

Spectral Imaging Model by XYZ+K Four-Band Filter

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Abstract

Spectral imaging has advanced to reproduce the real colors under different illuminants. Since the high-precision spectral camera is expensive, multi-band cameras with 6~16 channels have been widely used. On the other hand, human vision is basically tri-chromatic and perceives a color based on metamerism. According to matrix- R theory, any spectral input C is decomposed into fundamental C^* and metameric black B . Human vision perceives the fundamental C^* as visible component but bypasses the metameric black B as invisible component. On the conditional color matching under a fixed illuminant, we need only the fundamental. However both of fundamental and metameric black are necessary under the different illuminants, because the objective spectrum $C=C^*+B$ changes with the illuminant. This paper proposes a new spectral sensing model with XYZ+K four-band filters. The additional filter K is designed to rescue the “metameric black” and restores the spectral reflectance in combination with “fundamental” captured by XYZ colorimetric tri-color filters. The paper clarifies the mathematical design concept for the spectral sensitivity curve of filter K based on matrix- R theory and shows how the proposed XYZ+K sensing model works well when applied to the parametric or non-parametric spectral estimators keeping with the colorimetric color reproduction. The paper also introduces a simulation on spectral image restoration under the different illuminants.

Introduction

Since high-precision spectral camera is expensive, the multi-band system¹⁻³ supported with spectral estimation algorithms⁴⁻⁷ from low-dimensions have been developed actively. Singular Value Decomposition (SVD), Principal Component Analysis (PCA) or Wiener estimation methods provided a mathematical solution to this direction. However, the most of multi-band cameras have the following questions in the spectral design of multi-band filters.

[1] The spectral sensitivity design rule is unclear and choice of filter set is often empirical.

[2] The multi-band signals are not compatible with XYZ colorimetric reproduction.

LabPQR is a spectral CMS model for answering these questions. Lab plus PQR reflects the colorimetric plus metameric black components. However, it needs multi-band camera for encoding the spectral input to LabPQR⁸. Once encoded to LabPQR, it works excellent in the colorimetric to spectral inverse estimation with trained color chips.

The proposed model introduces a minimum set of XYZ+K four-band filters, where an additional filter K is designed to save the “metameric black” and restores the spectral reflectance in combination with “fundamental” captured by XYZ colorimetric tri-color filters. Different from LabPQR, the model needs neither spectral input camera nor encoding process but directly recovers the spectral input from the XYZ+K four-band signals.

The projection from spectral to tri-color space is many-to-one and non-invertible. Cohen⁹ derived the matrix R to project n -dimensional spectrum onto 3-dimensional HVS and clarified how it extracts the “fundamental” and ignores the “metameric black”. But both are necessary for the spectral imaging under the different illuminants.

In the previous paper^{10, 11}, the author proposed a simple method for the spectral estimation from XYZ using a strong correlation with the metameric black. It was useful for the specified color media such as photography or inkjet prints with the trained parameters but was substantially impossible to discriminate the metamers, because the XYZ system is blind to the metameric blacks. Since the “fundamental” is carried by XYZ value, first of all, the new model captures it by XYZ filters and next tries to rescue the lost “metameric black” by the additional filter “ K ”¹².

The paper clarifies a spectral design of filter “ K ” based on matrix R theory and shows how it’s useful for spectral recovery in compatible with colorimetric reproduction.

Fig.1 illustrates the basic concept of proposed XYZ plus K filter system for spectral imaging.

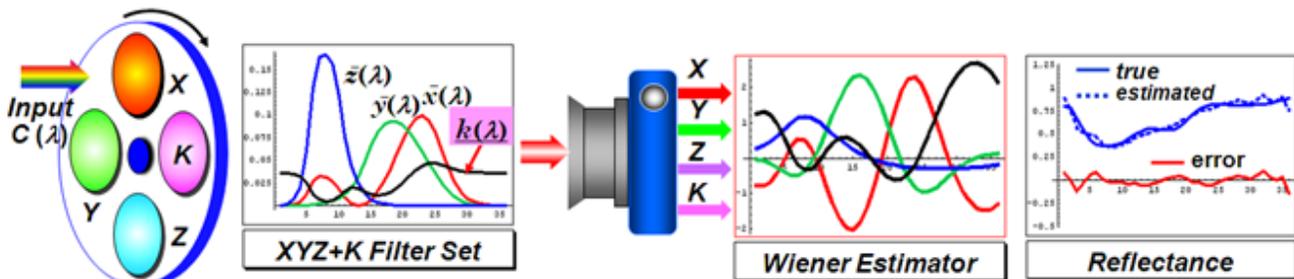


Figure 1 Conceptual Model of Spectral Recovery from XYZ plus K Filter system

Four-Band Model: Why XYZ + "K"?

