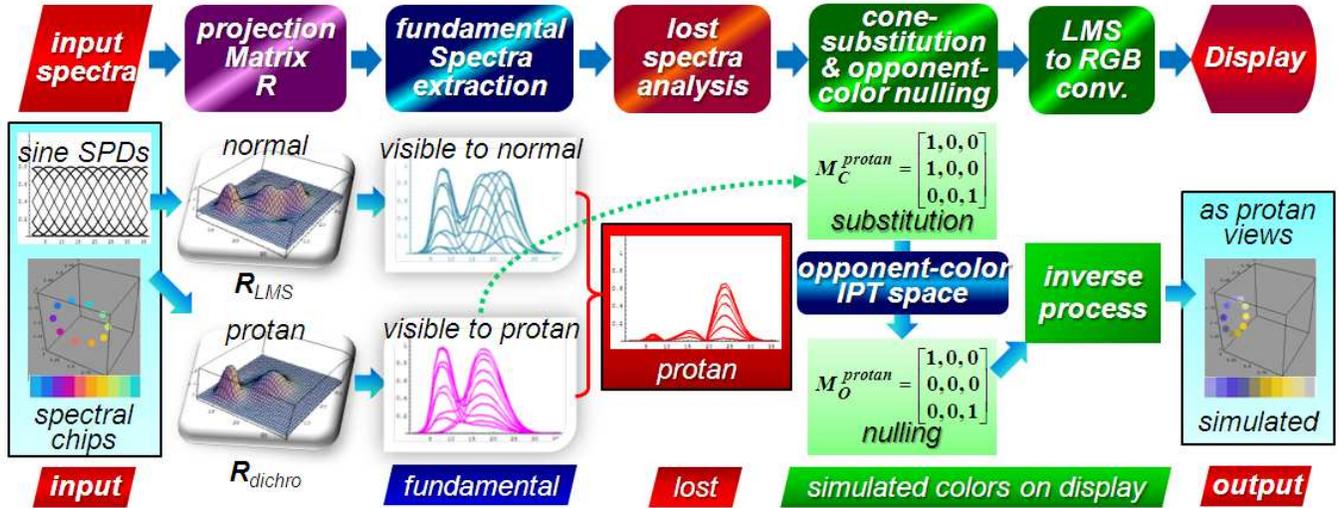


Figure 1 Overview of proposed model (in case of dichromatic vision for protanopes)

### Projection to Dichromatic Fundamentals

A color matching function (*cmf*)  $A$  transforms a  $n$ -dimensional spectral input  $C$  into the tri-stimulus vector  $T = XYZ$ . While, according to “matrix  $R$ ” theory [6],  $C$  is decomposed into the *fundamental*  $C^*$  and *metameric black*  $B$  in the HVSS as

$$C = [C(\lambda_1) \ C(\lambda_2) \ \dots \ C(\lambda_n)]^t, \quad t = \text{transpose} \quad (1)$$

$$C = C^* + B, \quad C^* = RC, \quad B = (I - R)C \quad (2)$$

$I$  denotes unit matrix and the projector  $R$  onto HVSS is derived from CMF  $A$  as

$$R = A(A^t A)^{-1} A^t \quad (3)$$

$A$  is the  $n \times 3$  matrix of 1931 CIE  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  *cmf*.

The *fundamental*  $C^*$  is the essential spectrum to the normal trichromatic vision that carries the the intrinsic color stimulus, perceived as the unique XYZ tri-stimulus sensation. The *metameric black*  $B$  is the residue insensitive to human vision and spans  $n-3$  dimensional null space.

