Scaling Lightness Perception and Differences Above and Below Diffuse White

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Abstract

The purpose of this research was to design and complete psychophysical experiments for scaling lightness and lightness differences for achromatic percepts above and below the lightness of diffuse white ($L^*=100$). Below diffuse white experiments were conducted under reference conditions recommended by CIE for color difference research. Overall a range of CIELAB lightness values from 7 to 183 was investigated. Psychophysical techniques of partition scaling and constant stimuli were applied for scaling lightness perception and differences, respectively. The results indicate that the existing L^* and CIEDE2000-weighting functions approximately predict the trends, but don't well fit the visual data. Hence, three optimized functions are proposed, including a lightness function, a lightness-difference weighting function for the wide range, and a lightness-difference weighting function for the range below diffuse white.

Introduction

The CIELAB L* cube-root-based function, recommended by CIE in 1976,[1] is commonly used to compute correlates of the lightness perception. However, it is possible to compute lightness values greater than that of diffuse white (L*>100), such as when encountering fluorescent materials or high-dynamic-range images. Specularities on glossy materials might also appear lighter than diffuse white when viewing them with different viewing geometries. However, the CIELAB equation has no established psychophysical meaning when L* values exceed 100. There are seldom psychophysical experiments for scaling lightness perceptions exceeding diffuse white. As a result, conditions such as measuring and viewing geometries of materials are carefully controlled to avoid specular reflection in traditional colorimetric applications. There has been a growing interest, for both material specification and HDR imaging, in how to calculate meaningful perceptual magnitudes for a wider lightness range. In addition, lightness discrimination data for a wider range are also necessary, since most of the visual tolerance data are derived from samples darker than diffuse white and the detailed relationship between lightness discrimination and lightness level, as modeled in CIEDE2000, requires more data for verification.

Four experiments are described in this paper. The first and second experiments are designed to scale lightness perception for a range above and below diffuse white and are denoted by "SL>100" and "SL<100", respectively. The third and fourth experiments are intended for scaling lightness differences for a range above and below diffuse white and are denoted by "DE>100" and "DE<100", respectively. The first three experiments, SL>100, SL<100, and DE>100, were conducted together under similar viewing

environments. The fourth experiment, DE<100, was conducted under CIE reference viewing conditions.

Scaling Lightness Perception

Three terms from the *International Lighting Vocabulary* are used to scientifically define the lightness and brightness.[2] These are:

Luminance: A physical measure of the stimulus with unit of cd/m^2 .

Brightness: Attribute of a visual sensation according to which an area appears to emit more or less light.

Lightness: The brightness of an area judged relative to the brightness of a similarly illuminated area that appears white or highly transmitting.

The lightness scale can be approximately described by Stevens' power-law[3] to correlate the perceptual magnitude and the stimulus that evokes it. Prior to Stevens, the power relationship was also suggested by Godlove[4] to describe one of the most well-known and utilized lightness scales, the Munsell Value scale.[5] Generally, the power relationship represents the common feature of response compression in lightness scales. This is illustrated in Fig. 1 with the example of Munsell Value scale, where the CIELAB lightness equation exhibits a good approximation. CIELAB L* is scaled by a factor of 0.1 in the figure because Munsell Value is approximately 0.1L*. The CIELAB lightness equation, shown in Eq. 1, is a modification from the simple cube-root power function to avoid infinite slope at zero luminance and improve fitting the Value scale. It is worth to noting that a simple cube-root power function does not well describe the Value scale.



Figure 1: The typical response compression of lightness perception illustrated by the Munsell Value scale, CIELAB L*, and a simple cube-root function.

$$L^{*} = 116f(Y/Y_{n}) - 16$$

$$f(x) = \begin{cases} x^{1/3} & x > 0.008856 \\ 7.797x + 16/116 & x \le 0.008856 \end{cases}$$
(1)

Y and Y_n are the luminances of the stimulus and the diffuse white, respectively.