According to the *matrix-R* theory, a n -dimensional color spectrum C is decomposed into the *fundamental* C^* (visible) and the *metameric black* B (invisible) as

$$\begin{aligned} C &= C^* + B, \quad C^* = RC, \quad R = A(A^t A)^{-1} A^t \\ B &= R_K C, \quad R_K = I - R; \quad I = \text{identity matrix} \end{aligned} \quad (1)$$

Where, R denotes $n \times n$ projector onto HVSS derived from CIE color matching matrix A .

The tri-stimulus value $T=XYZ$ carries the *fundamental* C^* that is the essential spectrum to human vision, while the *metameric black* B is neglected as invisible with zero-stimulus.

Letting a XYZ input be T_{65} taken by $sRGB$ camera for an input spectrum C_{65} under the illuminant D_{65} , the *fundamental* C_{65}^* is invertible from T_{65} by pseudo-inverse projection as

$$\begin{aligned} C_{65} &= C_{65}^* + B_{65}, \quad T_{65} = A^t C_{65} = A^t C_{65}^* \\ C_{65}^* &= P_{inv} T_{65}, \quad P_{inv} = A(A^t A)^{-1} \end{aligned} \quad (2)$$

Eq. (2) means the XYZ tri-color filter tacitly captures the *fundamental* spectrum C_{65}^* .

While, the *metameric black* B_{65} is obtained by applying the projector $R_K = I - R$ to input C_{65} as

$$B_{65} = R_K C_{65}, \quad R_K = I - R \quad (3)$$

The proposed model is equipped with a popular XYZ and the additional forth filter K to capture the *metameric black*. The filter K should have the spectral transmittance just same as projector R_K . However it's unrealistic to imitate R_K exactly, because R_K is a 2-D matrix with $n-3$ freedom of dimension ($n=36$, $\Delta\lambda=10$ nm: $\lambda=380\sim 730$ nm).

Hence, the proposed model drastically reduces the dimension of R_K to minimum 1-band filter K . Since the *fundamental* is exactly

restored from XYZ filter designed to satisfy *Luther condition*, the final spectral error mainly depends on the estimation accuracy of the *metameric black*. Fig.2 illustrates why the forth filter should be " K " if compatible with colorimetric reproduction and if allowed to add one more filter at least, and shows how to estimate the spectral input from $XYZ+K$ filter set.

Design of Forth Filter "K" based on Matrix R

Although it's hard to save the exact the *metameric black* B_{65} only by the 1-D filter K , the model approximates the projector R_K by paying attention to the following structural features.

(a) The j -th row vector B_j in R_K is composed of the *metameric black* for the single spectrum $\delta(\lambda_j)$.

$$\begin{aligned} R_K &= [B_1, B_2, \dots, B_j \dots B_n]^t \\ B_j^t &= R_K \delta(\lambda_j), \quad \delta(\lambda_j) = [0 \dots 0, 1, 0 \dots 0]^t; \quad j\text{-th element} = 1 \end{aligned} \quad (4)$$

(b) The j -th element $B_{65}(\lambda_j)$ in B_{65} is given by the inner product of row vector B_j and input C_{65} as

$$\begin{aligned} B_{65} &= R_K C_{65} = [B_{65}(\lambda_1), B_{65}(\lambda_2), \dots, B_{65}(\lambda_n)]^t \\ B_{65}(\lambda_j) &= \langle B_j \cdot C_{65} \rangle = \sum_{k=1}^n B_j(\lambda_k) C_{65}(\lambda_k) \end{aligned} \quad (5)$$

Eq. (5) denotes the exact *metameric black* B_{65} is sensed by applying a 2-D optical filter with the transmittance $B_j(\lambda_k)$ of R_K to the camera input spectrum C_{65} .