Here, the projection matrix  $R_{LMS}$  from  $n$ -dimensional spectral space to 3-dimensional LMS space is derived by replacing the *cmf*  $A$  with *cmf*  $A_{LMS}$  as follows.

$$\begin{aligned} R_{LMS} &= A_{LMS} (A_{LMS}^t A_{LMS})^{-1} A_{LMS}^t \\ A_{LMS}^t &= M_{XYZ \rightarrow LMS} A^t \\ &= \begin{bmatrix} I(\lambda) \\ m(\lambda) \\ s(\lambda) \end{bmatrix} = \begin{bmatrix} I(\lambda_1) & I(\lambda_1) & \dots & I(\lambda_n) \\ m(\lambda_1) & m(\lambda_1) & \dots & m(\lambda_n) \\ s(\lambda_1) & s(\lambda_1) & \dots & s(\lambda_n) \end{bmatrix} \\ M_{XYZ \rightarrow LMS} &= \begin{bmatrix} 0.4002 & 0.7075 & -0.0807 \\ -0.2280 & 1.1500 & 0.0612 \\ 0.0000 & 0.0000 & 0.9184 \end{bmatrix} \end{aligned} \quad (4)$$

Where,  $I(\lambda)$ ,  $m(\lambda)$ , and  $s(\lambda)$  are the LMS cone sensitivities. Now, the matrix  $R_{LMS}$  is easily extended to the dichromatic version  $R_{dichro}$  by reducing the dimension from 3 to 2 as

$$\begin{aligned} R_{dichro} &= A_{dichro} (A_{dichro}^t A_{dichro})^{-1} A_{dichro}^t \\ A_{dichro} &= A_{protan} = [m(\lambda), s(\lambda)] \text{ for protanopes} \\ A_{dichro} &= A_{deutan} = [I(\lambda), s(\lambda)] \text{ for deutanopes} \\ A_{dichro} &= A_{tritan} = [I(\lambda), m(\lambda)] \text{ for tritanopes} \end{aligned} \quad (5)$$

Thus, the *fundamental*  $C^*_{LMS}$  for normals is given by

$$C^*_{LMS} = R_{LMS} C = RC = C^* \quad (6)$$

$C^*_{LMS}$  is the same as the *fundamental*  $C^*$ , as far as XYZ to LMS transformation is linear through matrix  $A_{LMS}$ .

Indeed, matrix  $R_{LMS}$  and matrix  $R$  are the same identity projection matrices irrelevant to any linear transformation as

$$R_{LMS} = R = R(R) = R(R(R)) \dots = R^m = R_{LMS}^m \quad (7)$$

As well, the dichromatic version  $R_{dichro}$  is also an identical projection matrix holding

$$R_{dichro} = (R_{dichro})^m, \quad m = 2, 3, \dots, \infty \quad (8)$$

Figure 2 illustrates these identity projection matrices.

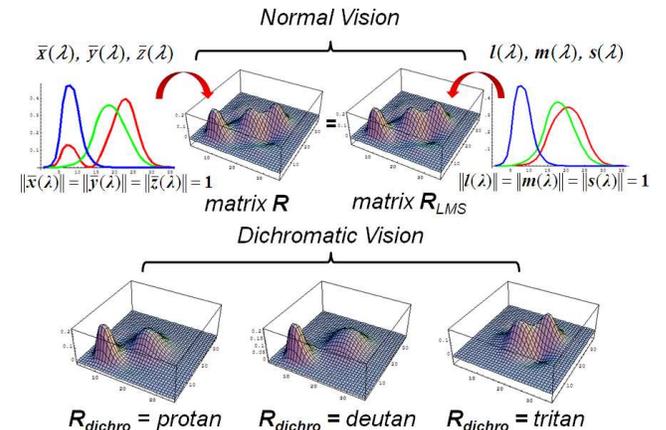


Figure 2 Identity projection matrices onto dichromatic HVSS

## Extraction of Lost Spectra for Dichromats

The *fundamental*  $C_{LMS}^*$  carries the visible spectrum essential to the normals. As well, the essential spectrum visible to the dichromats is extracted as the  $C_{dichro}^*$  by operating the projection matrix  $R_{dichro}$  in the equation (5) to a spectral input  $C$  as

$$\begin{aligned} C_{dichro}^* &= R_{dichro} C \\ &= C_{protan}^*, \text{ or } C_{deutan}^*, \text{ or } C_{tritan}^* \end{aligned} \quad (9)$$

