

Vectorial Quality Measure for Digital Camera in Opponent FCS

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Abstract

This paper reconsiders the Luther condition from a spectral and colorimetric point of view. The projection of spectral to colorimetric space is based on matrix \mathbf{R} theory. The basis of matrix \mathbf{R} spans a FCS (Fundamental Color Space).

The spectral colorimetric design for color sensors is discussed based on FCS by clarifying its geometrical structure in relation to the orthogonal color matching function and the FCS basis.

Since opponent-color model is useful for appearance-based color imaging, the paper highlights the opponent FCS with perfect orthogonality and introduces a vectorial quality factor Vq for the spectral-colorimetric evaluation of digital cameras. The spectral response to unit monochromatic stimuli is visualized as a 3-D locus that clearly shows the spectral mismatch with Luther condition. The measured Vqs are compared with Neugebauer's q -factors and also Vora & Trussell's ν -factors.

Introduction

The spectral sensitivities of color input devices such as digital camera or scanner are required to satisfy Luther condition in order to capture the correct tri-stimulus values.

The following quality measures for color scanning filters have been conveniently used to estimate the goodness.

- [1] Neugebauer's q -factor [1]
- [2] Vora&Trussell's ν -factor (generalized q factor) [2]
- [3] Tajima's weighted q -factor [3]

Though these factors are helpful for choosing the color filters from a colorimetric point of view, the details in the spectral mismatch are not always clear. From a point of spectral design in RGB camera, Worhtey and Brill analyzed the spectral sensitivity errors using LUM (Locus of Unit Monochromats) in FCS (Fundamental Color Space).

Unfortunately, the LUM error is not numerically measured but mostly discussed in the standard CIE FCS. From a point of color encoding, the FCS based on opponent-color model may be more interesting and attractive.

This paper discusses the spectral LUM responses in an orthogonal FCS with perfect opponent-color basis and measures the LUM error as a mean squared vectorial loci error. A new quality factor, what we call, Vq (Vectorial q -factor) is not a vector but a single scalar value reflecting the integrated spectral mismatches in the proposed orthogonal opponent-color FCS. The spectral mismatches with Luther condition for high-end digital cameras are introduced in comparison with q -factors or ν -factors.

Fundamental Color Space

FCS is defined as a color space spanned by the orthogonal normalized basis called "matrix- \mathbf{F} " derived from the matrix \mathbf{R} theory by J. B. Cohen [4].

In HVSS (Human Visual Sub-Space), n -dimensional color vector

\mathbf{C} is decomposed into fundamental \mathbf{C}^* and metamerick black \mathbf{B} through matrix \mathbf{R} as

$$\mathbf{C} = \mathbf{C}^* + \mathbf{B}, \mathbf{C}^* = \mathbf{R}\mathbf{C}, \mathbf{B} = (\mathbf{I} - \mathbf{R})\mathbf{C} \quad (1)$$

\mathbf{I} denotes unit matrix and \mathbf{R} is the projector onto HVSS (Human Visual Sub-Space) derived from CMF \mathbf{A} as

$$\mathbf{R} = \mathbf{A}(\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t \quad (2)$$

\mathbf{A} is the $n \times 3$ matrix of 1931CIE $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ CMF.

The matrix \mathbf{R} is the $n \times n$ symmetric matrix whose i -th column vector \mathbf{E}_i is composed of the fundamental for each single spectrum e_i at wavelength λ_i .

$$\mathbf{R} = [\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_i, \dots, \mathbf{E}_n] \quad (3)$$

$$\mathbf{E}_i = \mathbf{R}\mathbf{e}_i; \mathbf{e}_i = [0, 0, \dots, 1, \dots, 0]^t, i\text{-th element} = 1$$