Now, taking the scalar summation of the elements in the vector B_j as a weighting coefficient for C_{65} , Eq. (5) is simplified decisively as

$$\begin{aligned} B_{65} &\cong K^t C_{65}, \quad K = [K(\lambda_1), K(\lambda_2), \dots, K(\lambda_n)] \\ \text{where, } K(\lambda_j) &= \left\{ \sum_{k=1}^n B_j(\lambda_k) \right\} \end{aligned} \quad (6)$$

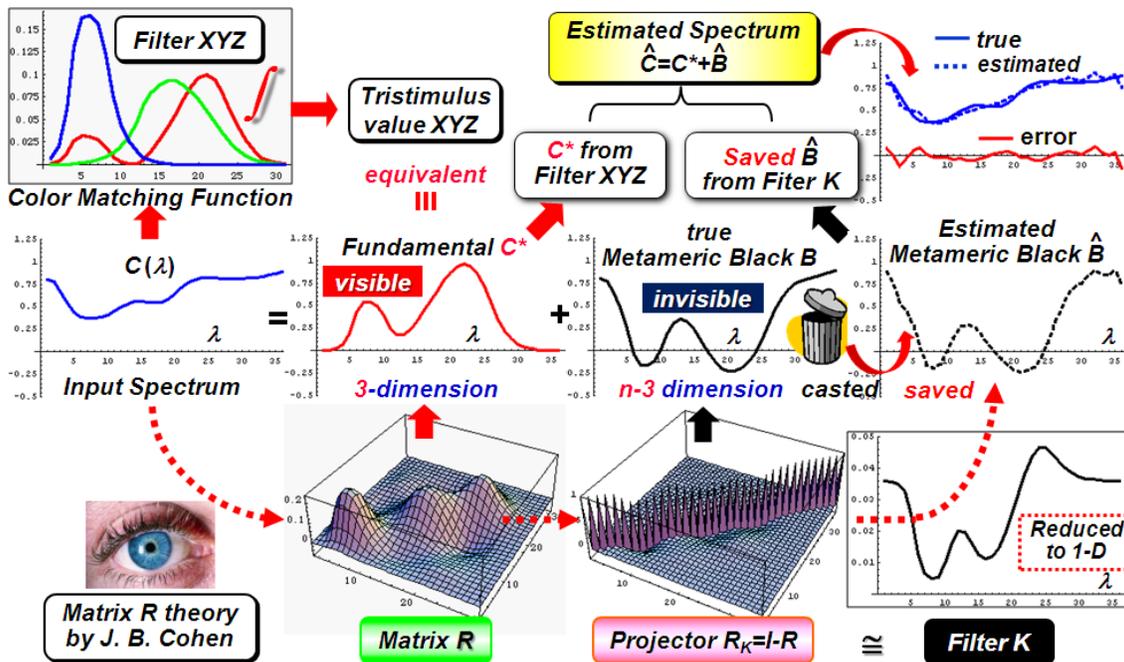


Figure 2 Why the forth filter should be K to save metameric black in compatible with XYZ?

The vector \mathbf{K} represents the transmittance of objective 1-D filter \mathbf{K} as a rough approximation for projector \mathbf{R}_K . Thus the *metameric black* \mathbf{B}_{65} is given by a simple product of the 1-D coefficient vector \mathbf{K} and the input spectrum \mathbf{C}_{65} . Geometrically, Eq. (6) means that the $n \times n$ matrix \mathbf{R}_K is reduced to $n \times 1$ filter \mathbf{K} by the integrated projection along the row (or column) direction. In other words, \mathbf{K} is equal to the *metameric black* \mathbf{B}_{EE} for the illuminant \mathbf{EE} with *Equal-Energy Spectral Power Distribution*. Indeed, in the case of $\mathbf{C}_{65}=\mathbf{EE}$, Eq. (5) becomes

$$\mathbf{B}_{EE} = \mathbf{R}_K \mathbf{EE} = \mathbf{K} \quad (7)$$

Because \mathbf{EE} includes all single spectra equally, $\mathbf{K}=\mathbf{B}_{EE}$ reflects all of the *metameric blacks* corresponding to the entire single spectra. In short, the filter \mathbf{K} is simply interpreted to pick up the *metameric blacks* of single spectra weighted by the input spectrum \mathbf{C}_{65} as given in Eq. (6).

Fig.3 illustrates the geometric structure of 1-D filter \mathbf{K} how it's related to the *metameric black* \mathbf{B}_{EE} and formed as the integrated projection of \mathbf{R}_K to the row (or column) direction. However, since the *metameric black* \mathbf{B}_{EE} has partly negative responses, we can't realize it as a physical filter \mathbf{K} . The

transmittance of optical filter \mathbf{K} should satisfy positivity in all ranges.

Assuming the spectral sensitivities of $X, Y, Z,$ and \mathbf{K} filter are linear independent, the forth filter \mathbf{K} may be composed by a linear transform of them without changing any basic property.

