Toward an automatic color calibration for 3D displays

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Abstract

This article considers the color correction of a 3D projection display installation. The system consists of a pair of projectors of the same model modified by INFITEC GmbH such that they can be used for projection of 3D contents. The goal of this color correction is to reduce the difference between the two mo-dified projectors such as the color difference between them does not disturb the user. Two new approaches are proposed and compared with the Infitec expert correction. One is based on an objective colorimetric match, the other on the optimization of a transform considering the color difference between the two signals.

Introduction

The concept of 3D projection is not new, at the early age of photography already, photographers have experimented this principle by taking two pictures of the same scene to give another dimension to their image. Each taken picture corresponding to the same scene as if it was viewed for one by the left eye and for the other by the right eye.

Today, 3D is becoming more and more popular in various domain (cinema, medical imaging, simulation, etc) and various technologies exist, requiring specific glasses (stereoscopic) or not (auto-stereoscopic) to see 3D content. The concept remains identical: displaying two images of a scene, one for the right eye and one for the left eye.

From a colorimetric point of view, the case of an auto-stereoscopic display is simple: the same light source is used to project/display images to both eyes. On the other hand, a stereoscopic display needs two projection systems and glasses (the spectacle glasses consist of a couple of filters that scatter the signal in two, one for each eye) to display 3D images. In some technologies, two different sets of primaries are necessary.

Improvements in filter technology has allowed to reduce the difference between the primaries of a pair of projectors [1], filters can have narrow band sizes (e.g. the use of two narrow red filters on the projectors decreases the difference between the two red primaries). But still, the difference is perceivable when an image or a film is observed without glasses: one projector is usually said to be reddish and the other greenish.

In our work, we used projectors modified by *INFITEC* GmbH. The projectors are modified with filters that are introduced in each projector to divide each primary red (R), green (G) and blue (B) in a reddish (or greenish) projector. The glasses are developed in parallel such that each spectacle has filtering properties according to the filters introduced in each projector. As a result our projection system is made of two projectors of very near set of primaries, near according to wavelength peaks difference between the red, green and blue primaries. The colorimetric properties of such a system has been studied [2], but no color correction is proposed yet, as far as we know, except the individual calibration of each projector followed by a manual expert correction.

We continue this article by presenting in detail this projection system and showing how the projectors primaries are modified with the filters. Based on this, we study different approaches for reducing the reddish and greenish effects. The first approach involves a manual process defined by *INFITEC* GmbH (the common process). We then show how an objective correction can be set up to reach an automatic colorimetric equivalence. The third method consists in optimizing the transform to maximize the dynamic range while keeping the difference unnoticeable.

Experimental setups

Projection system and measurement

A pair of modified JVC DLA-HD1 projectors constitutes our projection system. The light source is an Ultra-high pressure mercury lamp with three typical wavelength peaks for the three primaries.

With a *BLUE-Wave* Spectrometer from StellarNet we measure the light reflected by our screen (a wall painted by slightly grayish color). The measurements are made on a spectral window of $[190nm-1150nm]$ with a resolution of $0.5nm$, for our experiments we work only on the visible spectrum window of $[400nm-700nm]$ and each spectral reflectance measured is re-sampled to obtain a resolution of $1nm$ such that each reflectance is represented by a vector of $400:1:700=301$ discrete values.

In a first step we project an image completely white (i.e. digital value $\mathbf{c}=[r\ g\ b]^T=[1\ 1\ 1]^T$), for the red projector alone, the green projector alone and the two projectors together and measure the light reflected on the screen for each of these configurations. The direct measured curves are displayed in Fig. 1, Fig. 2 and Fig. 3. In each figure the red curves correspond to the projector with red filter, the green curves to the projector with the green filter. Our reference white $\mathbf{w}$ is the combination of both projectors projecting their maximum intensity in the same time, this is showed in the same figures by the black curve. Similarly our reference black $\mathbf{k}$ is the measurement of both projectors projecting a black image.