Effects on lightness perception

There were several lightness scale models proposed, [4,6-12] because the lightness perception is highly dependent on the luminance factor of the background, the level of illumination, stimuli configuration, sample size, and many other factors. As the optimal power-function exponent would be altered for different observers, experimental designs, and viewing condition, [2,13,14] those effects influencing lightness perception should be well understood and controlled when designing a lightness scaling experiment and analyzing the data. Fairchild[15] listed a series of color appearance phenomena that influence lightness perception. Simultaneous contrast, also referred to as lightness induction, is an effect that the perceived lightness of a stimulus changes with the relative luminance of the background. For instance, a gray stimulus looks lighter when it is placed on a darker background, and vice versa. Therefore, in order to control this effect, the backgrounds of stimuli are usually carefully controlled in a set of experiments.

Crispening is an effect where the perceived lightness difference between two stimuli is greater when viewed on a background with similar luminance. Semmelroth[16,17] developed equations to model the crispening effect for backgrounds with different luminance levels. According to his equations, lightness scales, illustrated in Fig. 2, exhibit a sharp increase in the slope of the curves due to crispening, and an overall shift of lightness magnitude due to simultaneous contrast.

The overall luminance level and the surround-relative luminance significantly influence lightness perception[18,19] as described by the Stevens effect and the Bartleson-Breneman equations, respectively. The perceived contrast, which correlates with the exponent of a lightness power-function, altered luminance level and surround relative luminance. In addition, the observers' state of adaptation also affects lightness perception.[18] Sixty seconds are the suggested duration for observers to adapt to a viewing condition when luminance is not significantly changed.[20] Moreover, the influences of sample size and stimulus configuration should be noted. Simultaneous contrast and crispening effects are more significant for smaller samples,[21] and the crispening effect disappears when samples have a small black frame.[22] Furthermore, response compression could significantly deviate from a power function for viewing conditions with changes of background and surround luminance level and stimulus configuration, thus requiring a more complex function to describe the visual results.[19]

In lightness scaling experiments, background has a very large influence on the results. It is necessary to obtain the background information and the viewing condition details when interpreting lightness scales and psychophysical results. It is desirable to choose data and scales obtained with similar viewing conditions when implementing a comparison.



Figure 2: Lightness scales with the influence of crispening for backgrounds with different relative luminance levels according to Semmelroth's equation.

Method of partition scaling

Experiments scaling lightness perception are intended to derive a ratio scale to correlate perceptual magnitudes of lightness attributes and physical measures of stimuli. The method of partition scaling, also known as method of bisection, has been successfully used to develop lightness scales. With two presented stimuli, A and B, the observer is asked to select a third stimulus, C, such that the lightness difference between A and B is equal to the lightness difference between B and C, or the lightness of C is halfway between A and B. A uniform lightness scale is obtained by conducting the experiment successively through the lightness range of interest.

Scaling Lightness Differences

Two psychophysical methods, constant stimuli and gray-scale comparison, are frequently used for scaling lightness differences. In the method of constant stimuli, observers are asked to judge whether the lightness difference of the anchor pair is greater or less than the lightness difference of the test pair. This method is also referred to as pass-fail judgement. In the method of gray-scale comparison, observers are asked to identify the lightness difference from a lightness difference gray-scale with equal ΔE^*_{ab} lightness difference steps that is closest to the lightness difference of a test pair. The method of constant stimuli is preferable, because its technique is based on fewer assumptions and provides marginally better precision.[23] Moreover, the method of constant stimuli is easier for observers to make judgments and for the experimenter to analyze results.

Five to seven samples are usually created along the ΔL^* direction for selected color centers to conduct lightness difference scaling using the constant stimuli method. A pilot experiment is typically executed to estimate the approximate tolerance threshold. Then, samples are created around the approximate tolerance threshold. The vector of each direction can be confirmed with principal component analysis.[24]

After conducting the visual judgement experiment, the responses of individual observer, encoded in 0 for smaller and 1 for larger than the lightness difference of the anchor pair, are corrected with a 3-by-1 low-pass filter (LPF) algorithm.[25] This LPF is based on the assumptions that, first, the observers tend to

have a monotonic visual judgement and, second, any lightness difference smaller than that of sample pairs in each group would tend to be chosen as smaller than that of the anchor pair, and vice versa. An example of this algorithm is shown in Fig. 3.



Figure 3: An example for applying the 3-by-1 low-pass filter algorithm to correct the non-monotonic responses.

