G_0 Colorants

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Abstract

Mixtures of colorants that faithfully render G_0 colors at maximum NCS chromaticness for the twenty-four (24) NCS aim hues were derived computationally. Candidate colorants were modeled with a Gaussian function smoothly varying as in natural materials distinguishing G_0 as the limit of object color independent of MacAdam's loci of pure spectral color. Spectra were synthesized using a gradient-based, nonlinear optimization with an objective function that maximized the color inconstancy of the mixture between two illuminants, CIE D65 and Illuminant A, thereby synthesizing a theoretical target set most sensitive to those changing conditions of illuminant that affect colorimetry.

Introduction and Background

Perhaps most fundamental to Evans' notion [1] of brilliance and its percepts of greyness and fluorence in the context of the perceptual gamut of real objects is his notion of G_0 as the point where the percept of grayness in a stimulus disappears. The notion that as brilliance proceeds from the perception of greyness through G_0, the mode of viewing changes from object mode where colour is said to have gray content in both the Nayatani [2] and Evans sense or veiled in the Hering sense [3]. Above G_0 the mode of viewing becomes what Nayatani later termed as pseudo-colour and Evans as fluorence where the perception of colour takes on an emissive, almost surreal character.

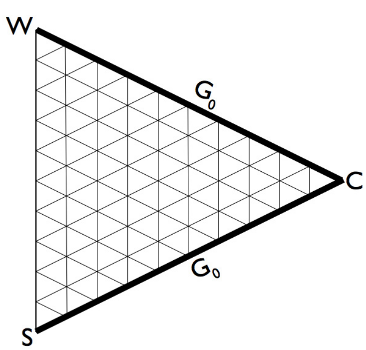


Figure 1: A constant NCS constant hue triangle illustrating G_0 where either NCS whiteness or blackness equals zero.

G_0 is derived from Nayatani's grayness percept gr given by:

$$gr = \min(S, W) \tag{1}$$

where S and W are The Swedish Natural Color System's (NCS) values for blackness and whiteness whose sum, along with NCS chromaticness, are normalized to a value 100. G_0 is then that value of grayness equaling zero. Figure 1 illustrates G_0 for a constant NCS hue triangle.

Recently, Okui *et. al.* [4] concluded from the work on the preferred gamut for wide gamut displays that "... the mode boundary between surface color and fluorent color … is similar to [MacAdam's] optimal color locus." Others, such as Li [5] readily equated the " … gamut of object color solid … with the color gamut of the MacAdam limit …". To this day, the belief persists that G_0 does not represent any real, physical response. Yet when mapped across all colors in a color system or space, that mapping totally contains the newly standardized ISO 12640-3 Reference Color Gamut [6] yet represents little more than half the volume [7] enclosed by the MacAdam Limits [8,9]. If G_0 is an estimate of the theoretical limit of object color or pure color, it is a poor one indeed. Instead, it seems more aligned with the gamut of real objects delineating its extent.

Purpose and Scope

If the loci of G_0 delineates the extent of object color, then unlike the theoretical colorants used by MacAdam, an optimized reflectance spectra synthesized from a more realizable set of colorants should be capable of their faithful rendering. The purpose here is to demonstrate such a set for each of the twenty-four (24) NCS aim hues at G_0 and maximum NCS chromaticness. And for each hue, the full extent of the locus of G_0 is rendered from these colorants.

Colors and Mixing

In a similar way as Chen [10] and because most spectra of naturally occurring materials vary smoothly, the reflectance spectra of a mixture of colorants were synthesized using Gaussian functions. Figure 2 illustrates the reflectance factors for such a series of colorants with varying spectral widths W with mean wavelength centered at a wavelength of zero for illustration purposes.

Optimization Method

As an optimization methodology, a perhaps traditional objective or cost function would have been to maximize color constancy between a set of illuminants as did Berns [11] and Ohta [12]. However, it was reasoned that such a function would be too constraining, and as was later confirmed, convergence was illusive for these high chroma, G_0 colors. Instead, an objective function $f(\mathbf{x})$ that maximized color inconstancy between two illuminants was thought to be more interesting providing a set of spectra most susceptible to those changing conditions (i.e., illuminant) that affect colorimetry. Hence, as optimization is a process of minimizing an objective function, the function that maximizes a color inconstancy index (CII) is given by:

$$f(\mathbf{x}) = \frac{1}{CII(\mathbf{x})} \tag{2}$$

where the parameter vector $\mathbf{x}$ represents the respective Gaussian mean wavelength and spectral width of the spectral reflectance factors for each of the synthesized colorants, typically three (3) that make up the spectra of the optimized mixture.