In detail, each *fundamental* for protanopes, deutanopes, and tritanopes is given by

$$\begin{aligned} C_{protan}^* &= [A_{protan} (A_{protan}^t A_{protan})^{-1} A_{protan}^t] C \\ C_{deutan}^* &= [A_{deutan} (A_{deutan}^t A_{deutan})^{-1} A_{deutan}^t] C \\ C_{tritan}^* &= [A_{tritan} (A_{tritan}^t A_{tritan})^{-1} A_{tritan}^t] C \end{aligned} \quad (10)$$

The lost spectral components are calculated as the differences in between the *fundamental*  $C_{LMS}^*$  for normals and the *fundamental*  $C_{dichro}^*$  for dichromats as

$$\Delta C_{dichro}^* = C_{LMS}^* - C_{dichro}^* \quad (11)$$

The spectral responses for the dichromats are estimated using the following three types of spectral color chips/

- [A] *Sinusoidal SPD generated by computer*
- [B] *IT8 Spectral Standard Color Chart*
- [C] *Munsell Spectral Color Chips*

### Virtual Spectral Chips: Sinusoidal SPDs

The sinusoidal SPD is a useful input for analyzing the spectral responses of human vision. Barlow [7] introduced the use of comb-filtered sine SPDs for an analysis of trochro- macy of human vision. Buchsbaum [8] and Brill [9] also used sine SPDs to explain the three degrees of freedom in HVSS.

Sine SPDs are described by

$$\begin{aligned} C(\lambda_i)_{jk} &= 0.5C_0 \{1 + m \sin(2\pi f_k (\lambda_i - 380) + \phi_j)\} \\ \text{where, } \lambda_i &= (380 + 10i) \text{ nm } (i = 1 \sim 36): \text{ wavelength} \\ 0 \leq C_0 &\leq 1: \text{ amplitude, } 0 \leq m \leq 1: \text{ modulation factor} \\ f_k &= 0.001k \text{ cycles/nm } (k = 1 \sim K): \text{ frequency} \\ \phi_j &= 2\pi(j-1)/J \quad (j = 1 \sim J): \text{ phase} \end{aligned} \quad (12)$$

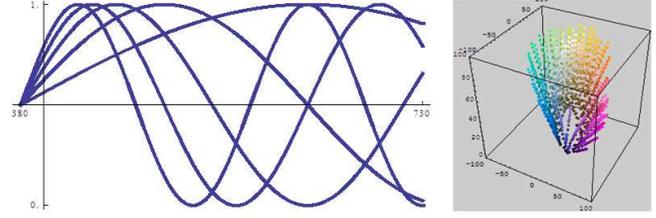
Human vision is most responsive to the sine SPDs in the frequency range of  $0.001 \leq f_k \leq 0.005$  cycles/nm.

A  $n$ -dimensional spectral color vector  $C_{jk}$  is generated for a combination of  $(f_k, \phi_j)$  setting the parameters  $C_0$  and  $m$  as

$$C_{jk} = [C(\lambda_1)_{jk}, C(\lambda_2)_{jk}, \dots, C(\lambda_n)_{jk}]^t \quad (13)$$

The color hue changes with the phase shift by  $\phi_j$ . Though the sine SPDs are virtual spectral color chips, they can cover the wide color gamut in HVSS and may be useful as a test stimulus to examine the spectral frequency responses for the trichromatic or dichromatic visions when the frequency  $f_k$  is swung low to high with the shifts of phase  $\phi_j$ .

**Figure 3** illustrates a basic spectral shape for  $0.001 \leq f_k \leq 0.005$  with  $C_0=m=1$  and generated 1000 chips for  $j=1-20$ .



**Figure 3** Sine SPD spectral shapes and 1000 chips in CIELAB

## Color Appearance Simulation Experiments

The color appearances for the dichromats are examined through the two simulation experiments as follows.