Probit analysis[26] is then applied to the filtered responses. By using Probit analysis, the assumption that the responses follow a cumulative normal distribution is made. This assumption can be tested and generally holds well for such experiments.[27,28] The chi-squared value is used to determine how well each set of responses agrees with the Probit model. If the calculated chisquared value is greater than a value for the number of samples minus two degrees of freedom and a significance of 10%, there is a significant difference between the data and the model. The difference can be attributed to random factors in the experiment and can be corrected with a heterogeneity factor. When the chisquared value is larger than the criterion or the color differences of samples do not vary along a vector quite well, such as when the first eigenvalue is smaller than 0.99, the 3D Normit analysis[29] can be applied instead of Probit analysis. As variations in the other two dimensions, chroma and hue in this experiment, are considered as noise, the samples are distributed only in the lightness direction.

Lightness weighting functions

Ideally, the lightness tolerances (T50s) should be identical at any position of the lightness scale when the lightness scale is perceptually uniform. However, this is not the case for the CIELAB lightness function. All the factors that affect lightness perception lead to this nonuniformity. One of the significant factors is the crispening effect. Due to this effect, the minimum lightness tolerance corresponds to the experimental background lightness.[30] Slight differences in the experimental design can cause significant differences in the degree of crispening,[22,30] which represents as the degree of curvature in the plot of lightness tolerance against lightness position. When the curvature is slight, this effect can be ignored, such as the lightness weighting function in CIE94.[31] When the curvature is noticeable, a curve of U or V shape could be used to fit the visual data.[30,32,33] The lightness weighting function of CIEDE2000[32,33] exhibits this behavior and is shown in Eq. 2.

$$S_{L, \Delta E_{00}} = 1 + \frac{0.015(L^* - 50)^2}{\sqrt{20 + (L^* - 50)^2}}$$
(2)

It should be noted that the minimum weighting of the function is at the position of $L^* = 50$, which is the lightness value of the background suggested by CIE.

Experimental Design

Experiment 1: Scaling lightness perception for a range exceeding diffuse white, SL>100

The method of partition scaling was used in both lightness scaling experiments. The stimulus configuration of the experiment is shown in Fig. 4. It was printed with glossy paper with achromatic inks, and fastened to the wall in a darkened room. Outside the configuration, there was a frame of black foam-core board. A DLP projector, above the observer with a geometry of 45%/0° to avoid the specular light, was used to illuminate this configuration and create a viewing environment with 600 lux. (The base projected value is digital count 100.) The three patches in the center were actually paper white only and modulated in luminance using the registered projector image. The luminance at the area of paper white was 1060 cd/m². Stimuli with luminances lower and higher than paper white were generated by altering projected values for the three patches different from the base projected digital count. Each patch was 2-by-2-inches with a projected onepixel black frame. The viewing angle was about 4.8 degree for each patch in this experiment with a viewing distance of approximately 24 inches. Outside the test patches, there are oneinch-width backgrounds with L* value of 50, and followed by twoinch-width backgrounds of paper white and half-inch-width gray scales. The reason to include the paper white and gray scales in the background is to help observers perceive the paper white as the reference diffuse white. The absolute tristimulus values of the stimuli for the 10° observer were calculated from the spectral power distributions measured using a PhotoResearch PR655 spectroradiometer. CIELAB values were calculated by taking the paper white as diffuse white, since it is most correlated to the visual judgement.



Figure 4: The configuration of both lightness scaling experiments. The gray background and gray scales are printed, and the stimuli of three patches are illuminated by the DLP on the paper white.

To scale the lightness for stimuli lighter than the white point, the middle patch was presented as paper white with base projected value, and treated as $L^{*}=100$, and the left patch was presented with lower projected values to create stimuli of L^{*} ranging from 90 to 20 at an interval of 10. The observer was asked to adjust the right patch until the lightness difference between the right and middle patches equaled the lightness difference between the left and middle patches. As a result, the estimated lightness value 110 can be obtained by analyzing the results of presenting $L^*=90$ at the left patch. Lightness values from 120 to 180 can be estimated in the same manner, by analyzing the adjustments for presenting $L^*=80$ to $L^*=20$ at the left patch.