Since the rank of \mathbf{R} is 3, it has only 3 independent vectors and the remaining $n-3$ are redundant. We can recreate \mathbf{R} by choosing arbitrary triplet from the column (row) vectors. The selected triplet is called "matrix \mathbf{E} " and $i=r$; g, b show the spectral primaries at wavelength $\lambda_r, \lambda_g, \lambda_b$ as follows.

$$\mathbf{R} = \mathbf{E}(\mathbf{E}^t \mathbf{E})^{-1} \mathbf{E}, \quad \mathbf{E} = [\mathbf{E}_r, \mathbf{E}_g, \mathbf{E}_b]^t \quad (4)$$

Indeed, Fig.1 shows the reconstructed matrix \mathbf{R} from the middle three entries $[\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3]$ at $\lambda=540, 550$, and 560 nm.

The FCS is a color space spanned by a triplet of basis vectors called "matrix \mathbf{F} ", which is orthonormalized version of matrix \mathbf{E} using Gram Schmidt method as

$$\mathbf{R} = \mathbf{F}\mathbf{F}^t;$$

$$\mathbf{F} = [\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3] = \text{GramSchmidt}[\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3] \quad (5)$$

$$\mathbf{F}^t \mathbf{F} = \mathbf{I}; \langle \mathbf{F}_j, \mathbf{F}_k \rangle = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{for } j \neq k \end{cases}$$

The symbol $\langle \mathbf{u} \bullet \mathbf{v} \rangle$ denotes the inner product of \mathbf{u} and \mathbf{v} . The selection of matrix \mathbf{E} is very important to construct orthonormal FCS as suggested by Brill, Finlayson, et al [5].

For example, Burns, Cohen, and Kuznetsov [6] created an orthonormal FCS called "R-L-V" choosing the matrix \mathbf{E} with quasi-orthogonal axes of Red, Luminosity, and Violet ($\lambda=770, 563$, and 380 nm) as illustrated in Fig.1.

Historically, the orthogonal CMF was firstly proposed by MacAdam [7] and also may be used as a matrix \mathbf{E} . Though the classical Guth's opponent CMF was not orthogonal in original, it was recently orthonormalized by Worthey et al [8] and used for evaluating spectral responses for digital camera. Kotera [9] reported that these CMFs span orthonormal FCSs close to R-L-V but slightly different one another as illustrated in Fig.2.