### Experiment [A] ~ Simulation based on Cone Substitution Hypothesis

In the Experiment [A], only the substitution hypothesis is considered and the colors seen by the dichromats are simply estimated from the *fundamentals* in eqs.(9) and (10).

The *fundamental*  $C_{dichro}^*$  is transformed back to the corresponding tri-color sRGB display signals as follows..

First, the dichromatic LMS color vector  $C_{LMS}^{dicro}$  corresponding to the perceived *fundamenta*  $C_{dichro}^*$  is calculated according to the substitution hypothesis as follows.

$$C_{LMS}^{dicro} = M_C A_{LMS}^t C_{dichro}^*$$

$$\text{where, } M_C = M_C^{protan}, \text{ or } M_C^{deutan}, \text{ or } M_C^{tritan}$$

$$M_C^{protan} = \begin{bmatrix} 1, 0, 0 \\ 1, 0, 0 \\ 0, 0, 1 \end{bmatrix}, M_C^{deutan} = \begin{bmatrix} 0, 1, 0 \\ 0, 1, 0 \\ 0, 0, 1 \end{bmatrix}, M_C^{tritan} = \begin{bmatrix} 1, 0, 0 \\ 0, 1, 0 \\ 0, 1, 0 \end{bmatrix} \quad (14)$$

Here, the matrix  $M_C$  works to transform the *fundamental* spectrum  $C_{dichro}^*$  into the tristimulus LMS signals which are dichromatic but reflect 3 channels

Next the color vector  $D_{sRGB}^{dicro}$  to be displayed on sRGB monitor is obtained by the inverse transforms as

$$D_{sRGB}^{dicro} = (M_{sRGB \rightarrow XYZ})^{-1} (M_{XYZ \rightarrow LMS})^{-1} C_{LMS}^{dicro} \quad (15)$$

### Experiment [B] ~ Simulation based on Cone Substitution and Nulling Hypothesis

Experiment [B] proceeds the next opponent-color stage based on the nulling hypothesis. The trichromatic cone responses are not transmitted directly to our brain, but encoded to the luminance-chrominance based opponent-color signals, so that this encoding stage is assumed to work effectively for dichromats as a neural system, even if missing one of the LMS cones. So many opponent-color models have been proposed, such as one-opponent-stage linear models by Ingling and Tsou, Guth's ATD80, Boynton, and two-opponent -stage linear models by De Valois, two-opponent-stage nonlinear models by Guth's ATD95, and so on. Here the opponent-color model IPT [4] (Ebner & Fairchild, 1998) is introduced, because IPT is simple but excellent in its color-opponency, hue linearity and color difference uniformities.

Now, the output  $C_{LMS}^{dicro}$  in equ. (14) after substitution processing, is transformed into the IPT opponent-color vector  $C_{IPT}^{dichro}$  as follows.

$$C_{IPT}^{dichro} = M_O(M_{LMS \rightarrow IPT})S_{IPT} \cdot C_{LMS}^{dichro}$$

$$M_O = M_O^{protan/deutan}, \text{ or } = M_O^{tritan} \quad (16)$$

$$M_O^{protan/deutan} = \begin{bmatrix} 1, 0, 0 \\ 0, 0, 0 \\ 0, 0, 1 \end{bmatrix}, M_O^{tritan} = \begin{bmatrix} 1, 0, 0 \\ 0, 1, 0 \\ 0, 0, 0 \end{bmatrix}$$

$S_{IPT}$  denotes the nonlinear scaling with power of 0.43 for each entry of  $X$ . Here,  $M_O$  works to make null one of the opponent chromatic mechanisms either (red–green for protanopes and deutanopes or (blue–yellow) for tritanopes.