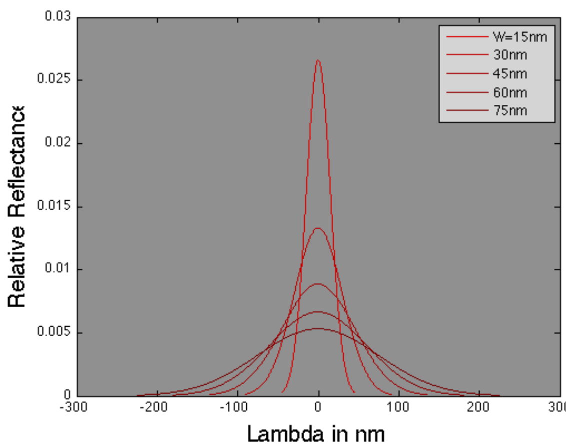


Figure 2: The range in standard deviation or width W of reflectance spectra synthesized for this problem. Tthe respective means are centered at a wavelength of zero for illustration purposes.

CIEDE94 was chosen as the basis for a color inconstancy index (CII) because of its simplicity and it's accounting for position along the chroma axis, and is given by [13]:

$$CII = \Delta E_{94}^{*} = \sqrt{\left(\frac{\Delta L^{*}}{k_L S_L}\right)^2 + \left(\frac{\Delta C_{ab}^{*}}{k_C S_C}\right)^2 + \left(\frac{\Delta H_{ab}^{*}}{k_H S_H}\right)^2} \tag{3}$$

where CII measures the distance $\Delta L^{*}, \Delta C_{ab}^{*}, \Delta H_{ab}^{*}$ in CIELAB lightness, chroma and hue between a sample under one illuminant (CIE Illuminant D65) and another (CIE Illuminant A) both transformed to constant chromatic adaptation [14]. The coefficients k_L, k_C are each set to two and k_H to unity emphasizing hue differences over lightness and chroma, and the positional factors S_L, S_C, S_H are given by:

$$\begin{aligned} &S_{L=1} \\ &S_C = 1 + 0.045 C_{ab}^{*} \\ &S_C = 1 + 0.015 C_{ab}^{*} \end{aligned} \tag{4}$$

for C_{ab}^{*} the geometric mean of the respective chroma values under each illuminant.

While both constrained and unconstrained nonlinear optimization methods were tried, the gradient method of the following form gave the most consistent results:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{F}(\mathbf{x}_k)^{-1} \mathbf{g}_k \tag{5}$$

where $\mathbf{F}(\mathbf{x}_k)^{-1}$ is the quasi-Newton approximation to the inverse Hessian of the Lagrangian, $\mathbf{g}_k$ the gradient of $f(\mathbf{x})$, and α_k chosen such that $f(\mathbf{x}_{k+1}) < f(\mathbf{x}_k)$ [15].

Process flow

Tristimulus values at the loci of G_0 at maximum NCS chromaticness for each of the twenty-four (24) NCS aim hues were computed using a method described previously [16] based on a method prescribed by Derrefeldt and Sahlin [17]. For each set of tristimulus values at maximum chromaticness, a mixture of colorants was synthesized that maximized color inconstancy; i.e., a set of colorants were synthesized for each trial of the objective function $f(\mathbf{x})$ from the optimization parameters $\mathbf{x}$ according to:

$$R_{\lambda_k n} = \frac{1}{x(n+3)\sqrt{2\pi}} e^{\frac{-(\lambda_k - x_n)^2}{2x_{n+3}^2}}, n = 1,2,3 \tag{6}$$

where λ_k is given in 10 nm steps from 380 to 730 nm, . The respective concentrations $\mathbf{c}$ of the three colorants for mixing are then given by:

$$\mathbf{c} = \mathbf{M}^{-1} \mathbf{XYZ}_{D65} \tag{7}$$

for XYZ_{D65} the tristimulus values of the G_0 color for CIE illuminant D65, and the linear relationship $\mathbf{M}$ between the colorants and tristimulus values given by:

$$\mathbf{M} = k \begin{vmatrix} \sum_\lambda S_{\lambda,D65} R_{\lambda,1} \bar{x}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,2} \bar{x}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,3} \bar{x}_\lambda \Delta\lambda \\ \sum_\lambda S_{\lambda,D65} R_{\lambda,1} \bar{y}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,2} \bar{y}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,3} \bar{y}_\lambda \Delta\lambda \\ \sum_\lambda S_{\lambda,D65} R_{\lambda,1} \bar{z}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,2} \bar{z}_\lambda \Delta\lambda & \sum_\lambda S_{\lambda,D65} R_{\lambda,3} \bar{z}_\lambda \Delta\lambda \end{vmatrix} \tag{8}$$

for $S_{\lambda,D65}$ the spectral power distribution of the CIE illuminant D65, $\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$ the CIE color matching functions for the 1931 2 degree observer, $R_{\lambda,n}$ the nth colorants spectral reflectance factor, $\Delta\lambda$ the wavelength interval (10 nm), and k a normalization constant. As these are theoretical colorants, concentrations can be both positive and negative so that colors like magenta can be more easily synthesized from the inclusion of a ideal white offset.

The spectral reflectance factor $R_{\lambda,Mix}$ of the mixture is then given by:

$$R_{\lambda,Mix} = c_1 R_{\lambda,1} + c_2 R_{\lambda,2} + c_3 R_{\lambda,3} \tag{9}$$

and the tristimulus values $\mathbf{XYZ}_A$ for Illuminant A by:

$$\mathbf{XYZ}_A = k \sum_\lambda S_{\lambda,A} R_{\lambda,Mix} \begin{vmatrix} \bar{x}_\lambda \\ \bar{y}_\lambda \\ \bar{z}_\lambda \end{vmatrix} \Delta\lambda \tag{10}$$

for $S_{\lambda,A}$ the spectral power distribution of CIE Illuminant A. Then, assuming CIELAB is most uniform in D65, these tristimulus values for Illuminant A are chromatically adapted to D65 using the CIECAT02 chromatic adaptation algorithm. Their respective CIELAB values are then calculated and the color inconstancy index evaluated according to Eq. 3.

Starting values

Convergence in any optimization process is greatly facilitated by a realistic set of starting values $\mathbf{x}_0$ particularly in the gradient-based method used here. These were obtained for each of the twenty-four (24) NCS aim hues by measuring the spectral reflectance factors of each color patch with blackness value $S = 5$ and chromaticness $C = 80$ using a Gretag-Macbeth Eye-One Pro spectrophotometer.

The resulting reflectance factor measurements were then fit by a unconstrained, nonlinear, least squares optimization for the mix of three synthesized colorants with an objective function that minimized root-mean-square residual error from the measured values. Both the measured values (dotted) and the fitted mixture (solid) are shown in Figure 3. Over all twenty-four (24) hues, the mean vector norm for the residuals of the fit was 0.015 with a standard deviation of 0.015 and a maximum of 0.05.

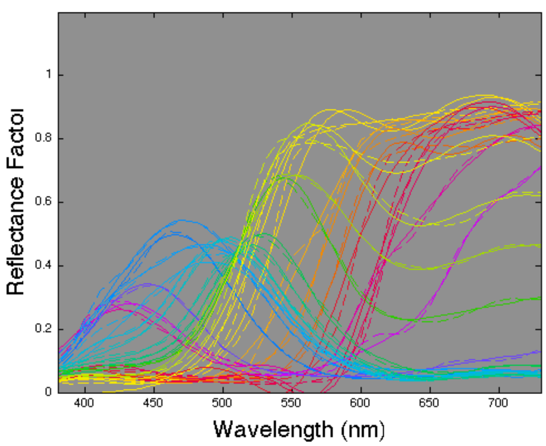


Figure 3: Starting values of reflectance spectra as the mixtures of three synthesized colorants (solid lines) and corresponding measured reflectance spectra for each of the twenty-four (24) NCS aim patches at S 05 C 80.

Objective function development and constraints

Early on in the trial and errors of developing the optimization process, it became clear that an objective function based on a color inconstancy index alone could not produce stable results. In a purely unconstrained optimization, the colorants tended to approach the monochromatic case with spectral reflectance factors approaching infinity. Constraining the lower bound of the spectral width of the synthesized colorants always produced a solution at the constrained value again with spectral reflectance factors greater then unity.