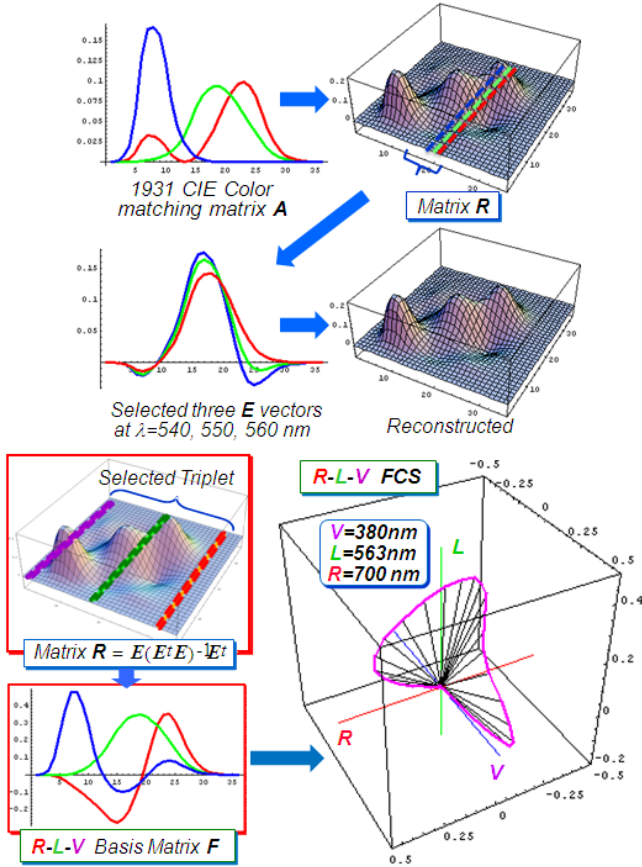


Figure 1 Construction of FCS

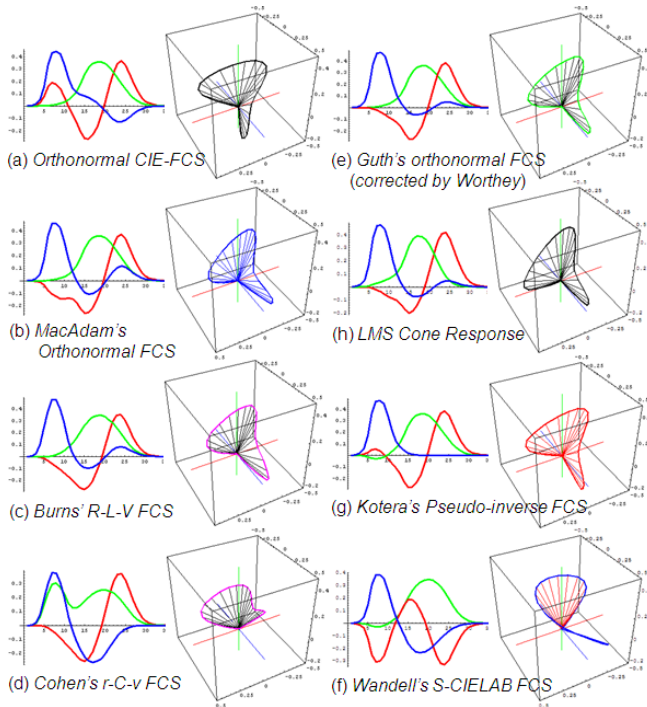


Figure 2 Typical basis matrix F and its FCS

Orthonormal Opponent FCS

Spectral Decomposition of Fundamentals

Setting the basis F_2 to an *achromatic fundamental* and (F_1, F_2) to arbitrary orthogonal *chromatic fundamentals* in matrix F , the matrix R is decomposed [4] as

$$\begin{aligned} R &= FF^t = R_A + R_C \\ R_A &= F_2 F_2^t, \quad R_C = R - R_A \end{aligned} \quad (6)$$

R_A and R_C are the orthogonal projectors to decompose the basic *fundamental metamer* C^* into the *achromatic* and the *chromatic fundamentals* C_A^* and C_C^* as

$$\begin{aligned} C^* &= C_A^* + C_C^* \\ C_A^* &= R_A C = R_A C^* \\ C_C^* &= R_C C = R_C C^* \end{aligned} \quad (7)$$

When the chromatic projector R_C is orthogonal to the achromatic projector R_A , the inner product between them should be zero as

$$\langle R_A \bullet R_C \rangle = 0 \quad (8)$$

Indeed, the matrix F in “R-L-V” FCS is composed of orthonormal basis functions with achromatic luminosity axis and satisfies the condition of Eq. (8). As well, FCSs derived from CMF by MacAdam or Guth also satisfy the same condition.

Orthonormal Opponent FCS

As well known, Luminance/Chrominance color models with opponent-color axes assigned to “Red-Green (R-G)” and “Yellow-Blue (Y-B)” have been widely used for color imaging, analysis, and picture coding. YIQ used in NTSC Television is a opponent-color system by a simple linear transformation from XYZ. CIELAB is a most popular uniform color space mapped on the R-G and Y-B Cartesian opponent-color coordinates. Hence, a foundation of FCS with orthogonal and opponent-color structure is a lot of fun in practical use.