To illustrate the terms greenish projector and reddish projector in those figures we have superposed on the primaries curves the closest corresponding CIE color matching function (CMF) curves for the CIE 1931 standard observer , i.e the $\bar{x}(\lambda)$ is displayed in Fig. 1 for the red primaries of both projectors. In these figures the CIE CMFs have been scaled such that we can observe how the peak of each function matches or not with the peak of each projector primary.

Later, to compute colorimetric values of the displayed color from the various RGB digital values input, we normalize the spectral signal using the following formula:

$$\mathbf{S}=\frac{\mathbf{s}-\mathbf{k}}{\mathbf{w}-\mathbf{k}} \tag{1}$$

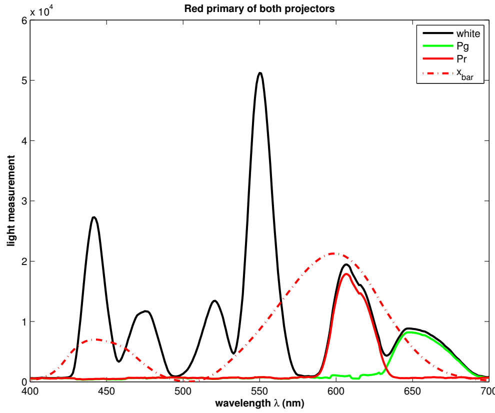

Figure 1. *Light measurements of white (both projectors projecting white), the red primaries for both greenish (in green) and reddish (in red) projectors and the* $\bar{x}(\lambda)$.

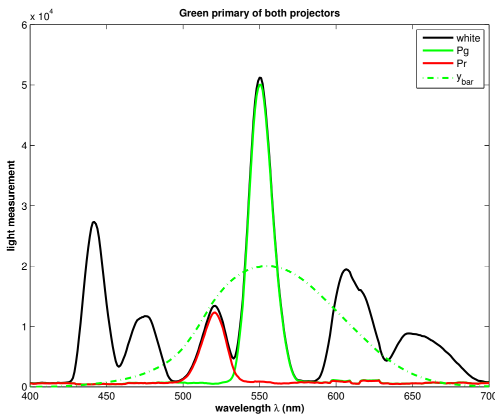

Figure 2. *Light measurements of white (both projectors projecting white), the green primaries for both greenish (in green) and reddish (in red) projectors and the* $\bar{y}(\lambda)$.

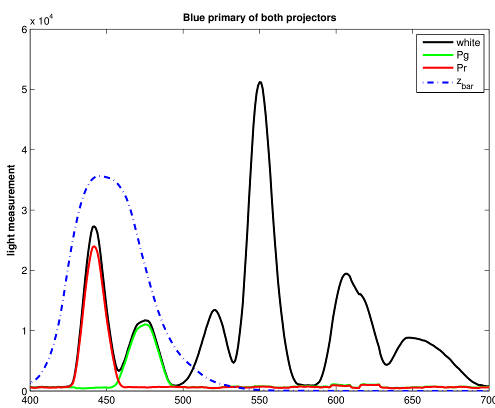

Figure 3. *Light measurements of white (both projectors projecting white), the blue primaries for both greenish (in green) and reddish (in red) projectors and the* $\bar{z}(\lambda)$.

that includes an offset correction **k** and where **s** is the measure from our spectrometer. In the figure showing the light measurements of pairs of primaries we can observe the poor overlap between the primaries which illustrates the sharpness of the filters. Also Fig. 4 displays the same information but in a chromaticity diagram and reveals the differences between the primaries, especially for the green channels.