If the mixture of colorants were scaled to be no more than unity, the resulting tristimulus values would also be correspondingly scaled. Restoring these values to reproduce the original G_0 color could only be accomplished by lowering the luminance factor Y/Y_n of the mixture computed as the maximum spectral reflectance factor R_{max} of the mixture for Y_n the luminance of diffuse white for matching purposes. Computed in this way and utilized in the objective function $f(\mathbf{x})$ as a power function shown below, the luminance factor serves to maintain the maximum spectral reflectance R_{max} to $R_{max} \leq 1$. This along with allowance for negative reflectance factors $R_{max} \geq -1$ produced stable results that avoided the simple, but uninteresting case of a spectral match under both illuminants..

$$f(\mathbf{x}) = \frac{\left[Y/Y_n(\mathbf{x})\right]^4}{CII(\mathbf{x})} = \frac{\left[R_{max}(\mathbf{x})\right]^4}{CII(\mathbf{x})} \quad (11)$$

Additionally, the optimization is constrained to a lower bound for the mean wavelength and spectral width of 380 nm and 45nm respectively and equal luminance in the adapted illuminant (D65) as hue/chroma shifts were the desired end result. Both positive and negative concentrations of each of the colorants were allowed along with an ideal white offset.

Results and Discussion

The optimization was performed on all twenty-four (24) NCS hues at maximum chromaticness thought to be the most difficult of the G_0 colors to synthesized with smoothly varying functions in wavelength. Overall, a mean color inconstancy index (CII) of 15.7 was achieved with a maximum value of 36.6, and a minimum value of 2.75.

Table 1: Maximum Color Inconstancy Index (CII) for the 24 NCS Hues at G_0 and maximum chroma

Hue	CII	Hue	CII	Hue	CII	Hue	CII
Y	3.2	R	36.1	B	20.9	G	25.5
Y10R	3.0	R10B	36.6	B10G	16.2	G10Y	27.0
Y30R	2.8	R30B	10.8	B30G	23.1	G30Y	11.3
Y50R	2.8	R50B	4.7	B50G	15.6	G50Y	10.7
Y70R	20.0	R70B	7.7	B70G	18.2	G70Y	8.4
Y90R	21.7	R90B	26.7	B90G	19.8	G90Y	3.7

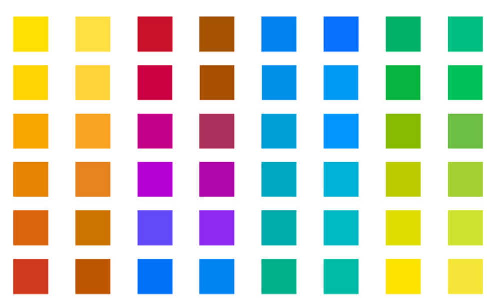
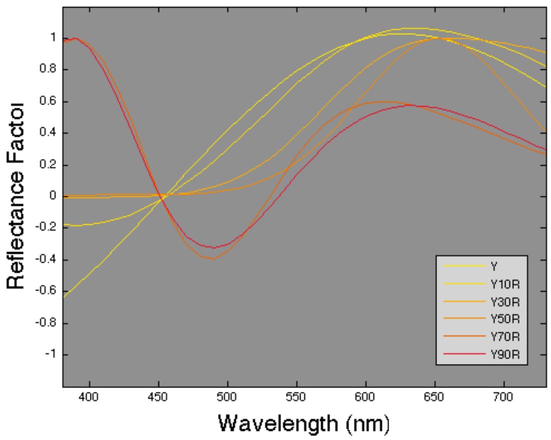
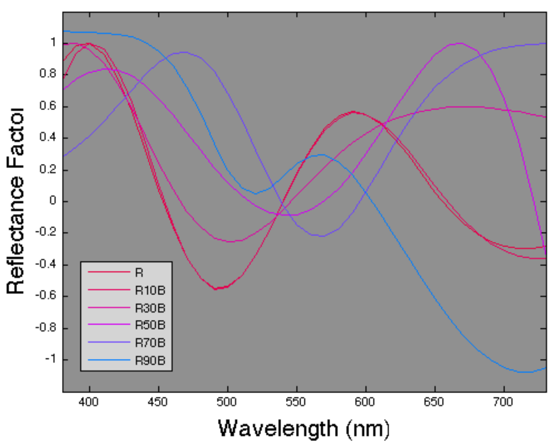


Figure 4:Ccolor patches corresponding to the twenty-four (24)NCS aim hues in D65 optimized for color inconstancy. The four sets of columns correspond to Table 1 with the patches rendered under D65 on the left and rendered under Illuminant A chromatically on the right.