Finally, the simulated color is given by the inverse processings to the sRGB display signals as follows.

$$D_{sRGB}^{dichro} = (M_{sRGB \rightarrow XYZ})^{-1} (M_{XYZ \rightarrow LMS})^{-1} \cdot (M_{LMS \rightarrow IPT})^{-1} C_{IPT}^{dichro} \quad (17)$$

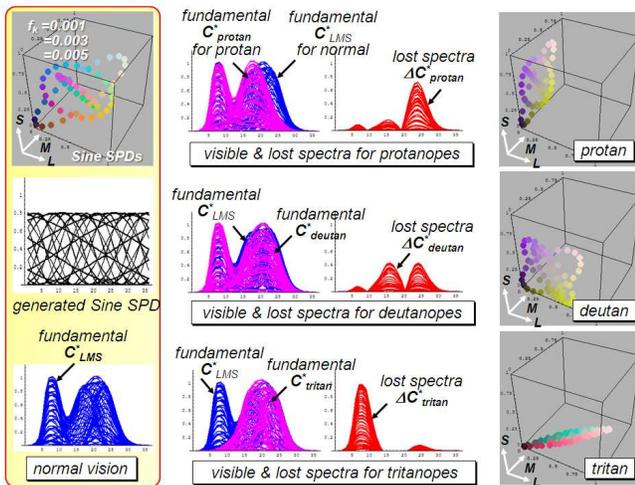
## Results

**Figure 4** shows the simulates results for the sine SPDs in Experiment [A].

As seen in the lost spectra for the dichromats against the normal vision, the *fundamental spectra*  $C_{protan}^*$  visible to protanopes and  $C_{deutan}^*$  visible to deutanopes are much overlapped, because the matrices  $R_{dichro}$  in **Figure 2** extract the common middle bands in the spectral distributions. As a result, the perceived colors for protanopes and deutanopes may be mutually similar. While, the visible spectra captured through the matrix

$R_{dichro}$  for the tritanopes are clearly separated and only their short spectral band is lost. Thus the tritanopes can see the more colorful scenes quite different from the protanopes and deutanopes. These effects are obviously imagined from the lost spectra and the simulated colors.

**Figure 5** summarizes the results for the Macbeth, IT8 and Munsell spectral color chips in the Experiment [A] and [B]. The simulated colors for dichromats with each spectral chips are



**Figure 4** Visible and lost spectra for the dichromats against the normal vision with sine SPDs in Experiment [1]

compared in the two experiments of [A] dichromatic spectral response captured as the fundamental based on LMS cone substitution hypothesis and [B] additional opponent-color process based on nulling hypothesis to void one of the red–green or yellow–blue opponent-color stages. The simulated colors are rather different with or without nulling process.

**Figure 6** illustrates the lost spectral distributions for the test color chips in **Figure 5**. The lost spectra are obtained by taking the differences in the visible spectra between the *fundamentals* for the normals and for the dichromats. As well as the sine SPDs in **Figure 4**, it's clear that the lost spectra for protan and deutan are distributed in the broad middle wavelength ranges with similar profiles each other, while those for tritan are concentrated mainly in the short wavelength regions with dissimilar profiles. This tells us why the simulated colors in **Figure 5** are resemble each other to the protan and the deutan, but dissimilar to the tritan.

**Figure 7** shows a simulated image comparison with other methods reported by P. Capilla et al [3]. Since the proposed model needs any spectral inputs for the image simulation but we have no spectral data for Picasso's "Dora Maar", the pseudo-spectral test image was generated by embedding the spectral inkjet chip in each pixel of sRGB [10]. The spectral chip matched to the pixel with minimum  $\Delta E_{ab}^*$  is picked up from the inkjet spectral palettes, each composed of  $n=36$  bands ( $\Delta\lambda=10$  nm in the range of 380–730 nm).

All the models in **Figure 7** look to have subtle differences one another. Among them, the last row by the Experiment [B] looks to be similar to the results by Brettel or Boynton, although the converted pseudo-spectral image is not exactly the same as the tri-color RGB image in the literature [3].

## Conclusions

The paper proposed a novel approach to analyzing the dichromatic color blind problem from a point of spectral responses based on the projection theory from the spectral space onto the 2-dimensional dichromatic HVSS. As a result, the following facts are clarified.