The chromatic projector R_C is further decomposed into two opponent-color components R_R and R_B by choosing an appropriate pair of (F_1, F_2) . Thus, the *chromatic fundamental* C_C^* is decomposed into two opponent-color *fundamentals* C_R^* and C_B^* corresponding to R-G and Y-B hue axes as follows.

$$\begin{aligned} R_C &= R_R + R_B \\ R_R &= F_1 F_1^t, \quad R_B = F_2 F_2^t \end{aligned} \quad (9)$$

$$\begin{aligned} C_C^* &= C_R^* + C_B^* \\ C_R^* &= R_R C_C^*, \quad C_B^* = R_B C_C^* \end{aligned} \quad (10)$$

To be perfectly opponent for C_R^* and C_B^* , it is desirable that the projectors R_R and R_B satisfy the following zero-sum conditions. That means the gray input with flat spectrum should be achromatic without any red-green or yellow-blue components.

$$\text{Sum}[R_R] = \sum_{j=1}^n \sum_{k=1}^n R_R(j, k) = 0; \quad R_R(j, k) \text{ is } jk \text{ element} \quad (11)$$

$$\text{Sum}[R_B] = \sum_{j=1}^n \sum_{k=1}^n R_B(j, k) = 0; \quad R_B(j, k) \text{ is } jk \text{ element}$$

Although the *matrix F* in “*R-L-V*” FCS surely satisfies the orthonormal condition in Eq. (5) and the orthogonality between achromatic and chromatic components in Eq. (8), the zero-sum condition in Eq. (11) didn’t hold good, and MacAdam’s or Guth’s FCSs as well.

In the “*R-L-V*” FCS, the fundamental of $\lambda_g = 563$ nm single spectrum is selected as the vector E_2 to reflect the *Luminosity*, but E_2 doesn’t exactly mean the *luminance* which is defined as a linear mixture of *R, G, B* components.

Since the entry of *matrix E* may be any linear combination of arbitrary column vectors $\{E_i\}; i=1\sim n$, for instance, the second vector E_2 may be given by the weighted sum of $\{E_i\}$ by illuminant *D65* that means the fundamental of *D65* itself.

$$E_2 = \sum_{i=1}^n D_{65}(\lambda_i) E_i = R D_{65}; D_{65} = SPD \text{ of illuminant} \quad (12)$$

$$D_{65} = [D_{65}(\lambda_1), D_{65}(\lambda_2), \dots, D_{65}(\lambda_n)]^t$$

The zero sum condition can be obtained by replacing the vector E_2 with the fundamental of white illuminant such as *EE* (*Equal-Energy*) or *D65*. Since *EE* is not popular in practice, *D65* is recommended as most widely used. Thus “*R-D65-V*” FCS is created by introducing the fundamental of *D65* into vector E_2 in Eq. (12) and a getting the *matrix F* by GramSchmidt. Now “*R-D65-V*” becomes a perfect opponent-color FCS with zero sum condition [10].