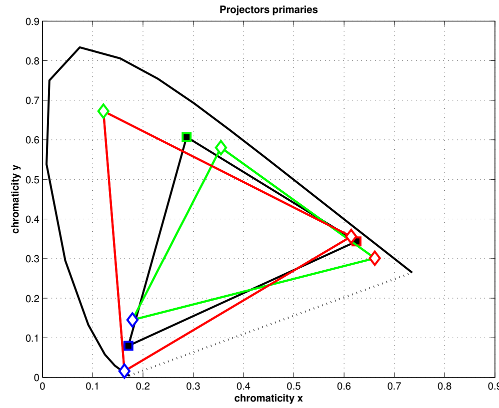

Figure 4. *Primaries describing the projector gamuts in the chromaticity diagram. In red, the reddish projector, in Green the greenish. The projector resulting of the sum of them seen without 3D glasses is plotted in Black.*

Study of displayable colors

We project and measure ramps of red, green, blue and gray to evaluate the intensity response curve of each primary. For the future experiments in this article we approximate the response curve of each primary by a power function $x^{\gamma} = x^{1.8}$.

The chromaticity values of each primaries are plotted for each projector in Fig. 5. This figure shows as well the gamut resulting in summing couple of primaries from both projectors and the common gamut between them.

Binocular color correction

As we can observed in the figures showing the spectral curves and chromaticity diagrams each projector presents a dominant tint. If the display is observed without glasses, the superposition of the full intensity images from both projectors should appear neutral/white to the human eye. However, the 3D effect appears while wearing the glasses. Looking at the image through one spectacle, one can easily perceive the dominant tint, the effect is decreased when the images arrive on each eye, but still the perceived image or color signal slightly deviates from the desirable color (especially for 'known' colors such as white snow, blue sky, human skin, etc). We can face three cases: The difference is large enough to cause binocular color rivalry [4], or it is small enough to generate a binocular color fusion [5]. In the case of a color fusion, we can face two cases: either it can be disturbing, such as in the snow or blue sky example, either it is unnoticeable or at least not disturbing.

The problem can be solved through a color correction that is similar to a gamut mapping problem: *How to modify the color rendering of each projector such that the differences between the displayed colors are smaller so that the color difference between the perceived images are not disturbing?*

We limit this work to simple and practical approaches, thus we consider a linear transform of the original RGB values. We can define the correction on the original RGB values used to control the projectors as follows:

$$\mathbf{c}' = \mathbf{M}\mathbf{c} \tag{2}$$

where $\mathbf{M}$ is a 3×3 correction matrix and $\mathbf{c}' = [r'\ g'\ b']^T$ and $\mathbf{c} = [r\ g\ b]^T$ the corrected and original normalized RGB values.

In this article we compare three approaches to evaluate the correction matrices (one for each projector): one approach controlled by the eyes of experts, one colorimetric objective transform and a third method that defines the transform in minimizing the colorimetric error while maximizing the resulting common gamuts.

All approaches require to linearize the intensity response curve of the projector, such as in Eq.3. We approximated the response curve with a power function, finding a gamma value of 1.8 for our installation.

$$\mathbf{c_Y} = [r_Y\ g_Y\ b_Y]^T = \mathbf{c}^\gamma \tag{3}$$

The colorimetric value of a displayed color can then be approximated assuming the primaries chromaticity constancy and a perfect additive mixing system:

$$\mathbf{C} = \mathbf{P}\mathbf{c_Y} \tag{4}$$

with $\mathbf{C} = [X, Y, Z]^T$ the tristimulus colorimetric values of the displayed color. And $\mathbf{P}$ the colorimetric transform associated to a projector such as:

$$\mathbf{P} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \tag{5}$$

where each column contains the **CIE XYZ** values of the primaries red, green and blue at their full intensity.

Following the same model, the green and red projectors, P_g and P_r are defined respectively by:

$$\mathbf{P_g} = \begin{bmatrix} X_{Rg} & X_{Gg} & X_{Bg} \\ Y_{Rg} & Y_{Gg} & Y_{Bg} \\ Z_{Rg} & Z_{Gg} & Z_{Bg} \end{bmatrix} \tag{6}$$

and

$$\mathbf{P_r} = \begin{bmatrix} X_{Rr} & X_{Gr} & X_{Br} \\ Y_{Rr} & Y_{Gr} & Y_{Br} \\ Z_{Rr} & Z_{Gr} & Z_{Br} \end{bmatrix}. \tag{7}$$

Considering these two displays, we can define a common gamut such as shown in Fig.5.