- [1] The visible spectra to the dichromats (protanopes, deutanopes, and tritanopes) are easily extracted by the 2-D version of *Matrix-R* notated as *matrix*  $R_{dichro}$ .
- [2] Since the *matrix*  $R_{dichro}$  is an identical and invariant mapping operator inherent in dichromatic vision, which is independent of any linear transformation to *LMS* or any illuminant, the captured spectra  $C_{dichro}^*$  by the dichromats are also device-independent. Indeed,  $C_{dichro}^*$  is the essential spectrum visible to all the dichromats and recursively reconstructed as

$$C_{dichro}^* = R_{dichro} C = R_{dichro} C_{dichro}^* = (R_{dichro})^m C_{dichro}^*$$

- [3] The lost spectra are easily obtained as a difference between the *fundamental*  $C_{LMS}^*$  for normals and the *fundamental*  $C_{dichro}^*$  for dichromats. These lost spectral profiles tell us why the color appearances are similar to the protanopes and deutanopes, but dissimilar to the tritanopes.

In the process by substitution hypothesis, any scaling for the cone signals may be necessary to keep an exact neutral color balance. This scaling problem is not considered at present but left behind as a future work.

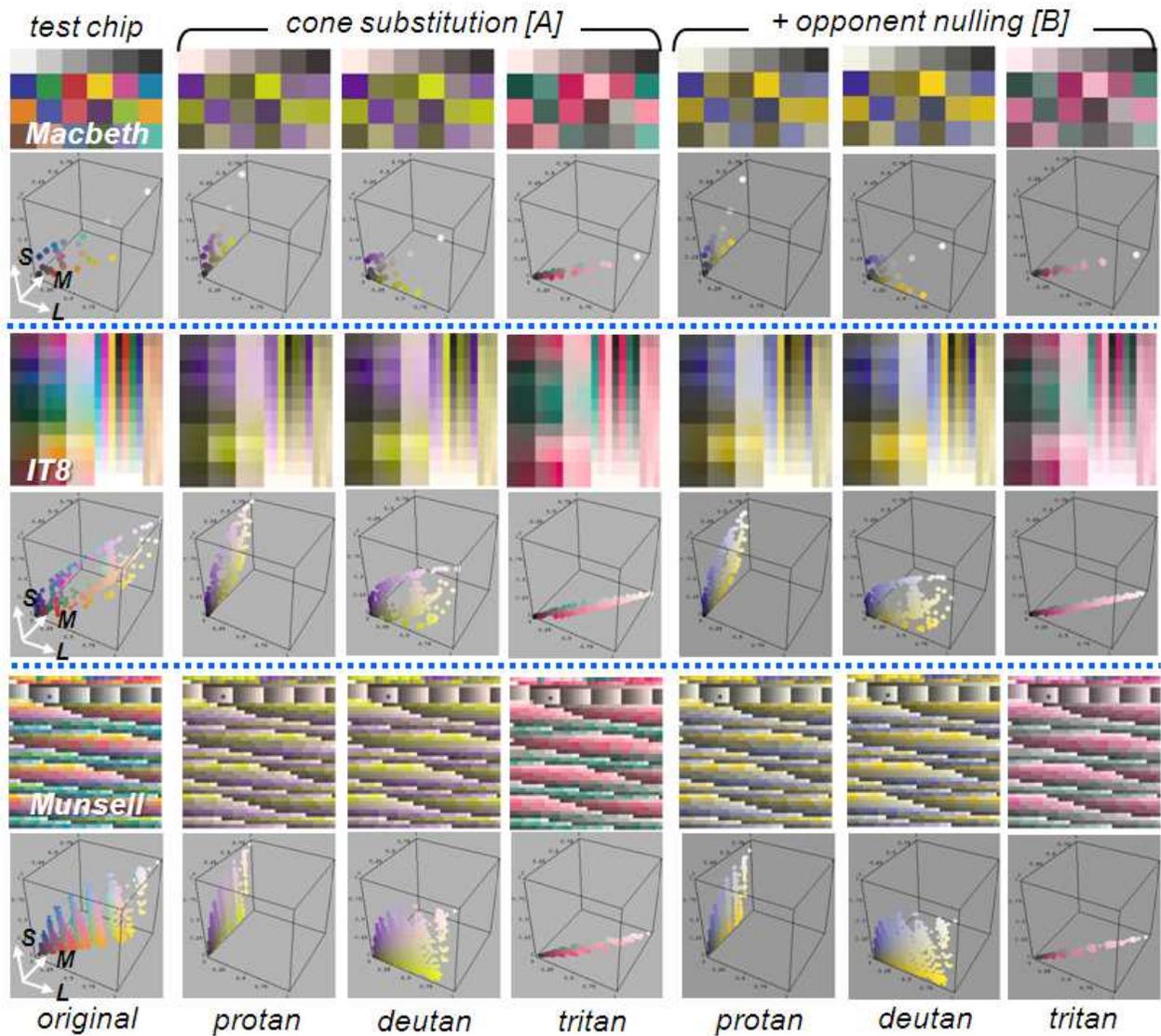


Figure 5 Simulated colors by [A] cone substitution and [B] with opponent-color nuling hypothesis