Letting the spectral response be $B_{EE}(\lambda)$ for the *metameric black* \mathbf{B}_{EE} , for example, we can design the spectral sensitivity of forth filter \mathbf{K} with positivity by simply subtracting the minimum bias as a constant level as

$$K(\lambda) = B_{EE}(\lambda) - \min[B_{EE}(\lambda)] \geq 0 \text{ for } \lambda = 380 \text{ nm} \sim 730 \text{ nm} \quad (8)$$

Otherwise, by a linear combination with CIE $\bar{x}(\lambda)$ of filter X as

$$K(\lambda) = B_{EE}(\lambda) + \bar{x}(\lambda) > 0 \quad (9)$$

Thus, a novel four-band $XYZ+\mathbf{K}$ filter set with the spectral sensitivity \mathbf{S} is formed as

$$\mathbf{S} = [\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda), K(\lambda)]' \quad (10)$$

Fig.4 illustrates the normalized spectral sensitivities of $XYZ+\mathbf{K}$ filter set in case of Eq. (9)

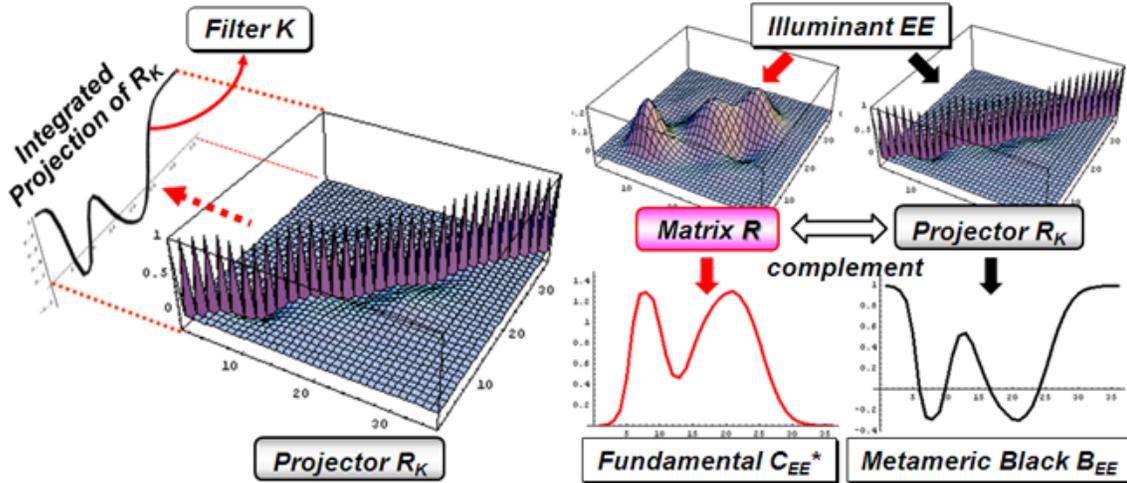


Figure 3 Basic profile of Filter \mathbf{K} approximated by integrated projection to 1-D from 2-D \mathbf{R}_K

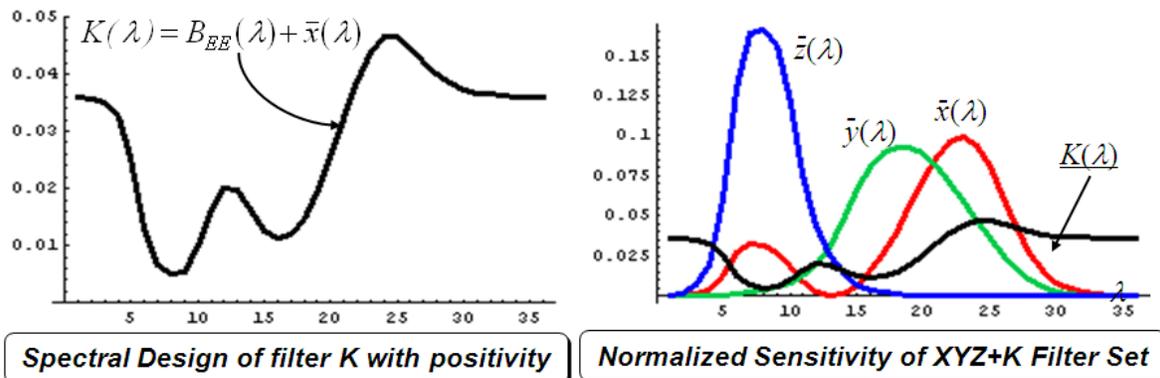


Figure 4 Spectral sensitivities of filter \mathbf{K} and normalized $XYZ+\mathbf{K}$ filter set

Spectral Estimation from XYZK Signal

The optoelectronic signal \mathbf{Q} from XYZ+K filter set is given by

$$\mathbf{Q} = [X, Y, Z, K]^t = \mathbf{S}^t \mathbf{C}_{65} + \mathbf{e}; \mathbf{e} = \text{system noise} \quad (11)$$

In order to estimate the input spectrum \mathbf{C}_{65} from the four-band signal \mathbf{Q} , the typical estimation algorithms are applied to the proposed model and its performance was evaluated as follows.