This gamut defines a virtual display that is the intersection of both projectors. We computed the primaries as proposed in [3] (see the Appendix for a more detailed description of this method), and defined the colorimetric transform based on matrix P_v.

$$\mathbf{P_v} = \begin{bmatrix} X_{Rv} & X_{Gv} & X_{Bv} \\ Y_{Rv} & Y_{Gv} & Y_{Bv} \\ Z_{Rv} & Z_{Gv} & Z_{Bv} \end{bmatrix} \tag{8}$$

From Eq. 4 we can formulate the problem for one projector such as: Finding the transform that gives $\mathbf{c_Y^*}$ from $\mathbf{c_Y}$, such as:

$$\mathbf{C} = \mathbf{P_v}\mathbf{c_Y} \tag{9}$$

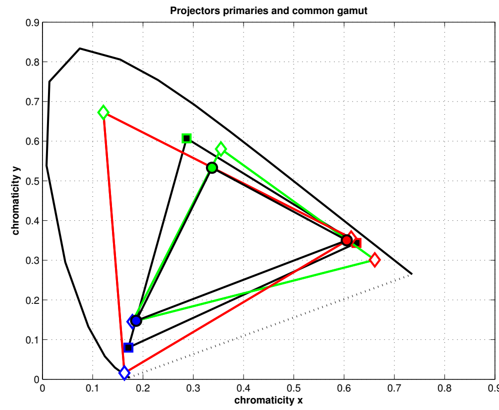

Figure 5. *The common color gamut in the chromaticity diagram is represented by the triangle with circle as vertices.*

and

$$\mathbf{C} = \mathbf{P_x}\mathbf{c_Y^*}, \tag{10}$$

$\mathbf{P_x}$ being either $\mathbf{P_g}$ or $\mathbf{P_r}$.

Then, the transform from $\mathbf{c_Y}$ to $\mathbf{c_Y^*}$ is defined by

$$\mathbf{P_x}\mathbf{c_Y^*} = \mathbf{P_v}\mathbf{c_Y} \tag{11}$$

thus

$$\mathbf{c_Y^*} = \mathbf{P_x^{-1}}\mathbf{P_v}\mathbf{c_Y} \tag{12}$$

thus

$$\mathbf{M} = \mathbf{P_x^{-1}}\mathbf{P_v} \tag{13}$$

where $\mathbf{M}$ is a 3×3 matrix which modifies the linearized RGB values as follows:

$$\begin{aligned} r_Y^* &= r_Y \times a_{11} + g_Y \times a_{12} + b_Y \times a_{13}. \\ g_Y^* &= r_Y \times a_{21} + g_Y \times a_{22} + b_Y \times a_{23}. \\ b_Y^* &= r_Y \times a_{31} + g_Y \times a_{32} + b_Y \times a_{33}. \end{aligned} \tag{14}$$

considering the constraints: $r_Y, g_Y, b_Y \in [0,1]$ and $r_Y^*, g_Y^*, b_Y^* \in [0,1]$.

Solutions

This section considers three methods to retrieve the matrix $\mathbf{M}$, $\mathbf{M}$ being either $\mathbf{A}$ for the greenish projector and $\mathbf{B}$ for the reddish projector.

In all cases, the correction on the digital values is applied as follow:

- normalization of the RGB values.
- $\mathbf{c_Y} = \mathbf{c}^\gamma$ which linearize the digital values
- Establishment of the correction matrix $\mathbf{M}$
- Correction: $\mathbf{c_Y^*} = \mathbf{M}\mathbf{c_Y}$
- the corrected values are de-gamma $\mathbf{c}' = \mathbf{c_Y^*}^{\frac{1}{\gamma}}$

Expert correction

This correction is performed manually by a color expert who establishes the parameters of M based on his experience and on multiple tries. This is the common way when buying an *INFITEC*GmbH system.