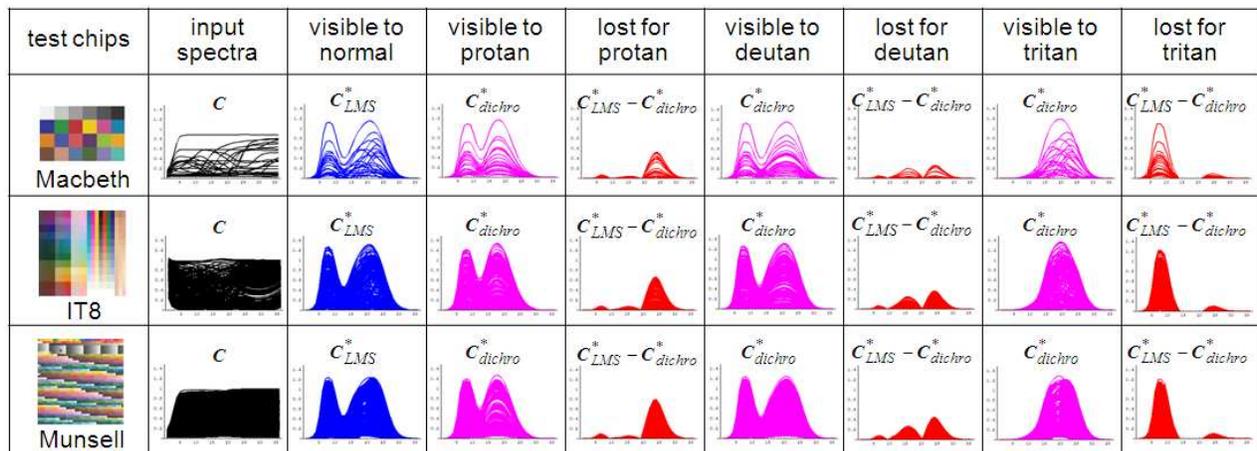


Figure 6 visible and lost spectra for each type of dichromats against normals

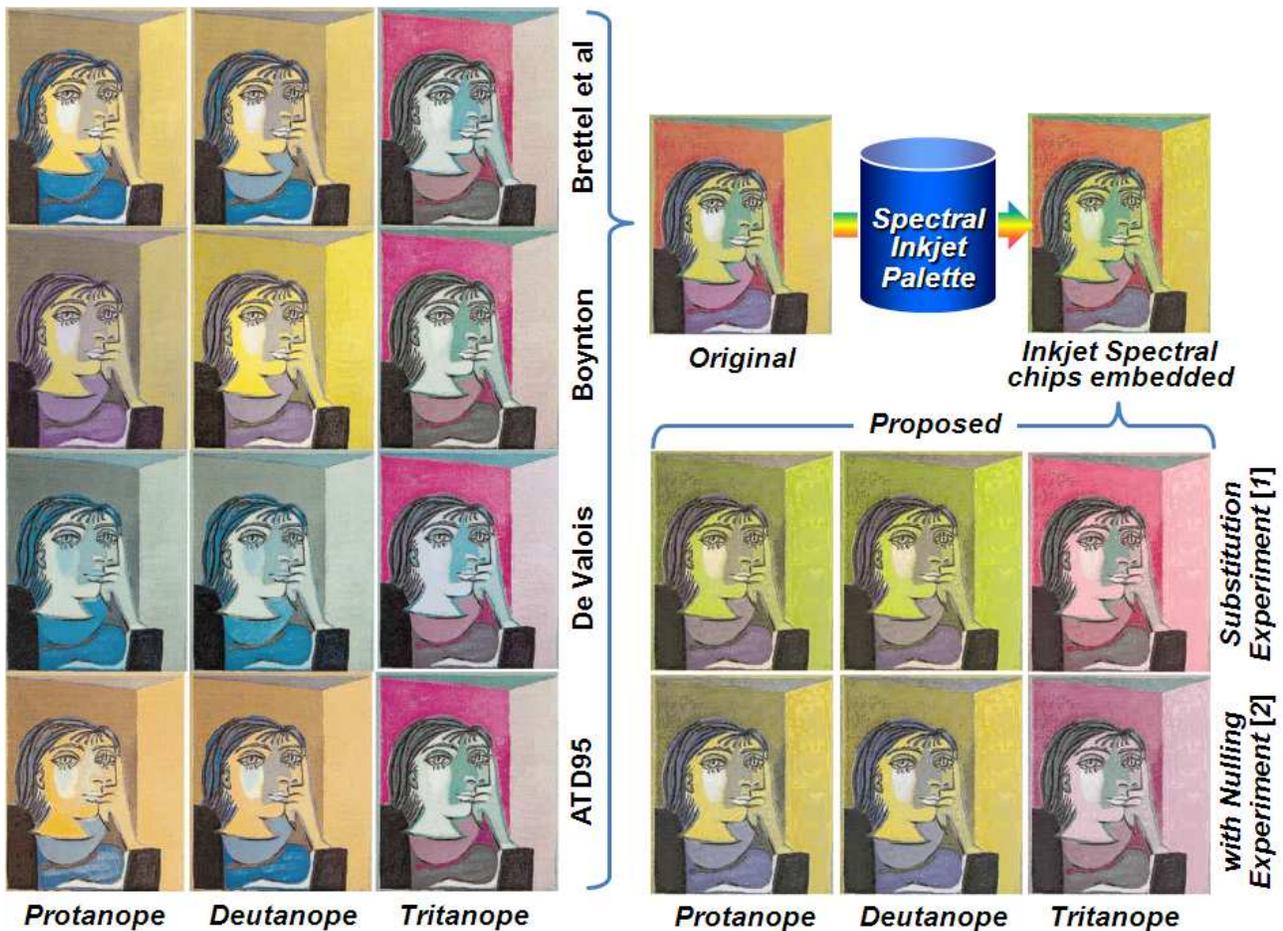


Figure 7 A simulated images for Picasso's "Dora Maar" in comparison with other methods

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**Hiroaki Kotera** joined Panasonic Corp., in 1963. He received Doctorate from Univ. of Tokyo. After worked at Matsushita Res. Inst. Tokyo during 1973-1996, he was a professor at Dept. Information and Image Sciences, Chiba University until his retirement in 2006. He received 1993 IS&T journal award, 1995 SID Johann Gutenberg prize, 2005 IEEE Chester Sall award, 2006 ISJ journal award, 2007 IS&T Raymond.C. Bowman award, and 2009 SPSTJ journal award. He is a Fellow of IS&T.