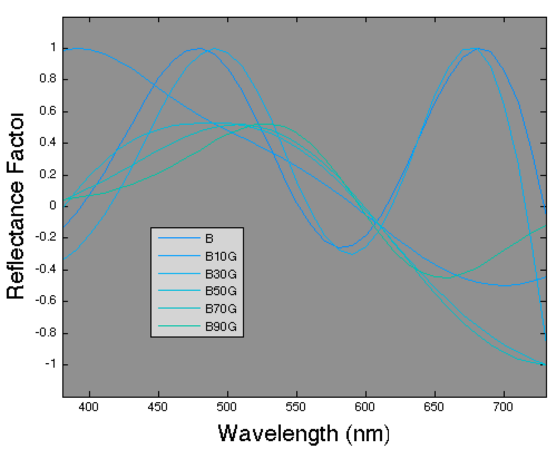
The Color Inconstancy Indices for each of the twenty-four (24) hues are given in Table 1 with corresponding representative color patches below in Figure 4. The four sets of columns in Figure 4 correspond to the columns in Table 1. Each show the respective G_0 colors at maximum chromaticness rendered under D65 on the left and under Illuminant A on the right. The colors were rendered to the full gamut of an HP LED DreamColor display to avoid clipping. While they cannot be reproduced here to their full extent, the color differences exhibited here are quite striking for many of the hues - particularly around red, green, and blue.



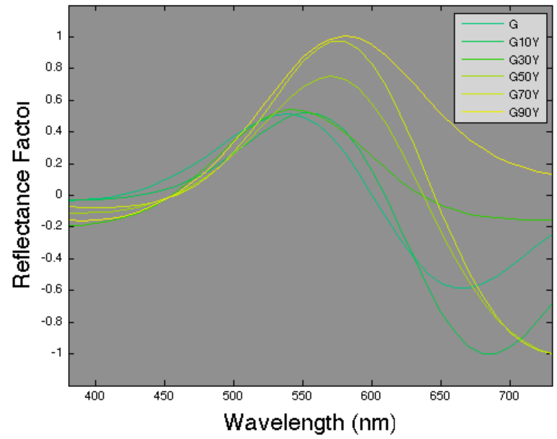
(a) Yellows



(b) Reds



(c) Blues



(d) Greens

Figure 6: The spectral reflectance of the mixture of three synthesized colorants optimized for color inconstancy at G_0 and maximum chromaticness for the twenty-four (24) NCS aim hues

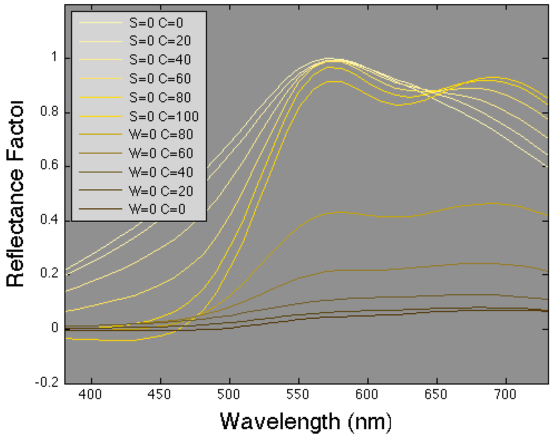
The reflectance spectra of the mixtures of optimized colorants are illustrated in Figures 6. The respective NCS hues are indicated in their legends and by their color. As shown, the spectra for most colors exhibited a negative component important for their convergence to other than a spectral match between the two illuminants. Each made good use of the area where the spectral energy of these two luminants are most different at the shorter end of the visible wavelengths and the longer.

Mixtures of colorants at the loci of G_0

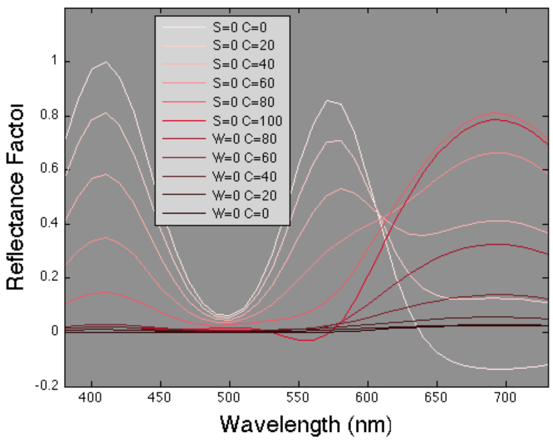
Each of these optimized sets of colorants can then be mixed according to Eq. 7 from the tristimulus values to render any point on the G_0 loci. Figure 5 illustrates the spectra of such mixtures for the NCS hues Y, R, B, and G. Corresponding NCS blackness S or whiteness W and chromaticness C values along the loci of G_0 are listed in the legends. The tristimulus values were computed as outlined under "Process Flow" in the above.