We start this approach by projecting a white image, i.e. all pixels are turned to one: $[r\ g\ b] = [1\ 1\ 1]$.

The final values of the matrix **A** and **B** are obtained in a loop process which stops when the feeling of non-correctness color decreases enough for the expert eyes.

The matrix **A** and **B** are defined as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{15}$$

and for each row i of **A** we have

$$\sum_{i=j}^{3} a_{ij} \leq 1 \tag{16}$$

and for each element of a_{ij} of **A**

$$0 \leq a_{ij} \leq 1. \tag{17}$$

The advantage of this approach is that we do not need to define the common gamut, and that it is probably taking the best of both projector since the process is stopped when the expert is satisfied. However this process can be time consuming, especially when we work with tiled displays installation, and let the colorimetric inaccuracy unknown.

Objective correction

This approach aims to perform an objective colorimetric correction. For that, we need to rewrite the colorimetric transform matrix. We consider that in most cases, the system is color managed and that the primaries of each display are known (either by measurement or by data provided by the manufacturer).

Let us define $\mathbf{P_v}$ as follow:

$$\mathbf{P_v} = \mathbf{P_x} + \Delta\mathbf{P_x} \tag{18}$$

with

$$\Delta\mathbf{P_x} = \begin{bmatrix} \Delta X_{Rx} & \Delta X_{Gx} & \Delta X_{Bx} \\ \Delta Y_{Rx} & \Delta Y_{Gx} & \Delta Y_{Bx} \\ \Delta Z_{Rx} & \Delta Z_{Gx} & \Delta Z_{Bx} \end{bmatrix} \tag{19}$$

The $\Delta\mathbf{P_x}$ parameters are retrieved from the computation of the common gamut.

We can thus rewrite Eq. 12 as:

$$\mathbf{c_Y^*} = \mathbf{P_x}^{-1}(\mathbf{P_x} + \Delta\mathbf{P_x})\mathbf{c_Y} \tag{20}$$

and Eq. 13 as

$$\mathbf{M} = \mathbf{P_x}^{-1}(\mathbf{P_x} + \Delta\mathbf{P_x}) \tag{21}$$

This solution maps all colors inside the common gamut. It is automatic, but can lead to a too much reduced dynamic range if the common gamut is too small.

Optimized correction

In the optimization approach, we optimize the transform **M** in order to have more freedom than in the objective colorimetric transform. That means that we accept to include some colorimetric errors in the transform, but we constraint this transform to lead to no disturbance for the observer. This means that we accept to introduce a colorimetric error that is judged to be negligible.

For each projector we want to solve the following optimization problem:

$$\min_{\Delta\mathbf{P_x}} fun(\mathbf{P_x}, \Delta\mathbf{P_x}) \tag{22}$$

where the function fun in the previous equation returns the average color difference ΔE^*_{ab} between the color displayed by a single projector and the color displayed in the common gamut for the same RGB digital values. We can rewrite:

$$\min_{\Delta\mathbf{P_x}} ||(\mathbf{P_x} + \Delta\mathbf{P_x})\mathbf{c_Y} - (\mathbf{P_v})\mathbf{c_Y}|| \tag{23}$$

We introduce in the optimization a tolerance criterion under the form of a ΔE^*_{ab} difference. A very large ΔE^*_{ab} should give us almost no modifications between $\mathbf{c_Y^*}$ and $\mathbf{c_Y}$ - some variation may occur depending of the starting guess value for the optimization. At the opposite introducing a tolerance of $\Delta E^*_{ab} = 0$ should bring us back to the analytical solution of the problem.

In theory at each iteration of the optimization process, each projector is reduced toward the optimal gamut, i.e. the common gamut defined by $\mathbf{P_v}$.