Pseudo-inverse Solution

Neglecting the noise \mathbf{e} , a solution to minimize the norm (energy) of \mathbf{C}_{65} is simply given by applying the well-known pseudo-inverse matrix \mathbf{S}_{inv} as

$$\hat{\mathbf{C}}_{65} = \mathbf{S}_{inv} \mathbf{Q}, \quad \mathbf{S}_{inv} = \mathbf{S}(\mathbf{S}^t \mathbf{S})^{-1} \quad (12)$$

for minimizing $\|\hat{\mathbf{C}}_{65}\| = (\hat{\mathbf{C}}_{65}^t \hat{\mathbf{C}}_{65})^{1/2}$

This is easy to calculate without any a priori knowledge, but isn't always a stable solution.

Smoothed-inverse Solution

Since the color objects in nature have the gentle and smoothed spectral shapes, minimum-norm estimation with the constrained smoothness provides the better solution.

Taking the second derivative of spectral input \mathbf{C}_{65} by applying the Laplacian operator \mathbf{D}

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{n-2} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_{65}(\lambda_1) \\ C_{65}(\lambda_2) \\ \vdots \\ C_{65}(\lambda_n) \end{bmatrix} = \mathbf{D} \mathbf{C}_{65} \quad (13)$$

Because $\boldsymbol{\delta}$ reflects "edge" component, its quadratic norm will be a smoothness measure as

$$\boldsymbol{\delta}^t \boldsymbol{\delta} = \sum_{i=1}^{n-2} \delta_i^2 = \mathbf{C}_{65}^t \mathbf{D}^t \mathbf{D} \mathbf{C}_{65} = \mathbf{C}_{65}^t \mathbf{N} \mathbf{C}_{65} = \|\mathbf{C}_{65}\|_N^2 \quad (14)$$

where, $\mathbf{N} = \mathbf{D}^t \mathbf{D}$

Now introducing Lagrange multiplier \mathbf{A} to minimize Eq. (14) under the smoothing matrix \mathbf{N} ,

$$J(\mathbf{C}_{65}) = \mathbf{C}_{65}^t \mathbf{N} \mathbf{C}_{65} + \mathbf{A}^t (\mathbf{S}^t \mathbf{C}_{65} - \mathbf{Q}) \quad (15)$$

The scalar term $J(\mathbf{C}_{65})$ is minimized when its first derivative is zero

$$\partial J(\mathbf{C}_{65}) / \partial \mathbf{C}_{65} = 2\mathbf{N} \mathbf{C}_{65} + \mathbf{S} \mathbf{A} = 0 \quad (16)$$

Solving Eq. (16) under Eq. (11), we get the solution by modifying \mathbf{N} to be non-singular as

$$\hat{\mathbf{C}}_{65} \cong \mathbf{S}_m \mathbf{Q}, \quad \mathbf{S}_m = \mathbf{N}^{-1} \mathbf{S} (\mathbf{S}^t \mathbf{N}^{-1} \mathbf{S})^{-1} \quad (17)$$

for $\mathbf{N} = \mathbf{N} + \epsilon \mathbf{I} (\epsilon \ll 1)$

Wiener-inverse Solution

Wiener-inverse is a popular estimator to minimize MSE under the known statistics for signal and noise as follows. The bias term \mathbf{b} is usually ignored as small, but here $\mathbf{m}_c \cong 0.3$ is used for general case¹³.

$$\hat{\mathbf{C}}_{65} \cong \mathbf{W}_{inv} \mathbf{Q} + \mathbf{b}$$

$$\begin{aligned} \text{where, } \mathbf{W}_{inv} &= \mathbf{R}_{CC} \mathbf{S} (\mathbf{S}^t \mathbf{R}_{CC} \mathbf{S} + \mathbf{R}_{ee})^{-1} \\ \mathbf{R}_{CC} &= \text{covariance matrix, } \mathbf{R}_{ee} = \text{noise variance} \\ \mathbf{b} &= \mathbf{m}_c - \mathbf{W}_{inv} \mathbf{S}^t \mathbf{m}_c \\ \mathbf{m}_c &= \text{mean vector of input spectra} \end{aligned} \quad (18)$$