Experiment 2: Scaling lightness perception for a range below diffuse white, SL<100

The estimated lightness values of 60 to 95, at an interval of 5, were also acquired with the same experiment setup of SL>100. In the experiment of SL<100, the right patch is presented as L*=100 and the left patch is presented with L*=90 to 20, at an interval of 10. The observer was asked to adjust the middle patch until its lightness was halfway between the left and right patches. This range of data can be used to correlate with the data or functions from other lightness scaling experiments.

Experiment 3: Scaling lightness differences for a range exceeding diffuse white, DE>100

The method of constant stimuli was used in both experiments on lightness difference perception, DE>100 and DE<100. The stimulus configuration of DE>100 is illustrated in Fig. 5. The condition, geometry, experimental setup, and viewing measurement method are the same as the first two experiments, SL>100 and SL<100. The stimuli of two lightness-difference pairs in the center are generated by the DLP projections on the paper white. The size of each pair is 2-by-5-inches for a viewing angle of about 4.8 degree at the distance of 24 inches. There were projected one-pixel black frames for the samples and a projected one-pixel black dividing line between each pair. The L* value of the background was 50. It should be noted that a diffuse white is used for CIELAB calculation in both DE>100 and DE<100 experiments to correlate the results from the two sets of experiments. The configuration is not a CIE reference condition for color difference experiments. The goal of this design was to help observers have more information about the illuminant and preserve adaptation to the diffuse white while presenting significantly lighter stimuli.

Nine lightness centers were chosen, where five were above, one near, and three below diffuse white. The range of the nine lightness centers was from L*= 68 to L*=183. Seven sample pairs were created with differences mainly along the L* direction for each lightness center. PCA was applied to analyze each set of fourteen patches for each lightness center. The first eigenvalues were greater than 0.99 for the samples of all lightness centers. This establishes the uni-dimensionality of the stimuli. The CIELAB values of the anchor pair were $(L^*,a^*,b^*) = (48.7, 0.2, 1.3)$ and (46.2, 0.2, 1.3), with a ΔE^*_{ab} of 2.53. Sixty-three sample pairs were presented with different random orders for each observer. The first ten samples presented, including six repeated testing pairs, were always darker than the paper white. Observers were asked to choose the pair with larger lightness difference. The ability of lightness discrimination becomes lower after looking at a high-lightness test pairs. The influence of this phenomenon on the final result is diminished by presenting samples with different random order for each observer.



Figure 5: The configuration of lightness difference scaling experiment (Experiment 3, DE>100). The gray background and gray scales are printed, and the stimuli of two lightness difference pairs are illuminated by the DLP on the paper white.

Experiment 4: Scaling lightness differences for a range below diffuse white, DE<100

A lightness difference scaling experiment for a range of lightness below diffuse white was also conducted by following the CIE reference conditions. The samples were printed with glossy paper and adhered to foam core boards to make them flat and avoid observable gloss. The sample size was the same as the stimuli used in the RIT-Dupont dataset [24, 25], that is, 2-by-5-inches for each sample pair on a 4-by-6-inches gray background, while each sample patch is 2-by-2.5-inches. The sample pair and gray background were printed together on glossy paper. A dividing line, with the same color of the background, was printed between the two patches of each pair in order to make the sample pair look similar to pairs prepared by sticking individual patches on the background. For the anchor pair and sample pairs of the lightness center close to the lightness of the background, black lines were printed as the dividing line and a the frame around the patches. It made the patches distinguishable from the background. The size of the lines was one pixel of the 1200-dpi printer.

The experiment was conducted under simulated D65 illuminant in a Macbeth Spectralight III light booth. To avoid stray radiation and approximate a 0/45 viewing geometry, the back and sides of the light booth were covered with black velvet. The spectral power distribution of a diffuse white under the light booth was measured using a PhotoResearch PR655 spectroradiometer. The spectral reflectance factors of the samples were measured using a BYK-Gardner 45/0 spectrophotometer with large aperture. Then, the tristimulus values for the CIE 10° standard colorimetric observer and the CIELAB values were calculated. The lightness value of the background was L*=50.2. The CIELAB values of the anchor pair were (L*,a*,b*) = (50.12, -0.7, 0.2) and (48.95, -0.7, 0.5), with a ΔE^*_{ab} of 1.21. The range of nine lightness centers was from L*=7 to L*=90. As a result, a wide range from L*=7 to L*=183 was covered by the two lightness-difference scaling experiments.