We are not aware of any clear threshold in the literature that should be used in this case. However, our experience is that for a white uniform patch, a ΔE^*_{ab} of 20 is not involving a critical disturbance.

Results and discussion

In the expert solution, we stop the process when correction matrices **A** and **B** give us visually satisfactory results. For our installation the matrices have the following values:

$$\mathbf{A} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

and

$$\mathbf{B} = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.1 & 0.8 \end{bmatrix}. \tag{25}$$

Using the projector model defined in Eq. 4 we can compute color differences ΔE^*_{ab} between the simulation of the color displayed by both projectors from the same RGB values.

We have defined a uniform grid in the RGB device color space for which we have calculated the color differences. The average difference obtained is around $\Delta E^*_{ab} \approx 20$. The difference is huge, but still smaller than when no correction matrix is applied, and is not disturbing to the user.

At the opposite, the analytical method give us extremely good results: a $\Delta E^*_{ab} = 0$. But this is not a surprise since by definition the computed matrix **M** map all the values to the common gamut. This method required to know the matrices of each projector, in our experiment we have measured these values for the red projector:

$$\mathbf{P_r} = \begin{bmatrix} 26.4 & 1.1 & 6.69 \\ 13.4 & 11.7 & 0.57 \\ 0.01 & 2.48 & 34.3 \end{bmatrix}, \quad (26)$$

and for these values for the green projector

$$\mathbf{P_g} = \begin{bmatrix} 6.03 & 9.2 & 2.35 \\ 2.3 & 18.1 & 1.75 \\ 0.002 & 0.18 & 16.2 \end{bmatrix}, \quad (27)$$

which allow to compute the common gamut defined by this matrix

$$\mathbf{P_v} = \begin{bmatrix} 6.55 & 7.95 & 3.28 \\ 3.33 & 15.17 & 2.11 \\ 0.01 & 2.51 & 16.28 \end{bmatrix}. \quad (28)$$

As we can see in Fig. 5 the common gamut reduced the dynamic of the displayable colors. It is to notice that the computational approach [3] used to define the common gamut leads to some errors. To continue on the reduction of the dynamic, the Table 1 illustrates the new range of digital values to control the projectors when the correction matrices are applied. If the green channel is not too attenuated (between 20% and 10% reduction) between the expert and objective corrections, we can observe that the others channels for the objective correction are almost divided by two when the expert correction keep at the minimum 80% of the available dynamic.

As we expected the optimization method locates itself between the analytical and expert method. The variation of the tolerance factor - a ΔE^*_{ab} value - having for effect to decrease or increase the common gamut between the two projectors. In Fig. 7 to Fig. 8 are displayed the pairs of optimized gamuts for each projector with different tolerance criterion value from $\Delta E^*_{ab} = 5$ to $\Delta E^*_{ab} = 2$. With no surprises the optimization with the tolerance criterion set to 0 gives similar results as the optimized gamut $\mathbf{P_v}$ shown in Fig. 5.

We can notice that the virtual gamut can be outside of the projector gamuts. This is due to some freedom we took over constraints during the optimization process.

The table compare the reduction of color dynamic when the different correction matrices are applied to the rgb digital command $[1\ 1\ 1]^T$**.**

Method	Proj.	M	r	g	b
Expert	$\mathbf{G/R}$	$\mathbf{A/B}$	1.0	0.8	1.0
			0.8	1.0	0.9
Objective	$\mathbf{G/R}$	$\mathbf{P_x^{-1}P_v}$	1.0	0.88	1.0
			0.51	1.0	0.46
Optimization	$\mathbf{G/R}$	$\mathbf{P_x^{-1}}\cdot$	1.0	0.66	1.0
$\Delta E_{ab} = 20$		$(\mathbf{P_x} + \Delta\mathbf{P_x})$	0.34	1.0	0.44

Conclusion

We have presented three different approaches for the color calibration of a 3D projection system. The projection system is constituted of two identical projectors, both equipped with color filters such that each eye receives one of the stereoscopic images. The use of filters divides the set of RGB projector primaries in two subsets of RGB primaries with narrow wavelength peaks, as result each eye sees different colors. The color calibration for this system aims to reduce the color difference between the two projectors.