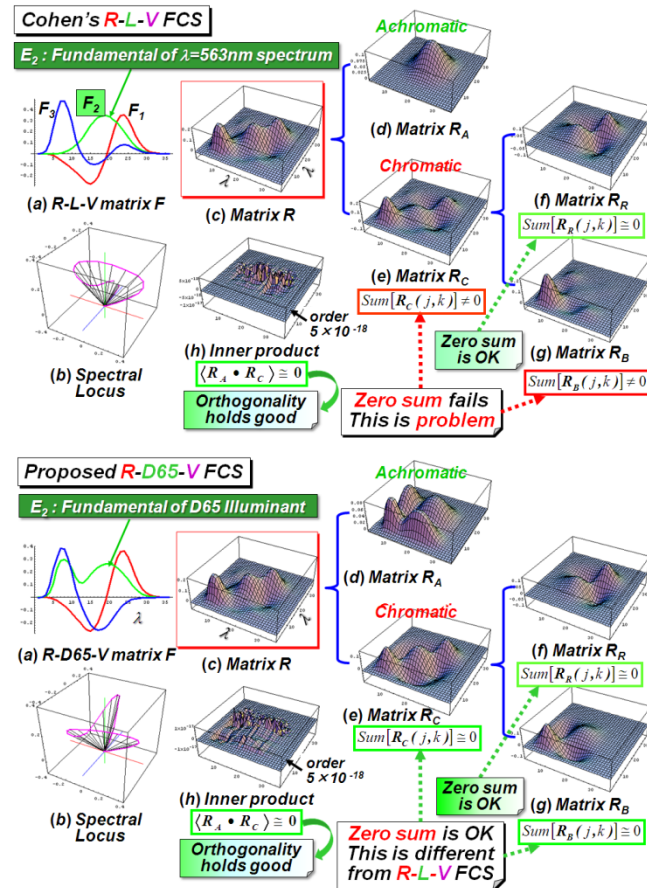


Figure 3 Opponency tests for FCS by decomposition of matrix *R*

Fig.3 summarizes how the *matrix R* is decomposed into the achromatic R_A and chromatic R_C components and further into the *opponent-color matrices* R_R and R_B . The upper half is the decomposition for “*R-L-V*” and the lower half is for the “*R-D65-V*” FCS. Although R_A and R_C are orthogonal each other in both models, the zero-sum conditions in the matrix R_B , hence R_C don’t hold good for “*R-L-V*”, while they are almost maintained in the “*R-D65-V*” with the negligible small errors. The zero-sum condition in *Yellow-Blue fundamental matrix* R_B is the definitive difference between them. That is, “*R-L-V*” is imperfect but “*R-D65-V*” is one of the perfect orthogonal opponent-color FCS.

Quality Measure of Color Scanning Filter

q-factor

Neugebauer’s *q-factor* has been easily used for estimating the goodness of RGB color scanning filters. It’s convenient for measuring the RGB channel errors independently.

Letting the spectral sensitivity in *R, G, and B* channel be $S_{Cam} = [S_R(\lambda), S_G(\lambda), S_B(\lambda)]$, the *q-factor* is given by

$$q_j = \sum_{k=1}^3 \langle S_j \cdot A^{ort_k} \rangle^2 / \|S_j\|^2; j = R, G, B \quad (13)$$

$$A^{ort} = [A^{ort_1}, A^{ort_2}, A^{ort_3}] = GramSchmidt[A]$$

Where, A^{ort} denotes an orthogonal CIE CMF and q_j simply means the power of cosine between the $j=R, G, B$ spectral sensitivity vector and the corresponding CMF, taking the value of $0 \leq q_j \leq 1$. If the $S_j(\lambda)$ is a linear transform of CMF, the Luther condition is perfectly satisfied with $q_j=1$.

Though Neugebauer used MacAdam’s orthogonal CMF as A^{ort} , this paper applied the orthonormalized standard CIE1931CMF by GramSchmidt.

Camera FCS by Matrix R_{Cam}

The device FCS such as digital camera is structured from the camera *matrix* R_{Cam} corresponding to *matrix R* by just replacing the CMF *A* with camera sensitivity S_{Cam} as

$$R_{Cam} = S_{Cam} (S_{Cam}^t S_{Cam})^{-1} S_{Cam}^t \quad (14)$$

Now the deviation from the Luther condition is measured by the difference between matrices R_{Cam} and matrix *R*.

Since a linear transformation of CMF also belongs to the family of CMFs, a linear matrix operation for camera sensitivity S_{Cam} may be allowed without affecting the Luther condition. Hence the basis matrix F_{Cam} of camera FCS is derived from the following two steps.

[Step1] Get best fit camera sensitivity to CIE CMF

Least Squares’ method

The linear transformation matrix M_{Fit} for getting the camera sensitivity S_{Fit} closest to CIE.CMF *A* is obtained by the method of least squares as

$$[S_{Fit}]^t = M_{Fit} S_{Cam}^t \text{ for } \min [E \{ \|S_{Fit} - A\|^2 \}] \quad (15)$$

$$M_{Fit} = [A^t S_{Cam}] [S_{Cam}^t S_{Cam}]^{-1}$$

Fit First method

According to "Fit First" method [13] by Worthey and Brill, the same S_{Fit} is obtained by operating the camera matrix R_{Cam} directly on A like as

$$S_{Fit} = R_{Cam}A = [S_{Cam} (S_{Cam}^t S_{Cam})^{-1} S_{Cam}^t]A \quad (16)$$

It's easy to prove that Eq. (15) equals Eq. (16).