In Figure 5(d), NCS G, illustrates well behaved additivity representative of a classic tint ladder formed by adding degrees of white to the mix. Figure 5(a), NCS Y, illustrates additivity to a lesser extent with reversals at the longer wavelengths bolstered by the optimization at maximum chromaticness ($S = 0, C = 100$). Figures 5(b) and 5(c) illustrate this effect to an even greater degree.

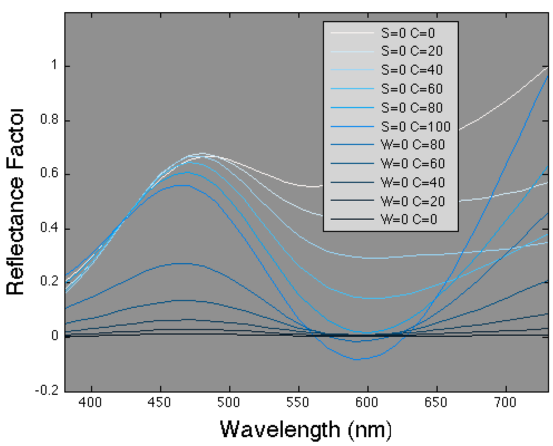
When mapping NCS S and C (i.e. G_0) to CIEXY as described in the above, curvature in the CIEXYZ surface formed by the loci of G_0 results. Hence, the Principle of Additivity is affected as, in most of these cases, three colorants are being mixed, and simply adding white to the mixture does not always generate a traditional tint ladder.



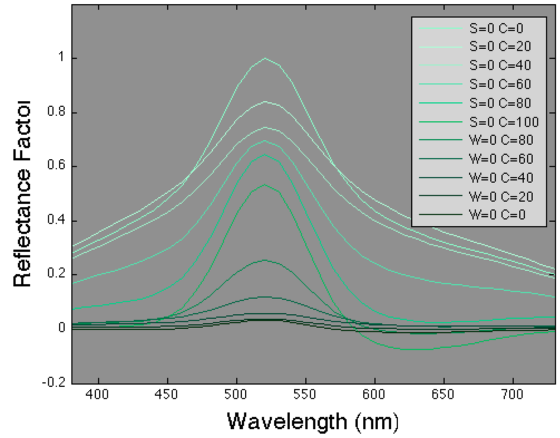
(a) Y



(b) R



(c) B



(d) G

Figure 5: The spectral reflectance of mixtures along the loci of G_0 given in the legend for the NCS aim hues Y, R, B, and G from the synthesized colorants optimized for color inconstancy at maximum chromaticness.

Summary and Conclusions

Mixtures of colorants that faithfully render G_0 or zero grayness at maximum NCS chromaticness for the twenty-four (24) NCS aim hues were demonstrated under D65. The spectral reflectance for each colorant was modeled with a Gaussian function smoothly varying as in natural materials distinguishing G_0 as the limit of object color independent of MacAdam's loci of pure spectral color. The mixtures were synthesized using a gradient based, quasi-Newton, nonlinear optimization with an objective function that maximized a color inconstancy index (CII) between two illuminants, CIE D65 and Illuminant A. The CII chosen was based on ΔE^*_{94} and gave twice as much influence on the result to chroma over hue and lightness differences.

Overall, a mean color inconstancy index (CII) of 15.7 was achieved with a maximum value of 36.6, and a minimum value of 2.75. Hence, these mixtures synthesize a theoretical target most sensitive to the changes in those conditions that affect colorimetry (i.e. particularly, changes in illuminant) and are then useful for evaluating the performance of color imaging devices computationally at the early design stage of their development. And while these colorants cannot be synthesized with real materials (negative reflectance factors), they can be computationally formed, tristimulus values computed from their mixture, and rendered for viewing on display media with sufficient gamut.

Each of the optimized sets of colorants were mixed to tristimulus values of any point on the locus of G_0 of each otheir corresponding NCS hue triangle. The resulting mixtures obeyed the Principle of Additivity only in very limited cases. Generally speaking, mapping the surface of G_0 in NCS to CIEXY results in curvature of the CIEXYZ surface, and the Principle of Additivity affected. Hence, simply adding degrees of white to the mix at maximum chromaticness to duplicate the result does not always produce a traditional tint ladder.

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