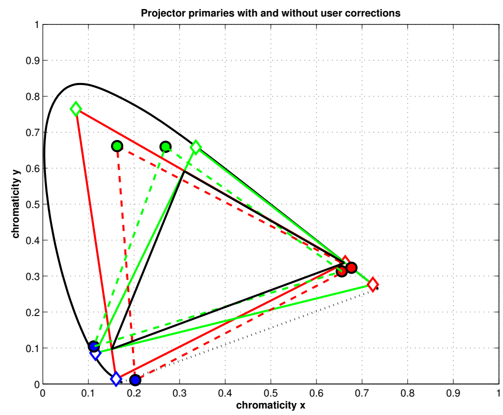

Figure 6. *Primaries of the projectors in the chromaticity diagram before and after manual correction applied. In red, the reddish projector, in green the greenish projector and in black the common gamut. The triangles plotted in dashed line red and green represent respectively the corrected reddish and greenish projectors.*

The first approach, called expert method, find the calibration parameters based on an expert appreciation and the confirmation by another panel of experts. This method does not require to measure the system primaries and the colorimetric accuracy remains unknown.

The two others methods need the primaries measurement or estimation and can automatize the color calibration. The objective method requires to compute the common color gamut of the system. While this method is colorimetricaly the most accurate it reduces also strongly the color gamut, thus the dynamic range of the display.

Finally the optimization method offer the possibility to optimize the gamut to a given threshold for which the human observer accepts to receive two different color signals. It is to note that even if we used CIE colorimetry for practical reasons, the fact that each eye receives a different image makes this choice to be disputed. Further works include the funding of the best threshold though psychophysical experiment.

References

[1] Helmut Jorke and Markus Fritz, INFITEC - A new stereoscopic visualisation tool by wavelength multiplex imaging, http://www.infitec.net/news.html.

[2] Gadia, D. and Bonanomi, C. and Rossi, M. and Rizzi, A. and Marini, D., Color management and color perception issues in a virtual reality theater, Proc. of the SPIE, .(2008).

[3] Pagani, A. and Stricker D., Spatially uniform colors for projectors and tiled displays, J. of the SID, 15, 9 (2007).

[4] Andrews T. J. and Lotto R. B., Fusion and Rivalry Are Dependent on the Perceptual Meaning of Visual Stimuli, J. of Current Biology, 14 (2004).

[5] Olson T., Optimizing Anaglyph Colors, Proc. of the 17th CIC (2009).

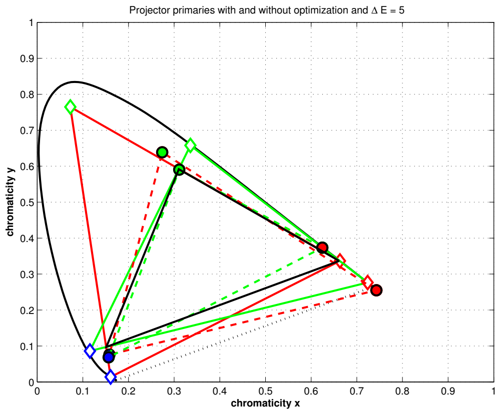

Figure 7. *Primaries of the projectors in the chromaticity diagram. In red, the reddish projector, in green the greenish and in black the common gamut. The gamut displayed in dashed red and green lines with nodes as circles correspond to modified gamut by optimization for a tolerance criterion ΔE^*_{ab} set to* 5.