[Step2] Get basis Matrix F for camera FCS

Next, we get the camera basis matrix F_{Cam} by orthonormalizing S_{Fit} with Gram Schmidt process as

$$F_{Cam} = GramSchmid[S_{Fit}] \quad (17)$$

Now the camera FCS is spanned by the triplet basis of F_{Cam} .

ν -factor in relation to Camera FCS

Neugebauer's q -factor is surely useful for judging the goodness of individual color filter independently but not for estimating a set of RGB in total. Hence ν -factor is proposed as a generalized q -factor extended to measure the total goodness for a set of multi-channels more than three.

The ν -factor is simply described as a deviation between the matrices R and R_{Cam} , in the sense of squared error of directional cosine between the projection matrices as

$$\nu(R, S) = \frac{Trace[RS]}{\alpha} \quad (18)$$

Assuming the spectral inputs are independent random variables, it's simply reduced to the average of q -factors as

$$\nu = (1/\alpha) \sum_{i=1}^{\beta} q_i \quad (19)$$

Where α denotes the rank of matrix R_{Cam} ($=3$) and β means the number of color channels ($=3$ for normal RGB camera).

Vectorial q -factor

In the past papers [11] [12], the colorimetric error for color scanner is evaluated as a spectral loci error using a virtual spectral target composed of Sine SPDs.

Recently, Worthey and Brill [13] advanced the same concept in the elegant manner by visualizing the LUM error in FCS. This clarified the defect in the spectral sensitivity design but missed to give the numerical measure unfortunately.

The error in camera LUM from HVSS is easily measured by the average spectral deviation, what we call, *vectorial q -factor* as

$$\begin{aligned} Vq &= 1 - MSE \left\| A^{Ort} - F^{Cam} \right\| \\ &= 1 - \left[\sum_{j=1}^3 \sum_{k=1}^n \{A_j^{Ort}(\lambda_k) - F_j^{Cam}(\lambda_k)\}^2 \right] / \left\| A^{Ort} \right\|^2 \end{aligned} \quad (20)$$

Estimation Results in Digital Camera

In practice, the spectral goodness is estimated for four commercial high-end digital cameras.

Evaluation in orthonormal standard CIE FCS

First the LUM errors between camera and human vision are estimated using the standard orthonormal CIE FCS as summarized in Fig.4. The numerical Vq values are listed in Table.1 in

comparison with q -factors and ν -factors. The camera LUMs are obtained starting from the measured spectral sensitivity S_{Cam} , calculating the best fit sensitivity S_{Fit} , and converting to the basis matrix F_{Cam} .

Evaluation in orthonormal opponent-color FCS

As well, the same estimation is performed using $R-D65-V$ orthonormal *opponent-color* FCS as shown in Fig.5. The results are little different from Fig.4.

The Vq values in opponent-color FCS show the severe assessment rather than in the orthonormal CIE FCS as listed in the last row in Table.1.

All of q , ν factors and Vq tell that camera C is the best and camera M is the worst. However the camera $N-1$ and $N-2$ (same maker) marked somewhat different scores in Vq from the conventional q or ν factors. The more detailed analysis is necessary but left behind as a future work.

Conclusions

The paper reconsidered the Luther condition from a spectral and colorimetric point of view. Recently a new concept of *spectral colorimetry* has been oriented to design the better multi-band camera incompatible with colorimetric and spectral reproduction.