Though the true covariance matrix \mathbf{R}_{CC} is obtained from trained samples, the Markov model is useful for non-parametric Wiener estimation. Assuming a strong correlation with coefficient ρ between the adjacent spectral components, the covariance matrix \mathbf{R}_{CC} is modeled by

$$\mathbf{R}_{CC} = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \dots & \rho^{n-1} \\ \rho & 1 & \rho & \rho^2 & \dots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \rho & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \dots & \dots & \dots & 1 \end{bmatrix} \quad (19)$$

Finally, removing the illuminant \mathbf{D}_{65} from $\hat{\mathbf{C}}_{65}$, the spectral reflectance $\hat{\mathbf{r}}$ is calculated by

$$\hat{\mathbf{r}} \cong \text{diag}[\mathbf{D}_{65}]^{-1} \hat{\mathbf{C}}_{65}, \quad \mathbf{D}_{65} = [D_{65}(\lambda_1), D_{65}(\lambda_2), \dots, D_{65}(\lambda_n)]^t \quad (20)$$

Experimental Results

Spectral Reflectance Estimation for Color Chips

The proposed model was tested for the spectral recovery of the typical color chips;

- (A) Macbeth chart, (B) IT8 chart, (C) Inkjet chip,
- (D) Acrylic paint chip in SOCS database

The estimation accuracy depends on the performance of inverse-projection operator from the low-dimensional signal \mathbf{Q} ($n=4$) to the high-dimensional spectrum ($n=36$). The parametric Wiener-inverse operator is fitted to the trained color samples, while the capability of non-parametric operator depends on how it can well imitate the parametric one without training. Fig.5 shows some examples of inverse-projection operator and the covariance matrix \mathbf{R}_{CC} .

Fig.6 illustrates a result in the spectral reflectance estimation for #125 chip (skin color) in IT8 and #24 acrylic paint chip (permanent green light) in SOCS. The performance was improved from left to right, resulting best in parametric Wiener-inverse method for all of tested chips.

Spectral Estimation Error

Table1 summarizes the spectral estimation errors. The goodness of the XYZK filter will be measured by to what extent the Wiener-inverse reaches SVD, because SVD will be ideal for describing the target by the best-fit eigen functions under the limited order of freedom. The parametric Wiener-inverse (shadowed lists with asterisk mean trained by own chip) resulted in the best and its MSE reached nearly to the same level as 4-th order SVD.

This tells the proposed XYZK filter is designed reasonably and works excellent in spite of only four bands.

Table2 shows CIELAB ΔE_{ab}^* under the changing viewing illuminants. The XYZK model clearly shows the error smaller than SVD, even if using the non-trained Wiener. It's a great advantage superior to SVD due to the compatibility with XYZ.

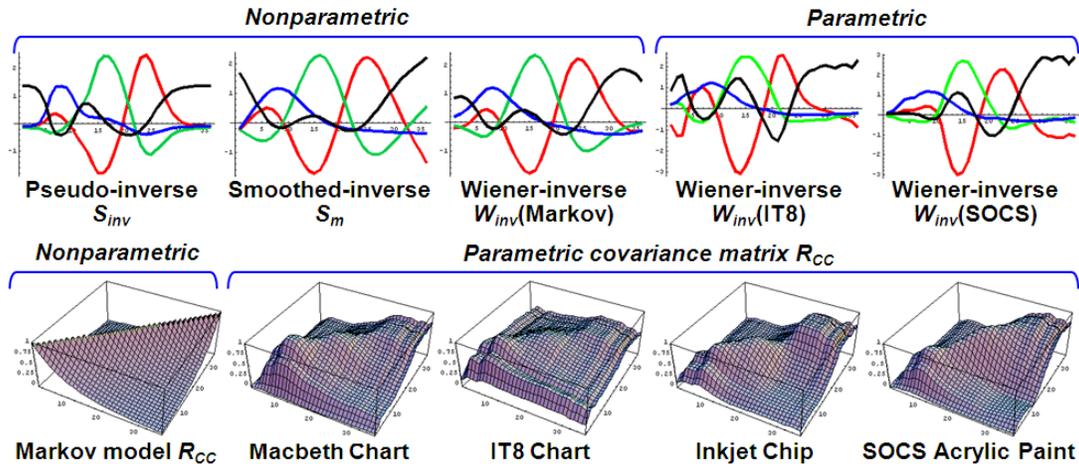


Figure 5 Spectral estimation operators (upper) and covariance matrices (lower)