Results and Discussions

Experiment 1 and 2 (SL>100 and SL<100)

The same group of fifty color-normal observers participated in both lightness-scaling experiments. The total range of estimated lightness values is from 60 to 180. (From 60 to 95 at an interval of 5 in the second experiment, and form 110 to 180 at an interval of 10 in the first experiment.) For each estimated lightness value, the luminance values adjusted by observers in the experiments are averaged. The estimated lightness values are then plotted as a function of the averaged luminance and shown in Fig. 6, with an optimized function to fit the visual data, shown in Eq. 3. Error bars are 95% confidence intervals based on the standard errors of the mean estimates.

$$L_{opt} = 105.12 \times (Y/Y_n)^{0.3847} \tag{3}$$

The R-squared value of this simple model is 0.986. Although the optimized functions based on models with various offset terms[34] have slightly better performance, they do not exhibit an intersection with the origin. More visual data, particularly in the range below a lightness value of 60 with similar experimental setups, might be required to correct and further refine this lightness function. The CIELAB lightness function is shown in the figure as well, since that applies best for visual data on a medium-gray background. There was a doubt whether the lightness scale exceeding diffuse white will keep following the power-law or become flattened very quickly. Figure 6 shows that response compression continues for the tested range and a power-based function can well describe the results. The CIELAB lightness function follows the data qualitatively, but falls outside the error bars for some parts of the range. This indicates that, in some extent, the calculated L* with values larger than 100 are meaningful.



Figure 6: The visual data, optimized lightness scale, and CIELAB lightness function.

Experiment 3 and 4 (DE>100 and DE<100)

The two difference scaling experiments were also conducted with fifty color-normal observers. Thirty-seven observers participated in both experiments. High repetition of observers between experiments 3 and 4 helps reduce the influence of interobserver variability. The lightness centers and T50s are shown in Table 1. The standard errors of DE>100 and DE<100 are 0.04 and 0.07, respectively. The *T*50s of the two experiments and the 95% fiducial limits are plotted in Fig. 7. There are three lightness weighting functions shown in the figure, the lightness weighting functions for CIEDE2000 (Eq. 2), optimization of DE<100 (Eq. 4), and optimization of the wide lightness range (Eq. 5).

$$S_{L, \text{ DE} < 100} = 3.59(L^* / 100)^2 - 3.71(L^* / 100) + 2.12$$
⁽⁴⁾

$$S_{L, \text{ Wide}} = 2.12(L^* / 100)^2 - 1.8(L^* / 100) + 1.79$$
(5)

Table 1. The color centers and *T*50s from two experiments of scaling lightness differences.

DE>100: tristimulus of diffuse white (922, 1060, 1096)												
Color Centers			+ T50			- T50						
L*	a*	b*	L*	a*	b*	L*	a*	b*				
183.7	2.4	-4.4	197.6	2.5	-3.3	169.9	2.3	-5.6				
164.6	2.8	-6.3	176.0	3.8	-5.3	153.3	1.7	-7.4				
146.7	1.7	-6.1	155.9	3.1	-6.3	137.5	0.3	-5.9				
127.0	0.4	-5.1	134.4	0.5	-5.5	119.6	0.4	-4.7				
106.5	-1.0	-4.8	113.1	-1.0	-5.7	99.9	-0.9	-3.9				
97.5	-0.8	-3.7	103.5	-0.9	-3.3	91.5	-0.7	-4.0				
88.2	-0.5	-3.1	93.9	-0.4	-3.1	82.4	-0.6	-3.0				
78.0	-0.5	-1.4	82.6	-0.5	-1.5	73.4	-0.4	-1.2				
68.2	-0.3	-0.8	72.1	-0.2	-1.4	64.3	-0.4	-0.2				