The key point of proposed approach lies in the use of opponent-color FCS with zero-sum conditions, that means

- [1] *Colorimetrically*: Luther condition corresponds to the *fundamental* based on the matrix R theory of human vision
 - [2] *Spectrally*: the opponent-color FCS is based on the spectral decomposition of *chromatic fundamental* with the perfect opponent-color axes that reflect the LUM error spectrally important to color appearance in relation to the camera design.
- Furthermore, it's desirable to discuss the *device metamerism* from a point of spectral imaging. The FCS based approach to this problem is also left behind as a future work.

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Author Biography

Hiroaki Kotera joined Panasonic Co., in 1963. He received Doctorate from Univ. of Tokyo. After worked in image processing at Matsushita Res. Inst. Tokyo during 1973-1996, he was a professor at Dept. Information and Image Sciences, Chiba University until his retirement in 2006. He received 1993 journal award from IS&T, 1995 Johann Gutenberg prize from SID, 2005 Chester Sall award from IEEE, 2006 journal award from ISJ and 2008 journal award from SPSTJ. He is a Fellow of IS&T.

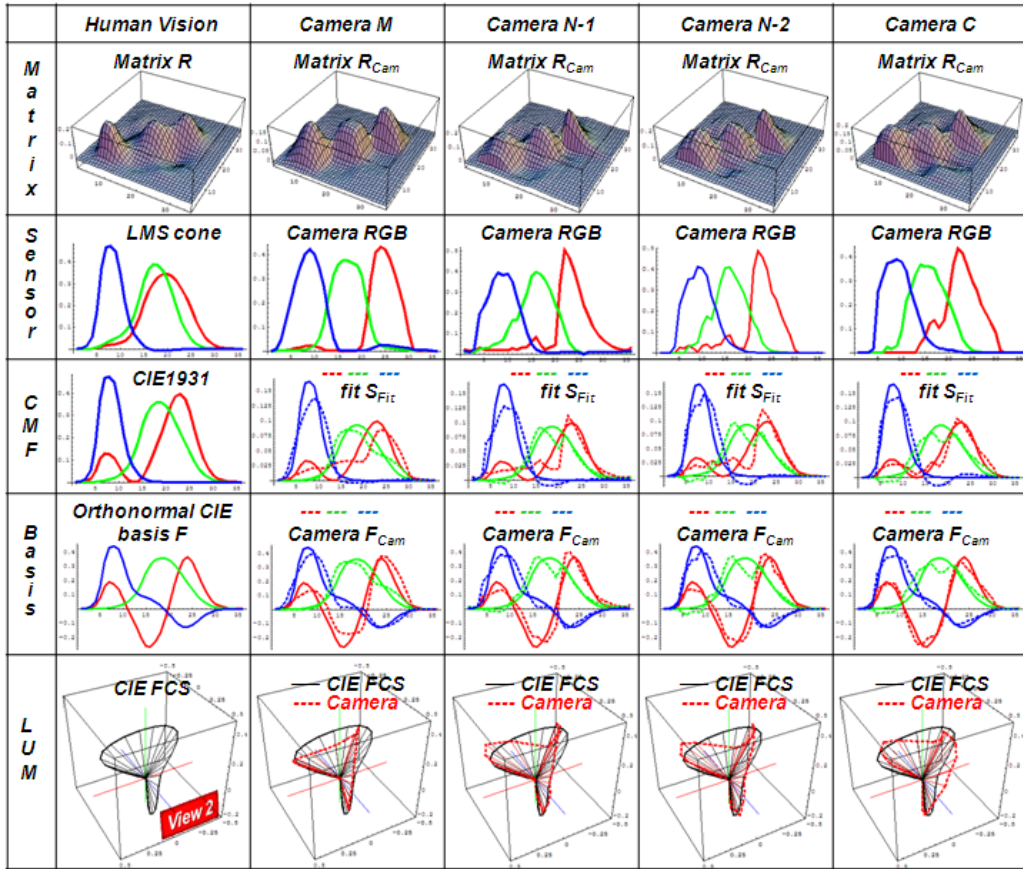


Figure 4 Results in spectral errors for digital cameras estimated in orthonormal standard CIE FCS