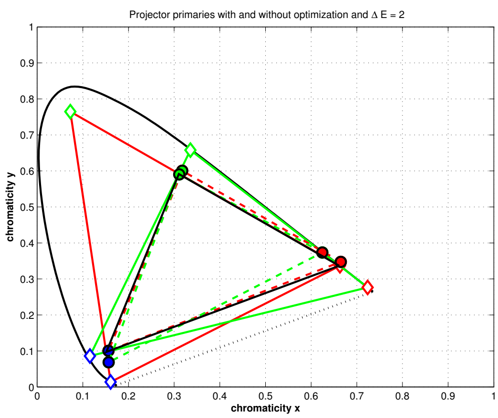

Figure 8. *Primaries of the projectors in the chromaticity diagram. In red, the reddish projector, in green the greenish and in black the common gamut. The gamut displayed in dashed red and green lines with nodes as circles correspond to modified gamut by optimization for a tolerance criterion ΔE^*_{ab} set to* 2.

[6] Ikeda M. and Sagawa K., Binocular color fusion limit, J. Opt. Soc. Am., 69, 2 (1979).

Appendix

computation of common gamut

In the approach described by Pagani and Stricker [3], the search of the common gamut between a group a of projectors follows two steps. The first one determines if a color is displayable or not by all projector. The seconde one is local and searches for extreme points of the common gamut, i.e. to define what are the red, green, blue, white and black of the common gamut.

This appendix describes only the general step of the algorithm called "displayability test". This step answers the following question: is the chromaticity xy displayable by a projector defined by its matrix $\mathbf{P}$ as defined in Eq. 5 and in which luminance range? The second step is just a simple optimization process well described in the original paper.

Displayability test

To perform this test we start from the general projector model:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{P} \begin{pmatrix} r \\ g \\ b \end{pmatrix} + \begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix} \tag{29}$$

where r, g and b are in $[0,1]$ and the $[X_k\ Y_k\ Z_k]^T$ is the offset. Then re-writing the previous equation as follows:

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \mathbf{P}^{-1} \begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix} \tag{30}$$

Since we make the displayability test for the chromaticities values xy, we need then to switch to the CIE Yxy space to clearly separate the luminance and the chromaticities. Let be L the luminance of the color, the direct conversion from (X,Y,Z) to (L,x,y) is given by:

$$(X,Y,Z) \rightarrow \left(L\frac{x}{y}, L, L\frac{1-x-y}{y} \right) \tag{31}$$

which allows to re-write equation 30 as follows:

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix} = L\mathbf{P}^{-1} \begin{pmatrix} \frac{x}{y} \\ 1 \\ \frac{1-x-y}{y} \end{pmatrix} - \mathbf{P}^{-1} \begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix}. \tag{32}$$

Knowing the limits of $[r\ g\ b]^T$ and L we have six inequalities to solve for a given xy which determine if this chromaticity is displayable or not.

We remind that we know in the previous equation the bounding values for r, g, b, x, y and L. The six inequalities can be then extracted by re-writting Eq. 32:

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix} = L \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} - \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{33}$$

and finally solving

$$\begin{aligned} 0 &\le La_1 - b_1 \le 1, \\ 0 &\le La_2 - b_2 \le 1, \\ 0 &\le La_3 - b_3 \le 1. \end{aligned} \tag{34}$$

If the limits obtained for L satisfy that L is a positive real value, then the displayability test for the chromaticities values xy is positive.

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Author Biography

Jérémie Gerhardt received his Master in image processing from the University Pierre et Marie Curie (2002) and his PhD in image and signal processing from the Ecole Nationale Supérieure des Télécommunications (ENST) in Paris (2007). Since then hes has worked in the Fraunhofer FIRST in Berlin, Germany. His work has focused on spectral color imaging, in his beginning on spectral color reproduction with multi-ink printer and yet on color correction for tiled displays installation.

Jean-Baptiste Thomas received his bachelor in Applied Physics in 2004 and his Master in Optics, Image and Vision in 2006, both from the University Jean Monnet. He received his PhD from the University of Bourgogne in 2009. He is now employed as a researcher at the Gjøvik University College. His research focuses on color in displays and multi-display systems.