1													
DE<100: tristimulus of diffuse white (388, 409, 465)													
Color Centers			+ T50			- T50							
L*	a*	b*	L*	a*	b*	L*	a*	b*					
90.4	-0.5	-1.5	92.2	-0.5	-1.6	88.5	-0.5	-1.4					
80.0	-0.5	-0.9	81.8	-0.5	-1.1	78.2	-0.5	-0.8					
69.7	-0.6	-0.6	71.4	-0.6	-0.6	67.9	-0.6	-0.6					
58.6	-0.7	-0.5	60.3	-0.6	-0.6	56.8	-0.7	-0.4					
48.8	-0.7	0.3	50.2	-0.8	0.1	47.5	-0.6	0.5					
39.1	-0.3	1.4	40.5	-0.4	1.3	37.8	-0.2	1.6					
28.9	0.0	1.0	30.2	0.1	1.2	27.7	0.0	0.9					
19.3	-0.1	0.2	21.2	-0.2	0.3	17.4	-0.1	0.1					
8.8	0.6	0.1	11.0	0.4	-0.2	6.6	0.7	0.4					
6.9	0.6	0.1	9.3	0.7	0.6	4.5	0.5	-0.5					



Figure 7: The visual discrimination data (T50s and fiducial limits) derived from experiments of DE<100 and DE>100, in addition to the lightness weighting functions for CIEDE2000, optimization of DE<100, and optimization of the wide lightness range.

The polynomial model was used as preliminary descriptive summary of the data, not as a final model for consideration in color difference equations. The curve of DE<100 follows a trend of U or V curve shape, exhibiting the non-uniformities of CIELAB lightness-difference scale. However, the lowest point is around L*=30, not L*=50 that is close to the background. That might be caused by the observer variability and the nonuniformity of CIELAB lightness scale that gives too large ΔL^* values for lightness differences for dark and light samples. It should be noted that the lowest lightness weighting is not necessary to be at the position of L*=50, when the crispening effect is largely diminished or cancelled out. The crispening effect will not be significant for complex stimuli configuration, such as cross-media image reproduction. Hence, a lightness weighting function eliminating the crispening effect might be appropriate. Although the configuration in this experiment was not a complex stimuli one, the crispening effect, for the sample pairs with lightness values close to the background, could be cancelled out by the black lines around the patches. As a result, the visual data in this experiment are appropriate for the application in cross-media image reproduction.

Color centers around L* 70, 80, and 90 are included in both DE<100 and DE>100. The results around this range don't overlap. Several differences between the experiment 3 and 4 might lead to this diversity, such as the experimental setup, modes of viewing, and color differences of the anchor pair.

In the experiment of DE>100, observers took longer when judging the lightness difference of the lightest samples. That is because of the tendency for observers to adapt to the higher luminance level. Although that means the reference white might change, it more reasonably represents the real world. For example, it is not uncommon to judge the relative brightness of various light sources in an environment or to look at surface colors that exceed the lightness of diffuse white such as areas on metallic or pearlescent automotive finishes. After paying attention to the such stimuli, we can not only clearly see perceptions of lightness above diffuse white, but are often concerned with color tolerances for such colors. As the observers adapt to a higher luminance level, the perceived lightness difference of the anchor pair becomes smaller. In this case, the judgments could be different when based on memory and when based on contemporaneously-perceived lightness difference of the anchor pair. This is one of the reasons that might lead to larger fiducial limits in the range exceeding diffuse white.

Conclusions

The compressive shape of lightness perception as a function of luminance factor can be approximately described by a powerlaw-based function. This compressive shape has been shown to hold for a range exceeding diffuse white with the psychophysical experiments described in this paper. Experiments on lightness difference scaling were also conducted to derive the color difference tolerance data for the range both below and above diffuse white. The trend of a U or V curve of lightness weighting functions still held for the normal range and wide range lightness. Optimized lightness function and lightness weighting functions were derived to fit the visual data in a descriptive sense. The optimized lightness function mainly corrects the under-prediction of the CIELAB lightness function for the range exceeding diffuse white. The optimized lightness weighing function for the range below diffuse white shows similar prediction with CIEDE2000 lightness weighting function for normal lightness range. However, two optimized lightness-weighting functions predict better than the CIEDE2000 function for the range exceeding diffuse white. More visual data are necessary to verify and improve the functions and more fully specify lightness and color appearance outside the range of normal, diffuse, reflecting objects that is so necessary for modern HDR and wide-color-gamut imaging systems.

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