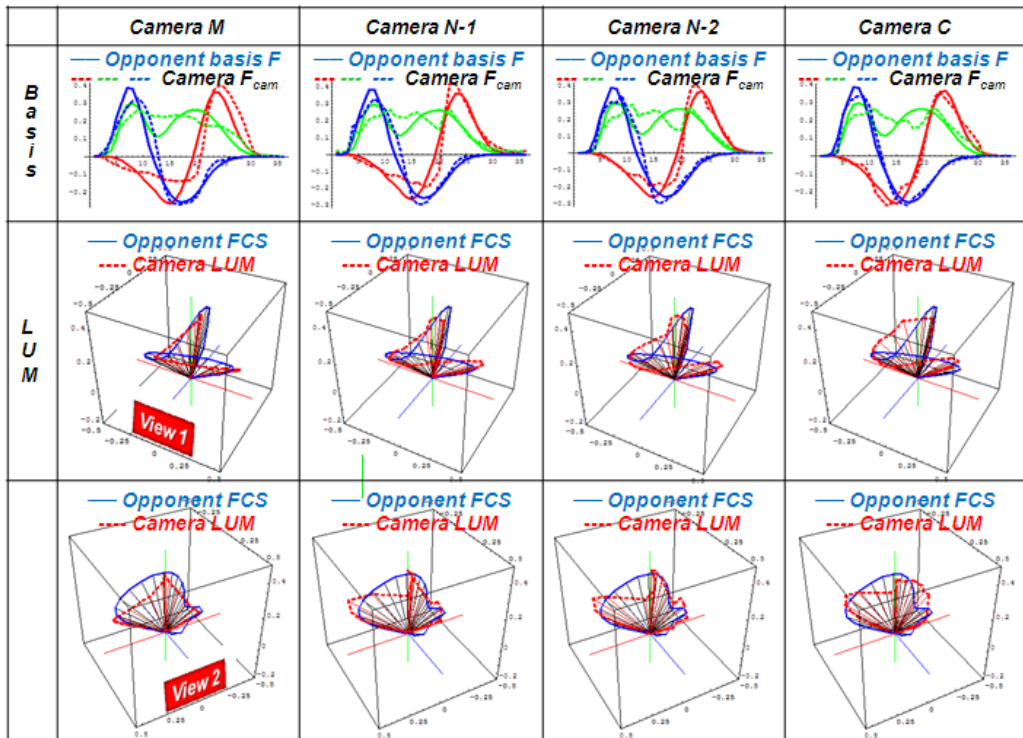


Figure 5 Results in spectral errors for digital cameras estimated in R-D65-V opponent-color FCS

Table 1 Measured quality factors for digital cameras

<i>Cameras</i>		<i>M</i>	<i>N-1</i>	<i>N-2</i>	<i>C</i>
<i>measures</i>					
<i>q-factor</i>	<i>q-Red</i>	<i>0.839</i>	<i>0.868</i>	<i>0.867</i>	<i>0.955</i>
	<i>q-Green</i>	<i>0.953</i>	<i>0.953</i>	<i>0.927</i>	<i>0.932</i>
	<i>q-Blue</i>	<i>0.920</i>	<i>0.900</i>	<i>0.914</i>	<i>0.944</i>
<i>-factor</i>		<i>0.904</i>	<i>0.907</i>	<i>0.902</i>	<i>0.944</i>
<i>Vectrial Vq</i>	<i>Orthonormal CIE FCS</i>	<i>0.909</i>	<i>0.929</i>	<i>0.920</i>	<i>0.940</i>
	<i>Opponent-color R-D65-VFCS</i>	<i>0.869</i>	<i>0.890</i>	<i>0.888</i>	<i>0.927</i>