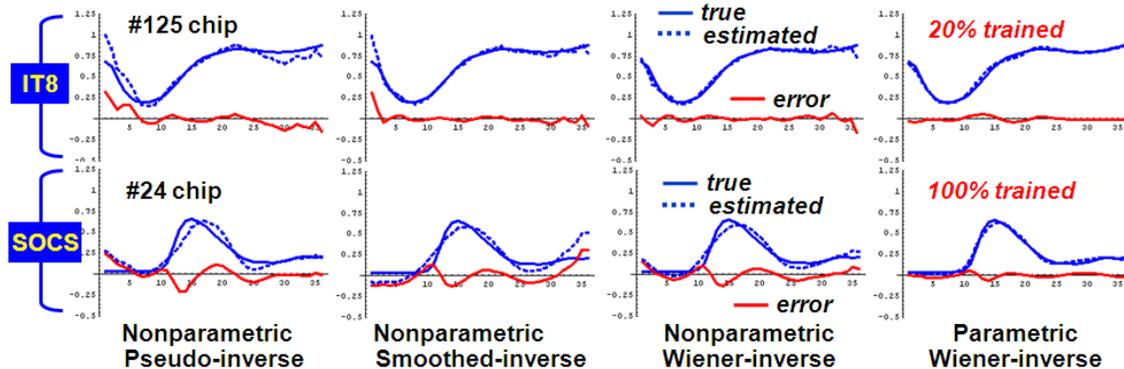


Figure 6 Example of spectral reflectance estimated by parametric/nonparametric typical models

Spectral Imaging under different Illuminations

Finally, the spectral image rendition was tested under the different illuminants. Here the pseudo-spectral test images¹⁴ are generated by embedding the spectral inkjet chip in each pixel of sRGB image taken under D_{65} . The spectral chip matched to the pixel with minimum ΔE_{ab}^* is picked up from the inkjet spectral palettes, each composed of $n=36$ bands ($\Delta\lambda=10$ nm). Fig.7 shows the images recovered from XYZK signal Q by non-trained Wiener-inverse. It looks just the same as true original when viewed under the same illuminant D_{65} . The color changes under the other illuminants are also reflected to the colorimetric color reproduction with color differences less than those by SVD.

Conclusions

A novel XYZ+K four-band spectral sensing model is proposed. Based on matrix-R theory, the additional filter K is designed to save the *metameric black* lost by XYZ filter. The model worked nice in the parametric *Wiener-inverse* spectral estimation. Its accuracy almost reached SVD as an ideal model. Though the non-trained *Wiener-inverse* is surely worse than that of trained, it resulted in the colorimetric reproduction superior to SVD. The spectral image restoration is simulated under different illuminants with pseudo-spectral test images that the $n=36$ -band inkjet spectral chip is embedded in each pixel and the model worked

successfully.

References

1. Y. Yokoyama et al, Proc. CIC5, 169-172, 1997
2. F. H. Imai et al, Jour. IS&T, 44, 280-287, 2000
3. M. Yamaguchi et al, Proc. IDW'00, 1115-, 2000
4. F. H. Imai and R. S. Burns, Proc.CIC7, 21-, 1999
5. N. Tsumura, et al, J. O. S. A., A16, 9, 2169-, 1999
6. Y. Murakami et al, Appl. Opt., 41, 23,4840-, 2002
7. N. Shimano, J. O. S. A., A 24, 3211-3219, 2007
8. M. W. Delhak and M. R. Rosen, Jour. IS&T, 50, 53-63, 2006
9. J. B. Cohen, Col. Res. Appl., 13, 5, 5-39, 1988
10. H. Kotera, Proc. AIC2007, 134-137, 2007
11. H. Kotera, Proc. Color Forum Japan, 129-132, 2007
12. H. Kotera, Japan Patent Application No. 2008-296255
13. C. E. Mancill, USCIPI Report, No.663, 1975
14. H. Kotera, Proc. CIC11, 358-363, 2003

Author Biography

Hiroaki Kotera joined Matsushita Electric Industrial Co., in 1963. He received Doctorate from Univ. of Tokyo. After worked in image processing at Matsushita Res. Inst. Tokyo during 1973-1996, he was a professor at Dept. Information and Image Sciences, Chiba University until his retirement in 2006. He received 1993 journal award from IS&T, 1995 Johann Gutenberg prize from SID, 2005 Chester Sall award from IEEE, 2006 journal award from ISJ and 2008 journal award from SPSTJ. He is a Fellow of IS&T.

Table 1 Spectral reflectance estimation accuracy in MSE (Mean Square Error)

Estimation Method		Spectral reflectance error (MSE)				
		Macbeth (N=24)	IT8 (N=264)	IJP (N=1331)	SOCS (N=50)	
Proposed XYZK Model	Pseudo-inverse	0.108	0.0203	0.106	0.140	
	Smoothed-inverse	0.0925	0.0119	0.0696	0.0946	
	Non-trained Wiener	0.0790	0.0072	0.0549	0.0693	
	Wiener trained by	Macbeth	0.00638*	0.0680	0.0438	0.0123
		IT8	0.0826	0.00207*	0.0541	0.0798
IJP		0.0382	0.0250	0.0119*	0.0391	
SOCS		0.00841	0.0731	0.0518	0.00939	
SVD (4-th order)		0.00626	0.00176	0.00911	0.00907	

Table 2 Colorimetric errors in CIELAB ΔE_{ab}^* (mean) under different illuminants

Estimation Method	Test chips	Standard Illuminant			Fluorescent Lamp		
		D65	D50	A	Daylight	Cool White	Warm White
Proposed Non-trained Markov Wiener	Macbeth	≤ 0.1	0.99	3.02	2.19	1.99	3.16
	IT8	≤ 0.1	0.92	2.28	1.68	2.36	4.43
	InkJet	≤ 0.1	1.98	6.23	3.31	3.32	6.59
	SOCS	≤ 0.1	1.78	5.44	2.96	3.57	6.49
Proposed Trained Wiener by each	Macbeth	≤ 0.1	0.56	1.71	1.06	1.59	2.62
	IT8	≤ 0.1	0.39	1.07	0.98	1.27	2.03
	InkJet	≤ 0.1	1.31	4.58	1.98	2.59	4.91
	SOCS	≤ 0.1	1.17	3.65	2.04	4.07	6.60
SVD (4-th order)	Macbeth	1.77	2.12	2.80	3.28	4.73	3.45
	IT8	2.59	2.56	3.04	4.19	5.32	3.45
	InkJet	5.75	6.00	6.60	7.48	8.50	8.14
	SOCS	5.12	5.75	6.75	7.42	9.27	8.78

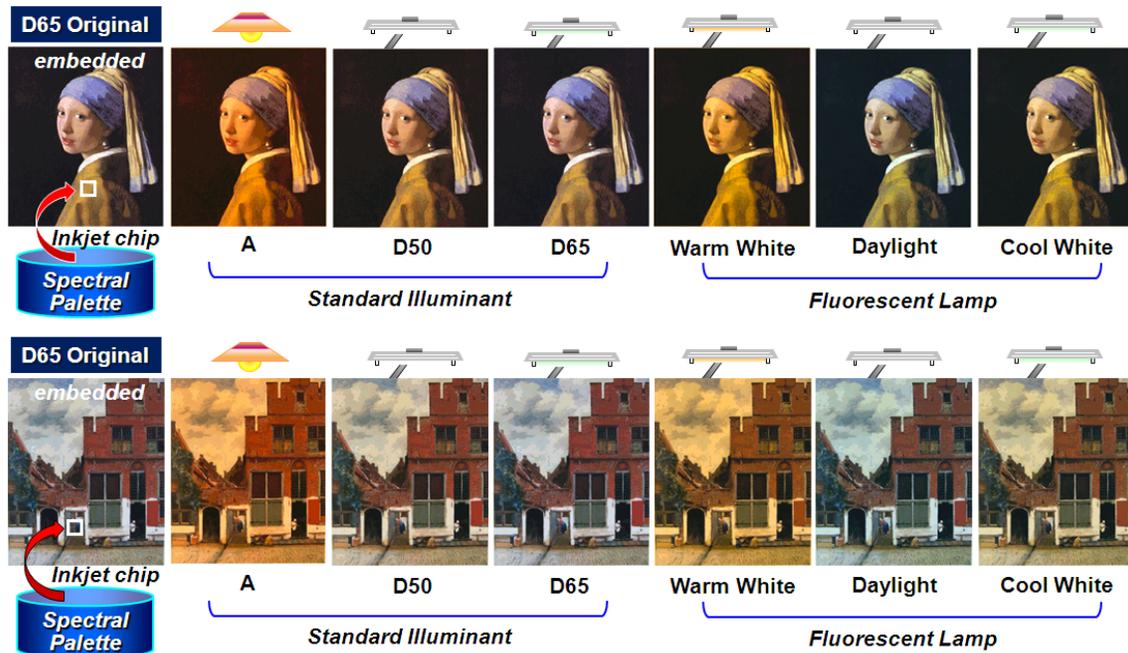


Figure 7 Colorimetric rendition of spectral image recovered from four-band XYZK low-dimensional signals